

## Chapter 2

# Non-Tobin's $q$ in Tests for Financial Constraints to Investment

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**Abstract** Liquidity constrained firms may be under two very well identified investment regimes, constrained and unconstrained. In this paper I derive theoretical investment equations for both regimes and discuss the consequences of ignoring the specific form of the liquidity constrained regime. I also show that expressing the investment equation as a function of Tobin's  $q$  is by no means necessary in theory and in practice, in particular, it is not required to test for liquidity constraints.

### Introduction

In this article I argue that so-called Tobin's  $q$  is not necessary at the theoretical nor at the empirical level to explain investment behavior. All possible questions of interest, such as tests for liquidity constraints, real effects of financial variables, and others, can be answered without relying on  $q$  as a concept. It is enough to solve a dynamic problem in which investment is the solution, a function of current and past state variables; this policy function can be directly estimated from the data.

The once prevailing Keynesian Tobin's  $q$ -theory explained investment as a function of a relative price  $q$  inside the IS-LM framework. In contrast, the neoclassical model of investment explained investment as a solution to a dynamic problem, that is, as a policy rule of investment as a function of current and past state variables. The prevailing Keynesian approach reacted to this challenge deriving  $q$ -theory from a choice-theoretic framework which explicitly takes account of adjustment costs associated with investment. In this assimilation, the definition of  $q$  was modified from Tobin's original formulation as a relative price to a variable that contained future investment opportunities. Instead of being a function of state variables, investment was now a function of  $q$ , the marginal value of capital over the price of capital.

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Moreover, to make this digression operational, the analysis focused on very special cases, so that “average  $q$ ” coincided with “marginal  $q$ ,” a distinction that was absent in Tobin’s original formulation. These special cases were elegantly derived, nevertheless they were restrictive and had several caveats that were immediately transmitted to the analysis of the data. The possible presence of financial constraints to firms’ investment raised the issue of the measurement of  $q$ . The significance of cash flow in investment regressions on  $q$  suggested the existence of financial constraints only if  $q$  was well measured, otherwise it was just a result of measurement error in  $q$  and cash flow capturing what  $q$  was supposed to capture, future investment opportunities. Thus, the discussion on whether firms are liquidity constrained was the discussion on the measurement of  $q$ .

In this paper, I show that the investment problem can be solved directly as a function of state variables and estimated from the data. What I call “Non-Tobin’s  $q$ ,” because it deviates strongly from Tobin’s definition of  $q$ , is a concept that has done more harm than good to the investment literature, obscuring the solution to a straightforward dynamic problem and opening the doors to several unfruitful discussions on the measurability of  $q$ .

The paper is organized as follows. In the next section I explain how the modern concept of  $q$  differs from the concept of  $q$  proposed initially by Tobin. In Section Model, I set out a model, characterize the optimal policy rule for investment under an unconstrained and a liquidity constrained regime; investment is a function of capital and productivity, the state variables of the problem, not of  $q$ . In Section A Tractable Special Case, I analyze a special case, originally analyzed by Hayashi, when there is homogeneity of degree one in the production function. In Section Estimation, I discuss the estimation of the models developed in the previous sections. The main conclusions of this paper are presented in Section Concluding Remarks.

## Background: $q$ and Investment

*The difficulty lies, not in the new ideas, but in escaping from the old ones, which ramify, for those brought up as most of us have been, into every corner of our minds. Keynes (1936)*

Keynes’s innovative ideas inspired much fruitful economic research that eventually became the mainstream way of economic thinking. Over time, as it normally happens, Keynes’s ideas became old, so that, paradoxically, his statement applies now to his own ideas: it is difficult to escape from them. New ideas appeared, but they did not fully displace Keynesian concepts. That is what happened with the Keynesian  $q$ -theory of investment proposed by Tobin.

The neoclassical theory of investment was based on micro-foundations and agents’ optimizing behavior. The investment function was the solution to a dynamic problem, thus, a choice variable as a function of state variables. However, the logic of the neoclassical approach never fully entered the subject of investment. The  $q$ -theory of investment was so influential that, under an alleged reconciliation, it managed to prevail over the neoclassical approach. Instead of inquiring directly on

the determinants of investment, the question was transferred to finding investment's relationship with a derived, endogenous, unobservable object like  $q$ .

It will be instructive to review Tobin's  $q$ -theory of investment in its original formulation, to illustrate to what extent it differs from the neoclassical approach.

### ***Tobin's $q$***

Tobin's  $q$  theory states that a firm will invest until the ratio between the stock-market valuation of existing real capital assets and its current replacement cost, that is,  $q$ , equals one. In Keynes's (1936) terminology  $q$  can be seen as the ratio of the marginal efficiency of investment to the rate of interest. Formally, the wealth definition in Tobin (1969, p. 19) was

$$W = qK + M/p,$$

where  $W$  is wealth,  $K$  is capital,  $M$  is money and  $p$  is the price of the final good, also called by Tobin "the cost of producing capital." As it can be seen,  $q$  is basically a relative price: the price of capital in terms of the final good. Unlike Keynes, Tobin allowed the value of existing capital goods, or of titles to them, to diverge from their current reproduction cost. Accordingly, the real rate of return from holding capital  $r_K$  equals  $R/q$ , that is, the marginal efficiency of capital relative to the reproduction cost over the relative price  $q$ . As Tobin (1969, p. 20) states:

"Suppose that the perpetual real return obtainable by purchasing a unit of capital at its cost of production  $p$  is  $R$ . If an investor must pay  $qp$  instead of  $p$ , then his rate of return is  $R/q$ ."

