Preface to the Second Edition

> *A mathematician, like a painter or a poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas. A painter makes patterns with shapes and colours, a poet with words. A painter may embody an 'idea,' but the idea is usually commonplace and unimportant. In poetry, ideas count for a great deal more; but as Housman insisted, the importance of ideas in poetry is habitually exaggerated... A mathematician, on the other hand, has no material to work with but ideas, and so his patterns are likely to last longer, since ideas wear less with time than words.*
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> *The mathematician's patterns, like the painter's or the poet's, must be* ***beautiful****; the ideas, like the colors or the words, must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly mathematics.*
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> —*G. H. Hardy,* A Mathematician's Apology*, 1940* [Har, pp. 24–25]

I grew up on books by Isaac M. Yaglom and Vladimir Boltyanski. I read their books as a middle and high school student in Moscow. During my college years, I got to know Isaak Moiseevich Yaglom personally and treasured his passion for and expertise in geometry and fine art. In the midst of my college years, a group of Moscow mathematicians, including Isaak Yaglom, signed a letter protesting the psychiatric imprisonment of the famous dissident Alexander Esenin-Volpin. Yaglom was fired from his job as professor for that.

In 1970, I visited Yaglom in his downtown Moscow apartment. We discussed problems I had then created about cutting triangles into triangles, which 20 years later became a foundation of my book *How Does One Cut a Triangle?* [S2]. This was an unforgettable mathematical meeting; Yaglom also showed me a powerful oil painting by the Russian avant-garde painter Robert Falk that he owned.

In 1974, the organizers of the *Conference on Mathematical Work with Gifted Students* at Leningrad University scheduled my plenary talk on problems of combinatorial geometry between the talks by Boltyanski and Yaglom. I was humbled to speak between two of the leaders of this field, but in his talk, Yaglom praised my applications of algebraic methods in geometry (on cutting triangles, see [S2] and its expanded edition [S10]); he called them a product of our time that could not have occurred earlier.

I left Russia for the United States in 1978. Shortly after, Yaglom visited my parents. My mother recalled asking him, "Why would you not leave Russia?" "I am too old, and all my friends are here," was Yaglom's answer.

Ten years later, at the 1988 International Congress on Mathematical Education in Budapest, I ran into Vladimir G. Boltyanski who informed me of Yaglom's recent passing on. I asked Boltyanski whether he would like to write a book together and dedicate it to Isaac Yaglom. Boltyanski answered my question with a question: "What do you need me for?" but he added, "Although, it may be more fun to write a book together."

In June of 1990, Vladimir came to Colorado Springs and spent three weeks in my home. As the result of this feverish joint effort, and eight more months on my own, editing and illustrating, the first edition of this book was born. It covered only four chapters out of some twenty-four that we had listed in Budapest as topics of mutual interest, but it was better than nothing, and the first edition appeared in early 1991.

I saw Volodya Boltyanski for the last time in 1993, seventeen years ago in Moscow. His last e-mail arrived from Mexico thirteen years ago, in May of 1997: he lived and worked there, and wanted to come to Colorado Springs to join me to write another book of *Etudes*. We tried, but his notice was too short, and we were unable to arrange Volodya's visit then. This was the last time I heard from him. When in 2007 Springer offered to publish a new expanded edition of this book, I tried to invite Boltyanski to join me in writing it. Regretfully, I did not know his whereabouts. Thus, the job of correcting, updating and substantially expanding this book fell upon me alone. I hope that now, at the age of 85, Volodya is alive and well, and continues to enjoy a healthy and productive life.

In his "extended review" in *The American Mathematical Monthly*, Don Chakerian complimented our choice of *Etudes*:

> Boltyanski and Soifer have titled their monograph aptly, inviting talented students to develop their technique and understanding by grappling with a challenging array of elegant combinatorial problems having a distinct geometric tone. The etudes presented here are not simply those of Czerny, but are better compared to the etudes of Chopin, not only technically demanding and addressed to a variety of specific skills, but at the same time possessing an exceptional beauty that characterizes the best of art . . . Keep this book at hand as you plan your next problem solving seminar.

The great expositor and promoter of this kind of mathematics, Martin Gardner gave *The Etudes* a nod, too:

> Alexander Soifer and Vladimir Boltyanski have produced a fascinating book, filled with material I have not seen before in any book.

For this expanded *Springer* edition, to the original four Chapters, I am adding five new shorter chapters. Let us take a look at their content.

In the eighteen years that followed, one of the many open problems in the book (Problem 5.3) has been solved in 2006 and published in *Geombinatorics* [Ka] by Mitya Karabash, a brilliant undergraduate mathematician from Columbia University (who entered a Ph.D. program of the Courant Institute of the Mathematical Sciences in the fall of 2008). To my surprise, he proved that an $m \times n$ rectangle can be tiled by L-tetrominoes of the same orientation if and only if mn is divisible by 8 and $m, n \neq 1, 3$. Mitya also proved that an $m \times n$ rectangle can be tiled by L-tetrominoes of the same orientation so that the tiling has 2-fold symmetry if and only if mn is divisible by 8 and m, n are both even, or mn is divisible by 16 and $m, n \neq 1, 3$. The new Chapter 5 is dedicated to Mitya's work.

Norton Starr of Amherst College was inspired by Problem 6.10, dealing with packing a parallelepiped with 3-dimensional trominoes, to look into more sophisticated packing of a cube with 3-dimensional trominoes and one 3-dimensional monomino, and determining where the monomino can be placed. Chapter 6 is dedicated to Starr's results, to be published in the October 2008 issue of *Geombinatorics*.

There has been a great progress in determining small Ramsey numbers, much of which was due to works by Geoffrey Exoo, Stanisław Radziszowski and Brendan McKay. Chapter 7 is dedicated to stating some of these results.

As Boltyanski and I predicted in the first edition of this book, the Borsuk Conjecture was disproved by Jeff Kahn and Gil Kalai in 1993 [KK]. This started a competition for a counterexample of the smallest dimension, which is the subject of Chapter 8.

Finding the chromatic number of the plane is my favorite unsolved problem in all of mathematics. Much (although not all) of my new *Mathematical Coloring Book* (published on November 4, 2008 by Springer [S7]) is dedicated to this problem. My desire to include some of my own and others' results in this book is therefore not surprising. They form Chapter 9, the longest of all the new chapters.

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Love of my children Mark Samuel Soifer and Isabelle Soulay Soifer has been recharging my creative engines and keeping me sane. Shmusik, Belya, I owe you so much!

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