

Contents

List of Figures	xxiii
------------------------------	-------

1. Mathematics in the French Revolution	1
1.1 The French Revolution	1
1.2 Some mathematicians	4
1.2.1 Monge	6
1.3 Descriptive geometry	8
2. Poncelet (and Pole and Polar)	11
2.1 Poncelet reminisces	11
2.2 Poncelet's mathematics	16
2.3 From Poncelet's <i>Traité</i>	18
2.3.1 Commentary	21
2.4 Pole, polar and duality	22
3. Theorems in Projective Geometry	25
3.1 The theorems of Pappus, Desargues and Pascal	25
3.2 Some properties of some transformations	35
3.3 Cross-ratio	39
3.3.1 Porismata	40
4. Poncelet's <i>Traité</i>	43
4.1 Poncelet's singular claims	43
4.1.1 Meeting	44
4.2 Cauchy responds	47
4.3 Other responses	51
4.4 Poncelet's more conventional methods	52

5. Duality and the Duality Controversy	53
5.1 Pole and polar	53
5.2 Gergonne versus Poncelet	55
5.2.1 Curves of higher degree	56
5.3 Gergonne	57
6. Poncelet and Chasles	63
6.1 What was done – differing opinions	63
6.2 Institutions and careers	66
6.3 Chasles	67
6.4 What was done?	69
6.5 Chasles, Steiner and cross-ratio	70
6.6 Extracts from Chasles' <i>Aperçu historique</i>	73
6.6.1 On descriptive geometry	73
6.6.2 On Monge and his school	74
6.6.3 On Monge's work	75
6.7 A quick introduction to modern projective geometry	76
6.7.1 The real projective plane	76
6.7.2 Projective spaces	78
7. Lambert and Legendre	79
7.1 Saccheri	81
7.2 Lambert	84
7.3 Legendre	86
7.4 Lambert	87
8. Gauss	91
8.1 Gauss	91
8.2 Schweikart and Taurinus	93
8.3 What Gauss knew	96
8.3.1 Gaussian curvature	98
9. János Bolyai	101
9.1 János and Wolfgang Bolyai	101
9.2 János Bolyai's geometry	104
9.3 János Bolyai's section 32	112
10. Lobachevskii	115
10.1 Lobachevskii and Kasan	115
10.2 Lobachevskii's new geometry	118
10.2.1 Lobachevskii's first foundations of geometry	120
10.2.2 Astronomical evidence	123

10.3 Lobachevskii's booklet of 1840	123
10.3.1 Opening remarks	123
10.3.2 Concluding remarks	126
11. To 1855	129
11.1 Minding's surface	129
11.2 The Bolyais read Lobachevskii	130
11.3 Final years of János Bolyai	131
11.4 Final years of Lobachevskii	132
11.5 Gauss's death, Gauss's <i>Nachlass</i>	134
12. Writing	137
12.1 Assessment and advice	137
12.1.1 Reading and writing the history of mathematics	137
12.1.2 Practice questions	141
12.2 References and footnotes	145
12.2.1 Appendices	146
12.2.2 Names	147
12.2.3 Your essays	147
12.3 Assessment question	147
12.3.1 Advice	147
13. Möbius	149
13.1 Möbius's <i>Barycentric calculus</i>	149
13.1.1 Barycentric coordinates	150
13.1.2 Projective transformations	153
13.1.3 Duality	155
13.1.4 Central projection from one plane to another	157
13.2 A note on duality	157
13.3 Möbius's coordinates	159
14. The Duality Paradox	161
14.1 Higher plane curves	161
14.1.1 Cubic curves	163
14.2 Plücker's resolution of the duality paradox	163
14.3 Confirmation by others	165
14.4 Plücker	166
14.5 Hesse	168