Thus, in Tobin's formulation the introduction of a relative price called  $q$  allows for a discrepancy between the interest rate and the rate of return on capital. Accordingly, he redefines the IS-LM space in terms of the rate of return on capital  $r_K = R/q$  rather than on the interest rate  $R$ , as it can be seen in Fig. 2.1 (Fig. 3 in Tobin's article).

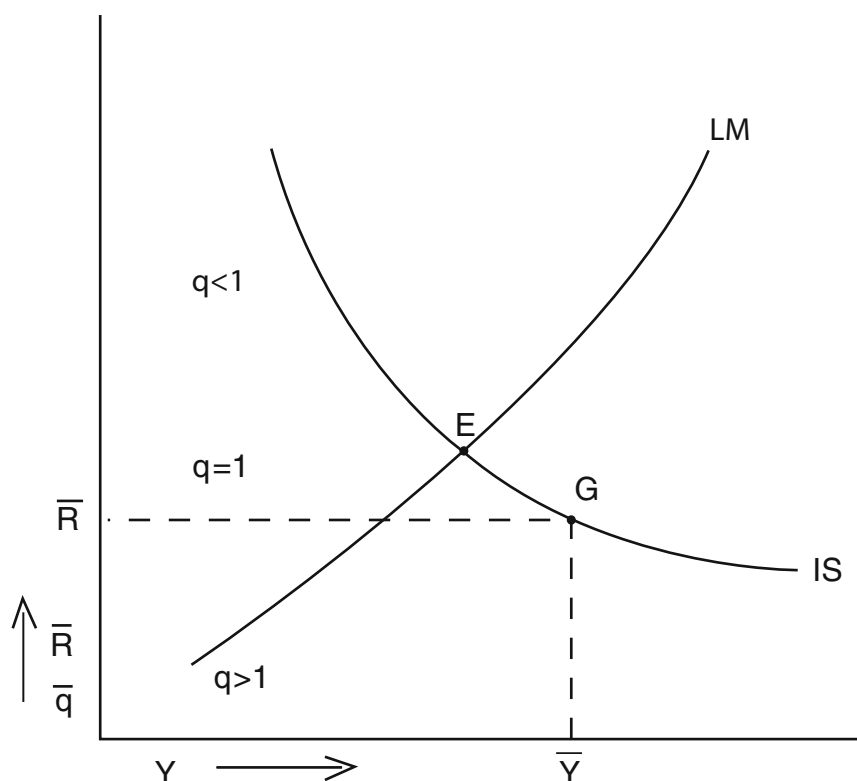
Only when  $q = 1$  these two rates are equal and investment becomes zero. It is in this sense that we can understand Tobin's (1969, p. 21) statement:

"The *rate* of investment – the speed at which investors wish to increase the capital stock – should be related, if to anything, to  $q$ , the value of capital relative to its replacement cost."

It is clear that investment will increase as a response to an increase in  $q$ , which is nothing else than the relative price of capital in terms of the final good, determined in an IS-LM equilibrium.

### ***Non-Tobin's $q$***

Unlike Tobin's and Keynes's investment theory, the neoclassical theory derived the investment function from the firm's optimizing behavior. Developed by Jorgenson



**Fig. 2.1** Original Tobin's  $q$  in the IS-LM space Tobin (1969, p. 22)

(1963) it was extended to allow for adjustment costs to capital or an installation function by Lucas (1967a,b); Gould (1968). As noted by Lucas and Prescott (1971),

“Explanatory variables in empirical studies of the demand for investment goods fall into three broad classes: variables measuring anticipated, future demand – sales, profits, stock prices indexes; variables measuring past decisions, the effects of which persist into the present – lagged capital stock and investment rates; and variables measuring current market opportunities – interest rates, factor prices, and, again, profits.”

Investment theory at the time was concerned with the latter two classes of variables. They propose, by contrast,

“an operational investment theory linking current investment to observable current and past explanatory variables, rather than to ‘expected’ future variables which must, in practice, be replaced by various ‘proxy variables.’ ”

Their formulation was a rigorous analysis of the capital investment decision in the presence of convex costs of adjustment, as such an important progress over Tobin's prevailing  $q$  theory. To formulate a model the researcher has to set up an optimizing dynamic model and solve for the choice variables, expressing them as a function of the state variables, which are current and past variables.

