Evidence on Time-Varying Factor Models for Equity Portfolio Construction

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1 Introduction

Many applicationers derive the variance-covariance matrix (VCM) for mean-variance optimization from some risk model or apply a simple historical estimate. A common problem to these approaches is the stability of the variance-covariance matrix. In turbulent market phases risk estimates from various risk models are well known to be unreliable. One reason for their poor risk forecasting ability is the fact that financial markets are subject to substantial structural change, applied risk models do not account for. In our paper we account for structural changes by deriving VCMs from time-varying estimates of the single factor model, i.e., the market model. We demonstrate the advantages of this approach with respect to risk estimation, portfolio selection and investment performance by means of simulated trading strategies.

The problem of choosing the adequate risk model has come in mind of scientific researchers and practioners only recently. While research has focused on forecasting returns for a long time there is a lack of evidence in evaluating the performance of different risk models and the consequences for portfolio optimization. Next to the well known sensitivity of the mean-variance optimization with respect to assumed expected returns the benefits promised by this approach also heavily depend on the accuracy in estimating the VCM (see, for example, [1] and [4]). Given the well known difficulty of estimating expected returns the most important improvement on MV optimization can be made in the VCM estimation which is mainly based on financial econometrics. However, on the performance of alternative risk models and optimization procedures there is only limited scientific evidence, such as [3, 9, 10, 13, 18] among others.

The capital asset pricing model (CAPM) due to [17] and [14] assumes stock returns to be a linear function of a single factor, namely the market return. Stock betas, i.e., stock return elasticities with respect to the market

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return, have been widely used to evaluate systematic risk, i.e., the return risk associated with market movements.

When estimating the CAPM it is common practice to assume stock betas to be invariant over time. However, this stability assumption has been questioned and a considerable amount of empirical evidence reports important beta variation over time (see among others, [2, 5, 8, 12, 16, 19], as well as [7]). Beta variation over time goes hand in hand with unstable correlations among stock returns and time-varying VCMs. This might have serious consequences for the outcomes of portfolio optimization which are not widely recognized by now.

In [6] we consider VCMs that are derived from time-varying beta estimates for mean-variance optimization. When estimating time-varying betas we rely on a time-varying market model given by

$$y_{i,t} = \alpha_{i,t} + \beta_{i,t} x_t + u_{i,t}, \quad u_{i,t} \sim N(0, \sigma^2_{u,i}), \text{ i = 1...N, t = 1...T},$$

with $y_{i,t}$ denoting the return of stock i at period t and x_t the market return, respectively. The error term $u_{i,t}$ captures specific risk of stock i measured by the standard deviation $\sigma_{u,i}$, and the slope coefficient $\beta_{i,t}$ measures the stock's return sensitivity with respect to x_t. The coefficient $\alpha_{i,t}$ denotes the stock specific return component at time t.

For estimating time-varying coefficients $\beta_{i,t}$ we employ three well established estimation approaches, namely (i) Moving Window Least Squares (MWLS); (ii) Flexible Least Squares (FLS) and (iii) the Random Walk Model (RWM). See [11] and [15] for an illustration of the estimation methods. We compare estimation results of these approaches with those, generated by the time-invariant Recursive Least Squares-approach (RLS).

Our empirical findings for the U.S. suggest that betas, stock correlations and, hence, VCMs are subject to significant variation in the short run as well as in the long run. In fact, important benefits arise from time-varying estimation of the market model when compared to time-invariant estimation via RLS.

Moreover, we examine the outcomes from mean-variance portfolio selection strategies based on variance-covariance matrices derived from these estimates. We obtain improved ex-ante risk estimates as well as portfolios that have superior risk and return characteristics while being well diversified. For the estimation techniques considered in this paper, we find the same ranking for nearly all investigated criteria. Due to our results, FLS is the best method. It is followed by RWM, MWLS and RLS. The FLS procedure delivers the most precise beta estimates as well as the most precise portfolio risk estimates. Moreover, efficient frontiers suggest higher returns for given volatilities, trading strategies show the highest Sharpe Ratios and finally, portfolios are the most diversified.

To summarize, the portfolio performances found in our empirical analysis indicate a strong need for the application of time-varying estimation approaches for estimating correlations in risk analysis and portfolio construction. Due to our results, the FLS estimate is the favourable method to do so.

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