

Preface

This book is an example-based introduction to techniques, from elementary to advanced, of using *Mathematica*, a revolutionary tool for mathematical computation and exploration. By integrating the basic functions of mathematics with a powerful and easy-to-use programming language, *Mathematica* allows us to carry out projects that would be extremely laborious in traditional programming environments. And the new developments that began with version 6 — allowing the user to dynamically manipulate output using sliders or other controls — add amazing power to the interface. Animations have always been part of *Mathematica*, but the new design allows the manipulation of any number of variables, an important enhancement. *Mathematica in Action* illustrates this power by using demonstrations and animations, three-dimensional graphics, high-precision number theory computations, and sophisticated geometric and symbolic programming to attack a diverse collection of problems..

It is my hope that this book will serve a mathematical purpose as well, and I have interspersed several unusual or complicated examples among others that will be more familiar. Thus the reader may have to deal simultaneously with new mathematics and new *Mathematica* techniques. Rarely is more than undergraduate mathematics required, however.

An underlying theme of the book is that a computational way of looking at a mathematical problem or result yields many benefits. For example:

- Well-chosen computations can shed light on familiar relations and reveal new patterns.
- One is forced to think very precisely; gaps in understanding must be eliminated if a program is to work.
- Dozens (or hundreds or thousands) of cases can be examined, perhaps showing new patterns or phenomena.
- Methods of verifying the results must be worked out, again adding to one's overall understanding.
- Different proofs of the same theorem can be compared from the point of view of algorithmic efficiency.

- One can examine historically important ideas from the varied perspectives provided by *Mathematica*, often obtaining new insights.

The reader will find examples of these points throughout the book. Here are two specific cases: Chapter 17 contains a discussion of the four-color theorem that seeks to turn Kempe's false proof of 1879 into a viable algorithm for four-coloring planar graphs. And Chapter 15 shows how a certain published construction in computational geometry — the construction of a three-dimensional room that contains a point invisible to guards placed at every vertex — must be changed if it is to be correct.

Another point worth mentioning is that a detailed knowledge of some of *Mathematica*'s internal workings can lead to novel solutions to programming problems. As an example, Chapter 12 shows how an understanding of the data computed by `ContourPlot` can lead to a simple and effective routine for finding all solutions in a rectangle to a pair of simultaneous transcendental equations.

The chapters are written so that browsing is possible, but there is certainly a progression from elementary to advanced techniques, and the novice is encouraged to read the chapters in order. Even advanced users will benefit from a careful reading of the first few chapters. Chapter 5 is devoted to the `Manipulate` command, which is used throughout the book. The output cannot be appreciated very well on the printed page and the reader is encouraged to load the files in the electronic supplement and work through the demonstrations in an interactive way.

From its beginning, twenty years ago, the pleasure and power of using *Mathematica* arose from the two somewhat separate features: the kernel and the front end. The kernel does the underlying computations and the front end is the device through which the user communicates with the kernel. There has always been some communication between the two, especially after version 3 with its many front end enhancements, but the two interfaces had the feel of separate entities. With version 6 the connection between the two became much stronger, as one can create `Manipulate` output that runs on its own. All the code in this book has been designed to work in both versions 6 and 7, with some small exceptions where use has been made of functions that are new in version 7. All the timings in the book are from a Macintosh laptop running a 2.16 gigaHertz Intel chip.

There have been important enhancements to the kernel as well. Some are minor, but together they contribute to making programming smoother, faster, and just more fun. A brief sampling: `Table` and `Do` commands now accept iterators that vary over a list, as in `Do[this, {x, {a, b, c}}]`, where `x` takes on the values `a`, `b`, and `c`; `RandomChoice[s]` generates a random choice from the list `s`; `Total[s]` sums the

elements of `s`; `Tally[s]` gives the elements of `s` with their frequencies; `ZetaZero[i]` gives the i th zero of the Riemann ζ function. Another nifty enhancement is the inclusion of comprehensive databases, some requiring a web connection to access. Thus one can get stock prices, currency exchange rates, and data about many structures in mathematics, physics, chemistry, geography, and even grammar. The data can then be integrated into *Mathematica* programs, thus allowing the user to conveniently analyze or present the data using all of *Mathematica*'s tools.

This book includes a CD with *Mathematica* files containing all the code that appears in the book, and much code that is not in the book. In some of the chapters one needs to load code from the disk to enable various sophisticated functions.

From a personal point of view I have to emphasize the huge impact that *Mathematica* has had on my teaching and research. I have been using the program for almost 20 years and it has led to many new ideas at the forefront of research, in a wide variety of fields. Math and science are much more fun when one can visualize concrete examples of the objects being studied; *Mathematica* gives us the chance to see even very abstract constructions, and thus to understand them more deeply.

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