It became clear that economic theory had to grow out from the optimizing behavior of the economic agents. The economic profession assimilated this methodological turn very rapidly, so that in the late seventies and early eighties several authors made efforts to reconcile the neoclassical approach with Tobin's and Keynes's approach. Under that line of research, Mussa (1977), Abel (1979, 1983), and Hayashi (1982) proposed dynamic models that allegedly showed that the neoclassical theory of investment was formally equivalent to Tobin's  $q$  theory of investment. They used models of the firm's present value maximization and obtained the optimal rate of investment as an increasing function of  $q$ . So, Abel (1985) defined  $q_t$  as the marginal valuation of capital divided by  $w_{n+1,t}$  (the shock to the adjustment cost function):  $q_t = V_{K,t}/w_{n+1,t}$ . Hayashi (1982), on its turn, defined Tobin's *marginal*  $q$  as  $q = \lambda/p_I$  and *average*  $q$  as  $h = V/(p_I K)$ , where  $\lambda$  is the present discounted value of additional future (after-tax) profits that are due to one additional unit of current investment. These definitions of  $q$  were totally different from Tobin's original formulation of  $q$  as a relative price.

Defined as the ratio between the marginal value and the price of capital,  $q$  was an object with "a remarkable information content" (Hayashi 1982):

"All the information about the demand curve for the firm's output and the production function that are relevant to the investment decision is summarized by  $q$ . Expectations about future course of the rate of investment tax credits  $k$  are also incorporated in  $q$  and do not affect the form of the investment function."

The relevant investment equation, for instance (13) in Hayashi (1982), had the form:

$$\frac{I}{K} = \beta(\tilde{q}; t).$$

This reasoning, however, was at its heart against the logic of solving a dynamic programming (DP) problem, by determining the policy rules showing how control variables depend on state variables, which are current and past variables observed by the optimizing agent, as Lucas and Prescott (1982) were proposing for investment. A variable that summarizes information about the future, that is, future state or choice variables is an intermediate object, helpful in the process of solving the DP-problem, but cannot be an argument of the solution itself. One needs to go beyond this intermediate step and find a direct function between choice and state variables. Stating investment as a function of this  $q$  cannot be the solution to the firm's DP-problem. Interestingly, in Tobin's original formulation of  $q$  investment is a quantity expressed as a demand function of a relative price, a legitimate state variable.

Therefore, the Keynesian  $q$  theory of investment remained basically unchanged and just assimilated the formal optimizing tools used by the neoclassical approach. Moreover, in this assimilation the way to solve a firm's DP-problem was changed by introducing an intermediate object in the policy rule. Thus, in investment theory the neoclassical work ended up being more a methodological than a conceptual contribution.

Once this intermediate object was introduced as a de facto argument in the investment equation, the focus of attention moved on to the issue of how operational the theory was and the observability of  $q$ . As  $q$ , now containing a derivative, was not

observable anymore, in practice it had to be replaced by proxy variables. To bridge the gap between unobservable marginal  $q$  and its most likely proxy, observed average  $q$ , Hayashi (1982) introduced additional assumptions into the investment model: if the firm is a price-taker with constant returns to scale in both production and installation function, then marginal  $q$  and average  $q$  are the same.<sup>1</sup>

These steps were not at all necessary, as investment can be explained without any object like  $q$ . Moreover, pursuing this intermediate object has led researchers to make restrictive assumptions and lose focus in the analysis of investment. This has been the case with testing for financial constraints, where the investment function, in practice, was finally restricted to be an investment regression. In the next sections I set up a simple dynamic model of investment with a specific form of the financial constraint, and discuss its solution and the inconvenience, both in theory and in practice, of introducing an object of the kind of  $q$  in the investment equation.

## Model

I start with the simplest model of investment without adjustment costs; then I incorporate convex adjustment costs to capital variations. Consider a firm that chooses investment to maximize the present discounted value of dividends:

$$E_0 \sum_{t=0}^{\infty} \frac{D_t}{(1 + \rho)^t}.$$

The firm's output just depends on capital

$$Y = \theta K^\alpha,$$

where the firm's productivity  $\theta$  follows a Markov process  $P(\theta'|\theta)$ . Capital accumulation satisfies the law of motion:

$$K' = (1 - \delta)K + p_K I,$$

where  $\delta$  is the depreciation rate. The firm can issue equity up to a certain exogenous level which depends on the firm's productivity:

$$D \geq \overline{D}(\theta),$$

where  $\overline{D}(\theta) \leq 0$ . That is, the reward function  $D$  can become negative up to a certain level.

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<sup>1</sup> In contrast, if the firm is a price-maker, then average  $q$  is higher than marginal  $q$  by what is legitimately called the monopoly rent.

### ***Frictionless Capital Adjustment***

In the simplest model with free adjustment to capital variations, dividends are defined as

$$D = \theta K^\alpha - p_K I,$$

the firm produces and invests. For this problem the Bellman Equation is

$$V(K, \theta) = \max_{K'} \left\{ \theta K^\alpha + p_K(1 - \delta)K - p_K K' + \frac{1}{1 + \rho} \int V(K', \theta') dP(\theta' | \theta) \right\}$$

subject to  $D \geq \bar{D}(\theta)$ .