15. The Plücker Formulae	173
15.1 Singular points	173
15.1.1 The non-singular cubic curve in the plane	174
15.1.2 The non-singular quartic curve in the plane	174
15.1.3 28 real bitangents	177
16. Higher Plane Curves	179
16.1 Non-singular points and tangents	179
16.2 Double points	180
16.3 Homogeneous coordinates	182
16.4 First and subsequent polars	183
16.4.1 The first polars of a circle	183
16.4.2 Inflection points	184
16.5 Hessians	185
16.6 Addendum	188
16.7 References	190
17. Complex Curves	191
17.1 Complex by necessity	191
17.1.1 Complex numbers in geometry	193
17.1.2 The introduction of complex curves	193
17.2 Elliptic functions	194
18. Riemann	195
18.1 Riemann	195
18.2 Riemann's publications	197
18.3 Riemann on geometry	198
18.3.1 Surfaces	199
18.4 Riemannian geometry	201
18.5 From Riemann's <i>Habilitationsvortrag</i>	202
19. Differential Geometry of Surfaces	211
19.1 Basic techniques	211
19.1.1 Geodetic projection	212
19.2 Introducing Beltrami's <i>Saggio</i>	215
19.2.1 Beltrami's <i>Teoria</i> of 1868	217
19.3 The <i>Saggio</i>	219
19.3.1 From Beltrami's <i>Saggio</i> of 1868	219
19.4 Legendre's error	225
19.5 References	225

20. Non-Euclidean Geometry Accepted	227
20.1 Beltrami's version	227
20.2 Gauss's posthumous contribution	228
20.2.1 Kant?	229
20.3 Felix Klein	230
20.3.1 Klein at Erlangen	232
20.3.2 ... and beyond	232
20.4 Klein's Cayley metric	233
20.5 Klein's unification of geometry	234
20.6 The Erlangen Program in the 1890s	236
20.7 Weierstrass and Killing	237
21. Writing	241
21.1 Assessment questions	241
21.2 Advice	242
21.2.1 Cremona	242
21.2.2 Salmon	244
21.2.3 Lobachevskii's account in 1840	245
22. Fundamental Geometry	247
22.1 The rise of projective geometry	247
22.2 Cremona	249
22.2.1 Cremona's projective geometry	251
22.3 Salmon	254
22.4 Anxiety – Pasch	255
22.5 Helmholtz	256
22.5.1 Free mobility	258
23. Hilbert	259
23.1 Hilbert	259
23.2 Hilbert and geometry	261
23.2.1 The <i>Grundlagen der Geometrie</i>	262
23.2.2 Desargues' theorem	263
23.3 Impact	266
23.4 References	267
24. Italian Foundations	269
24.1 Peano and Segre	270
24.2 Enriques	273
24.3 Pieri	276
24.4 Conclusions	277
24.5 Veronese	277

25. The Disc Model	281
25.1 Poincaré	281
25.1.1 A prize competition	282
25.1.2 Poincaré's discovery of non-Euclidean geometry	283
25.1.3 The Poincaré and Beltrami discs	285
25.2 Poincaré and Klein	287
25.3 Circumcircles	288
25.4 Inversion and the Poincaré disc	289
25.4.1 Inversion	290
25.5 References	297
26. The Geometry of Space	299
26.1 How to decide?	299
26.2 Poincaré's conventionalism	300
26.2.1 Enriques disputes	301
26.3 "Space and geometry"	304
26.4 Poincaré's arguments	305
27. Summary: Geometry to 1900	309
27.1 References	311
28. The Formal Side	313
28.1 Nagel's thesis	313
28.2 From Hilbert's <i>Grundlagen der Geometrie</i>	315
29. The Physical Side	321
29.1 Geometry and physics	321
29.2 Einstein	322
29.2.1 The special theory of relativity	322
29.2.2 The paradoxes of special relativity	325
29.3 Minkowski	326
29.4 Einstein, gravity and the rotating disc	326
29.5 Einstein's <i>Relativity</i>	328
30. Is Geometry True?	333
30.1 Truth	333
30.1.1 Mathematical truths	334
30.2 Proof	335
30.2.1 Frege versus Hilbert	336
30.3 Relative consistency	337
30.4 Poincaré on "Non-Euclidean geometries"	338

31. Writing	341
31.1 Assessment questions	341
31.2 Advice on writing such essays	342
31.3 How the essays will be graded	343
 A. Von Staudt and his Influence	345
A.1 Von Staudt	345
A.1.1 Von Staudt's <i>Geometrie der Lage</i>	346
A.1.2 Klein's response to von Staudt	349
A.2 Non-orientability	351
A.3 Axiomatics – independence	354
A.4 References	357
 Bibliography	359
 Some Geometers	377
 Index	379

Worlds Out of Nothing

A Course in the History of Geometry in the 19th Century

Gray, J.

2010, XXVI, 384 p. 71 illus., Softcover

ISBN: 978-0-85729-059-5