The corresponding Lagrange equation is then

$$\begin{aligned} L(K', \lambda) = \max_{K', \lambda} & \left\{ \theta K^\alpha + p_K(1 - \delta)K - p_K K' \right. \\ & + \frac{1}{1 + \rho} \int V(K', \theta') dP(\theta' | \theta) \\ & \left. + \lambda [\theta K^\alpha + p_K(1 - \delta)K - p_K K' - \bar{D}(\theta)] \right\}, \end{aligned} \quad (2.1)$$

so that the Euler Equation is

$$L_{K'} = -(1 + \lambda) p_K + \frac{1}{1 + \rho} E V_{K'} = 0.$$

Apparently, the solution to this problem is given by

$$\frac{E V_{K'}}{p_K} = (1 + \lambda) (1 + \rho).$$

However, the object  $\frac{E V_{K'}}{p_K}$  does not reveal anything. To really solve this problem we need to go further and take out the choice variables that are contained in the term  $E V_{K'}$ . Therefore, the solution to this problem is actually contained in the following condition:

$$\frac{E V_{K'}}{p_K} \equiv \frac{E [\theta' (1 + \lambda') | \theta]}{p_K} \alpha K'^{\alpha-1} + (1 - \delta) = (1 + \lambda) (1 + \rho),$$

that comes out from an application of the envelope theorem. That is, the expected marginal product of capital (MPK) in terms of capital goods, augmented by the depreciation rate, has to coincide with the interest rate, both adjusted by the shadow value of internal funds.

Consider the special case, when there are no liquidity constraints,  $\lambda_t = 0, \forall t$ . Then, the expected marginal product of capital net of depreciation simply equals

the interest rate:

$$\frac{E[\theta'|\theta]}{p_K} \alpha K'^{\alpha-1} - \delta = \rho.$$

In this case, there is no need for further concern, as there is a straightforward explicit solution for capital next period and, therefore, investment:

$$K'(K, \theta) = \left[ \frac{E[\theta'|\theta] \alpha}{p_K (\rho + \delta)} \right]^{\frac{1}{1-\alpha}},$$

$$I(K, \theta) = \left[ \frac{E[\theta'|\theta] \alpha}{p_K (\rho + \delta)} \right]^{\frac{1}{1-\alpha}} - p_K (1 - \delta) K.$$

Notice that capital next period does not depend on capital in the current period. Postulating an intermediate object like  $q = \frac{EV_{K'}}{p_K}$  would be a needless complication in a straightforward solution to this problem.

Now, let us focus on the case with liquidity constraints. The constraint may or may not be currently binding:

$$\alpha \frac{E[\theta' (1 + \lambda') |\theta]}{p_K} K'^{\alpha-1} + (1 - \delta) = \begin{cases} (1 + \rho), & \text{if } \lambda = 0, \\ (1 + \rho) (1 + \lambda), & \text{if } \lambda > 0. \end{cases}$$

If the liquidity constraint binds, the solution for investment is simply given by  $\theta K^\alpha - p_K I = \overline{D}(\theta)$ :

$$I(K, \theta) = \frac{\theta K^\alpha - \overline{D}(\theta)}{p_K}.$$

These two regimes are selected according to a productivity-specific threshold  $K^*(\theta)$  so that for  $K \leq K^*(\theta)$  this constrained solution applies, and when  $K > K^*(\theta)$  the interior solution regime shown above applies.

Hence, the solution for investment is given by two regimes that can be solved explicitly:

$$I = \min \left[ \underbrace{\left[ \frac{E[\theta' (1 + \lambda') |\theta] \alpha}{p_K (\rho + \delta)} \right]^{\frac{1}{1-\alpha}} - p_K (1 - \delta) K}_{\text{currently unconstrained}}, \underbrace{\frac{\theta K^\alpha - \overline{D}(\theta)}{p_K}}_{\text{constrained}} \right].$$

We learn the following from this exercise:

1. There is a threshold in capital that determines which regime applies. For a given productivity level, small amounts of capital are associated with binding liquidity constraints, while larger amounts with an interior solution
2.  $EV_{K'}$  is basically expected MPK and, as such, an endogenous variable; it contains investment, the solution to the dynamic programming problem



3. We can determine whether a firm is currently financially constrained: it will invest all output plus allowed equity,  $I = \frac{Y - \bar{D}(\theta)}{p_K}$ , that is, the firm's financial position does affect investment, moreover, in a very particular way
4. It is, however, less obvious to determine whether the firm will be constrained in the future, as we do not know the future  $\lambda$ s. The firm may be financially constrained in the future even if we reject that they are currently financially constrained.

In this simple model of investment with liquidity constraints we find some conclusions that will also apply to the specification with quadratic adjustment costs.

### ***Costly Capital Adjustment***

Now suppose that there are quadratic adjustment costs to capital, so that the reward function is:

$$D = \theta K^\alpha - p_K I - \frac{b}{2} \left( \frac{I}{K} \right)^2 K.$$

Then, the Bellman equation becomes

$$V(K, \theta) = \max_{K'} \left\{ \theta K^\alpha + p_K I - \frac{b}{2} \left( \frac{I}{K} \right)^2 K + \frac{1}{1 + \rho} \int V(K', \theta') dP(\theta' | \theta) \right\}$$

subject to  $D \geq \bar{D}(\theta)$ ,

which yields the following Euler equation

$$- \left[ p_K + b \left( \frac{I}{K} \right) \right] (1 + \lambda) + \frac{1}{1 + \rho} E V_{K'} = 0,$$

where  $E V_{K'} = \alpha E [\theta' | \theta] K'^{\alpha-1} + p_K (1 - \delta) + b (1 - \delta) \frac{E [I' | \theta]}{K'} + \frac{b E [I'^2 | \theta]}{2 K'^2}$ . Once again, this term is basically expected MPK augmented by the depreciation rate and the effect of adjustment costs. Notice that now investment appears in two terms: directly as  $\frac{I}{K}$  and inside of  $E V_{K'}$ . The object  $E V_{K'}$  is still endogenous. Unlike in the previous example, the Euler equation determines investment implicitly, not explicitly. We can express but not *explain*  $\frac{I}{K}$  as a function of  $E V_{K'}$ .

This time there is no explicit solution even if there are no constraints at all:  $\lambda_t = 0, \forall t$ :

$$\frac{\alpha E [\theta' | \theta] K'^{\alpha-1} + p_K (1 - \delta) + b (1 - \delta) \frac{E [I' | \theta]}{K'} + \frac{b E [I'^2 | \theta]}{2 K'^2}}{p_K + b \left( \frac{I}{K} \right)} = 1 + \rho.$$

In the numerator expected MPK is augmented by expected marginal capital adjustment costs, so that a recursive solution is needed, and the denominator includes

current marginal adjustment costs, so that, unlike the case with no adjustment costs, capital next period does depend on capital in the current period.

Now, suppose there are liquidity constraints. Then the Euler equation becomes

$$\frac{E \left[ (1 + \lambda') \left( \alpha \theta' K'^{\alpha-1} + p_K (1 - \delta) + b(1 - \delta) \frac{I'}{K'} + \frac{b I'^2}{2 K'^2} \right) \mid \theta \right]}{p_K + b \left( \frac{I}{K} \right)} = \begin{cases} (1 + \rho) p_K, & \text{if } \lambda = 0, \\ (1 + \rho) p_K (1 + \lambda), & \text{if } \lambda > 0. \end{cases}$$

Again, if liquidity constraints are not currently binding,  $\lambda = 0$ , we have an implicit and recursive solution. We only have an explicit solution for investment, if  $\lambda > 0$ . Indeed we have a quadratic equation that defines investment:

$$\theta K^\alpha - p_K I - \frac{b}{2} \left( \frac{I}{K} \right)^2 K - \overline{D}(\theta) = 0.$$

The solution for this equation is

$$I = -\frac{p_K}{b} K + \frac{1}{b} \sqrt{p_K^2 K^2 + 2bK [\theta K^\alpha - \overline{D}(\theta)]}. \quad (2.2)$$

Hence, there is only an explicit nonrecursive, static, solution when the liquidity constraint is binding.

This result does not basically change if we allow for short-term debt in the dividend definition,

$$D = \theta K^\alpha - p_K I - \frac{b}{2} \left( \frac{I}{K} \right)^2 - (1 + r)B + B'.$$

Now the firm pays back  $(1 + r)B$  contracted in the previous period and decides on  $B'$  for next period. However, the Euler equation shown above does not change, that is, there is no term that captures debt in the investment Euler equation. The consequence of this extension is that when the dividend constraint is binding,  $D = \overline{D}(\theta)$ , the equation for investment is modified in the following way:<sup>2</sup>

$$I = \frac{1}{b} \sqrt{p_K^2 K^2 + 2bK [\theta K^\alpha - (1 + r)B - \overline{D}(\theta)]} - \frac{p_K}{b} K.$$

And, if there are no adjustment costs it simply becomes

$$I = \theta K^\alpha - (1 + r)B - \overline{D}(\theta).$$

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<sup>2</sup> This is under the special case of no debt next period  $B' = 0$ . Generally speaking  $B'$  has to be solved from a system of two Euler equations, one equation for investment and another for debt.

Hitting the liquidity constraint means that the firm's financial position  $D - \bar{D}(\theta)$  determines investment. Once again, there are two exclusive regimes, one in which financial constraints are not binding and the current financial position of the firm does not matter, and one in which financial constraints do matter and the financial position of the firm critically affects investment.

I conclude this section remarking that constrained and unconstrained solutions are exclusive: expected MPK determines investment only when the solution is unconstrained; the firm's financial position determines investment when the liquidity constraint is binding. That is, it is either the expected MPK or the financial position, not both at the same time. Notice also that this analysis is performed without constructing what has been called Tobin's  $q$ .

## A Tractable Special Case

In the investment literature a very special version of this problem has been of particular interest, when both the production and the adjustment cost function exhibit homogeneity of degree one. In that case it has been stressed that the marginal and average value of the firm on capital, and thus, marginal and average  $q$  are the same. In the context of the simple model of investment with quadratic adjustment costs to capital, a similar result can be found:

**Theorem 1 (Hayashi 1982).** *For the case without liquidity constraints,  $\lambda_t = 0, \forall t$ , if  $\alpha = 1$ , then  $V$  is homogenous of degree one in capital, i.e.,  $V(K, \theta) = A(\theta)K$ , where  $A(\theta)$  is a function of  $\theta$ . Proof: In Appendix.*

**Corollary 1.** *If  $\alpha = 1$ , then  $\frac{V}{K} = V_K$ .*

**Corollary 2.** *If  $\alpha = 1$ , then  $\frac{I}{K} = -\frac{p_K}{b} + \frac{E[A(\theta')|\theta]}{b(1+\rho)}$ .*

In that case, there is an AK-value function, so that  $q = \frac{A(\theta)}{p_K}$ , that is,  $q$  only depends on the stochastic process, not on capital, in the present or in the future, and on the structural parameters of the DP-problem. Thus,  $q$  is still an intermediate object, a transformation or sufficient statistic for current productivity.

This result does not only mean that marginal and average  $q$  are the same, but also that  $q$  is fully exogenous to capital. This result is important theoretically, as it is only current productivity that is informative about future investment opportunities, and empirically, as it implies that there is no endogeneity bias in an OLS regression of investment over  $q$ .

This tractable case has been analyzed for the interior but not typically for the liquidity constrained solution, which is as tractable as the unconstrained solution. From (2.2) we obtain

$$\frac{I}{K} = -\frac{p_K}{b} + \frac{p_K}{b}s, \quad (2.3)$$

where  $s = \sqrt{1 + \frac{2b\theta K - \bar{D}(\theta)}{p_K^2 K}}$ . This equation is linear and indeed not too different than the previous interior solution. Instead of having  $q = \frac{A(\theta)}{p_K}$  here we have an  $s$ -term, a ratio between an explicit function of productivity, that comes from the adjustment cost function, and the user cost of capital. The  $q$ -term is informative about some expected value of the whole productivity process; the  $s$ -term is just informative about *current* productivity. The distinction is very subtle as both terms are in fact functions of current productivity, their difference being the specific forms that these functions assume.

Since this special case implies  $Y = \theta K$ , the  $s$ -term is observable and dependent on the value of output over the value of capital  $\frac{Y}{p_K K}$ . Thus, we have  $s = \sqrt{1 + \frac{2b}{p_K} \frac{Y - \bar{D}(\theta)}{p_K K}}$ , a quadratic transformation of an observable ratio.

As said above, the Euler equation for capital does not change if we extend the dividend definition allowing for, for instance, short-term debt. The  $s$ -term can be generally defined as  $s = \sqrt{1 + \frac{2b}{p_K} \frac{CF}{p_K K}}$ , where  $CF = Y - (1 + r)B - \bar{D}(\theta)$ ; it is an non-linear increasing function of  $CF$ . This derivation will prove useful in the discussion about the estimation and testing for financial constraints.

## Estimation

The estimation of investment under liquidity constraints has typically been made assuming convex adjustment costs. Hence, in the literature it is very common to derive the following equation from the Euler equation without any constraint:

$$\frac{I}{K} = -\frac{p_K}{b} + \frac{p_K}{b(1 + \rho)} \frac{EV_{K'}}{p_K} \quad (2.4)$$

and then postulate the following linear investment equation

$$\frac{I_{it}}{K_{it}} = \beta_0 + \beta_1 q_{it} + u_{it}$$

where  $q$  stands for  $\frac{EV_{K'}}{p_K}$ , and the random term  $u \sim N(0, \sigma^2)$  can be considered a measurement error in the investment-capital ratio. This equation accounts for investment in the absence of any friction other than capital adjustment costs.

The condition seen above, namely that  $\alpha = 1$ , solves two problems in estimating this investment equation: proxying for  $q$  by an observable and that  $q$  is an endogenous object. In that case, average and marginal  $q$  coincide, and one can safely proxy  $V_K$ , usually unobserved, by  $\frac{Y}{K}$ , more easily observed. On the other hand, in general  $q$  contains the endogenous variable  $\frac{I}{K}$ , which could imply that even if this were the correct specification of the investment equation, an OLS estimation yields biased estimates of  $\beta_0$  and  $\beta_1$ . However, the same condition that allows to proxy

marginal by average  $q$  implies that  $q$  is fully exogenous and only depends on current productivity. Certainly, it is an empirical matter to test whether  $\alpha = 1$  applies.

Following Fazzari et al. (1988) this benchmark equation is usually augmented by an extra term, forming thereby what has become the usual test for liquidity constraints:

$$\frac{I_{it}}{K_{it}} = \beta_0 + \beta_1 q_{it} + \beta_2 CF + u_{it}, \quad (2.5)$$

where  $CF$  stands for “cash flow.” In the absence of financial constraints, it is argued, only  $q$  should matter:

$$H_0 : \beta_2 = 0$$

Thus, if this null hypothesis is rejected and cash flow turns out to significantly affect investment, it is argued that financial constraints are present. Most of the discussion around this approach has been centered in measuring  $q$  adequately and interpreting what a significant  $\beta_2$  means. As established above, if the firm's financial position determines investment, then expected MPK does not. It cannot be that cash-flow and  $q$  determine investment together; it is either one or the other. If  $q$  ‘explains’ investment, cash-flow should not. If cash-flow explains investment, then  $q$  does not. Thus, the alleged test for liquidity constraints is not really based on the solution to the DP-problem, which rather suggests two different exclusive regimes.

These considerations notwithstanding, if the data contain currently constrained and currently unconstrained firms the estimation of (2.5) will yield mixed results. Liquidity constrained firms will make cash flow matter and diminish the importance of  $q$ , while liquidity unconstrained firms will make the  $q$  significant while undermining the importance of cash flow. To illustrate this point suppose that we estimate (2.5) with data in which there are  $\pi$  unconstrained firms and  $1 - \pi$  constrained firms. Now, we have a mixture of (2.4) and (2.3):

$$\frac{I}{K} = -\frac{p_K}{b} + \pi \frac{p_K}{b(1+\rho)} q + (1-\pi) \frac{p_K}{b} s, \quad (2.6)$$

where  $s$  was defined above as a nonlinear function of  $CF$ . Thus, (2.5) can be seen as an approximation to this expression. Even under the assumption that  $CF$  is a valid proxy for  $s$ , one can see that  $\beta_2 = (1-\pi) \frac{p_K}{b}$ , so that a significance test is basically informative about the proportion of constrained firms in the sample  $(1-\pi)$ .

To address this issue, the literature has divided firms into two groups, one which is a priori expected to be constrained, typically small firms, and the group which is expected to be unconstrained, larger firms. As seen above, there is a capital threshold  $K^*(\theta)$  that indicates the regime that firms are facing. Certainly, the researcher does not know this productivity-specific capital threshold. Moreover, strictly speaking this threshold is endogenous, dependent on the model's parameters, and should be determined as part of the estimation procedure. This exercise, performed across several sample splits and for several countries, shows that firms that are a priori expected to be liquidity constrained exhibit greater sensitivity of

investment to internal funds:  $\beta_2^c > \beta_2^u$ , where  $c$  stands for constrained and  $u$  stands for unconstrained.<sup>3</sup>

Thus, this estimation strategy addresses the issue of misclassification by conjecturing that coefficients of allegedly constrained firms may be just larger than those of allegedly unconstrained firms. It is an ex post validation of an a priori partition. However, notice that by the same token the investment-cash flow sensitivity has to be higher for constrained firms, it has to be true that  $\beta_1^u > \beta_1^c$ , that is, investment has to be more sensitive to  $q$  for unconstrained firms, if  $\pi^u > \pi^c$ . This issue has not been usually considered in the investment literature.

This estimation approach may be also problematic if the cash flow variable is correlated with investment, not because there are liquidity constraints but because  $q$  is mismeasured, so that it does not capture all investment opportunities. Then, cash flow might capture future investment opportunities not totally measured by  $q$  (Gomes 2001, Erickson and Whited 2000, Saltari and Travaglini 2003), or indicate other sources of misspecification in the investment model (Bond and Van Reenen 2007, Ejarque and Cooper 2004). Given this concern, Gilchrist and Himmelberg (1995) address the mismeasurement problem by proposing an alternative measure of  $q$  as following an AR(1) process. Then it is estimated using a VAR of firm fundamentals; nevertheless, cash flow enters significantly in the investment equation for constrained firms. On the other hand, interestingly, Gomes (2001) finds that even with liquidity constraints, standard investment regressions predict that cash flow is an important determinant of investment only if one ignores  $q$ . Conversely, he also obtains significant cash flow effects even in the absence of financial frictions. He suggests that cash-flow-augmented investment regressions work probably because of a combination of measurement error in  $q$  and identification problems. Alternatively, under the light of the derivations shown above, this result may just express that cash flow is strongly correlated with  $q$ , so that only one of these terms is significant both for the constrained and the unconstrained regime.

These measurement and estimation problems seem to arise from having  $q$  as the center of the theoretical concern as well as of the estimation strategy. An alternative approach has been to adopt estimation strategies that altogether do not require measuring  $q$ . In particular, in a General Method of Moments estimation the Euler condition for investment implied by a model of perfect capital markets typically strongly rejected for firms that are classified as constrained (Whited, 1994). Another alternative approach is to estimate the model's behavioral parameters using specific functions by the method explained by Rust (1994) and Eckstein and Wolpin

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<sup>3</sup> Similar results are obtained when the sample is partitioned on the basis of bond ratings (Gilchrist and Himmelberg 1995), firm size (Gertler and Gilchrist, 1994), membership of an industrial keiretsu in Japan (Hoshi et al. 1991). A detailed review of this literature can be found in Hubbard (1998) and Bernanke et al. (1999). In contrast with these results Kaplan and Zingales (1997) find that the coefficient on cash flow does not increase monotonically across groups of firms as the degree of financial constraint increases. Actually, firms that seem less constrained according to several criteria have a higher coefficient on cash flow, as compared to more constrained firms. However, as shown by Pratap (2003), this result can be rationalized by the presence of liquidity constraints when capital adjustment costs are non-convex.

(1989). Liquidity constraints are then identified from the dynamics of a firm's evolution as formalized by the dynamic estimation process. Pratap and Rendon (2003) recover the underlying model's parameters by a Maximum Likelihood procedure and perform a likelihood ratio test on parameterized dividend constraints. Similarly, Hennessy and Whited (2007) recover the behavioral parameters of their theoretical model by Simulated Method of Moments estimation. They assume a specific equity cost function and test for statistical significance of bankruptcy and equity costs.<sup>4</sup>

These alternative estimation approaches show that financial constraints are significant and not an artificial result of an erroneous measurement of  $q$ . As such they are encouraging about the feasibility of estimating investment models and, moreover, answer all questions of interest without using  $q$  at all. Moreover, these approaches also allow researchers to analyze counterfactual simulations of alternative economic scenarios.

## Concluding Remarks

In this paper I contend that Non-Tobin's  $q$  is a needless object both in the theoretical and practical analysis of investment.

Tobin's original formulation  $q$  is an observable relative price, the price of capital with respect to the price of a final good. In adapting his main ideas to fit into a micro-founded theory of investment, the definition of  $q$  was changed and became the marginal value of capital over the price of capital. The concept of  $q$ , however, is alien to the solution of a Dynamic Programming problem, in which choice variables, such as investment, have to be explained by current and past state variables. This modified or Non-Tobin's  $q$  was a derived, endogenous and unobservable object that brought more questions than answers to the investment literature.

To make the  $q$ -theory operational, restrictive assumptions, which were not usually empirically tested, were needed, so that  $q$  could be replaced in practice by proxy variables. Nevertheless, the issue of measuring  $q$  correctly never stopped being a concern and a possible source of biased and misleading results, especially in testing for financial constraints to firms. Cash flow variables, possibly capturing future investment opportunities that  $q$  should be capturing, were solidly significant across

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<sup>4</sup> At the same time that the literature is moving toward more structural approaches one can also distinguish the trend to move in the opposite direction, toward performing "natural" experiments. This method consists of exploiting a policy change that affected the flow of credit to an identifiable subset of firms. Then the researcher computes "difference-in-differences," that is, a twofold comparison between observed variables of "control" and "treated" firms, observed "before" and "after" the policy change. For instance, Banerjee and Duflo (2004) exploit a 1998 reform in India that increased the maximum size below which a firm is eligible to receive priority sector lending. Control firms are those that were already in the "priority" sector. The result is that bank lending and firm revenues went up for the newly targeted firms in the year of the reform, so they conclude that there are severe credit constraints. Under this approach, measuring  $q$  is optional, as it is not needed to determine the treatment effect.

investment regressions. This result suggested the presence of important financial constraints or of severe measurement problems in  $q$ . This ambiguity was addressed by estimation strategies that did not require measuring  $q$ , finding that financial constraints were indeed important. Focusing on the measurement of  $q$  proved to be a big detour from the main topic of interest, explaining investment. In fact, the detour started long ago, when Non-Tobin's  $q$  was proposed as the main determinant of investment.

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## Appendix: Proof of Theorem 1

I proceed inductively; showing that  $V'(K', \theta') = A'(\theta') K'$  implies  $V(K, \theta) = A(\theta)K$ .

Let  $V'(K', \theta') = A'(\theta') K'$ , then the Euler equation implies:

$$\frac{I}{K} = \frac{1}{b} \frac{1}{1+\rho} E[A'(\theta') | \theta] - \frac{1}{b} p_K \equiv B(\theta).$$

Thus, the investment-capital ratio only depends on current productivity, not on capital,  $\frac{I}{K}(K, \theta) = B(\theta)$ . Or, in other words, investment is homogenous of degree one in capital.

Then, the firm's value is:

$$\begin{aligned} V(K, \theta) &= \theta K - p_K \left( \frac{I}{K} \right) K - \frac{b}{2} \left( \frac{I}{K} \right)^2 K + \frac{1}{1+\rho} E[A'(\theta') | \theta] K', \\ &= \left[ \theta - p_K B(\theta) - \frac{b}{2} B^2(\theta) + \frac{1}{1+\rho} E[A'(\theta') | \theta] ((1-\delta) + B(\theta)) \right] K, \\ &= A(\theta)K. \end{aligned}$$

Thus, the value function is homogenous of degree one in capital.

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