

Chapter 2

The Need for and Possible Methods of Objective Ranking

Andrzej P. Wierzbicki

Abstract The classical approach in decision analysis and multiple criteria theory concentrates on subjective ranking, at most including some aspects of intersubjective ranking (ranking understood here in a wide sense, including the selection or a classification of decision options). Intuitive subjective ranking should be distinguished here from rational subjective ranking, based on the data relevant for the decision situation and on an approximation of personal preferences. However, in many practical situations, the decision maker might not want to use personal preferences, but prefers to have some objective ranking. This need of rational objective ranking might have many reasons, some of which are discussed in this chapter. Decision theory avoided the problem of objective ranking partly because of the general doubt in objectivity characteristic for the twentieth century; the related issues are also discussed. While an absolute objectivity is not attainable, the concept of objectivity can be treated as a useful ideal worth striving for; in this sense, we characterize objective ranking as an approach to ranking that is as objective as possible. Between possible multiple criteria approaches, the reference point approach seems to be most suited for rational objective ranking. Some of the basic assumptions and philosophy of reference point approaches are recalled in this chapter. Several approaches to define reference points based on statistical data are outlined. Examples show that such objective ranking can be very useful in many management situations.

Keywords Rational subjective ranking · Rational objective ranking · Objectivity · Reference point approaches

2.1 Introduction

While there exists a need for *objective ranking* in some management situations, the classical approach in decision analysis and multiple criteria theory concentrates solely on *subjective ranking*, at most including some aspects of *intersubjective*

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ranking. This is because, in a popular belief of management science, decision making is usually based on personal experience, memory, thoughts, thinking paradigms and the psychological states (sometimes called *habitual domains*, see [35]) of the decision maker. Management science maintains that all individual decisions are subjective; it might be only admitted that there are situations where the decision may have impact on many other people, in which case, showing a kind of objectivity is needed. Objectivity might be considered desirable but, since the *true state of nature* and the *perceived state of nature* usually are not the same, and people use their perceived state of nature to make decisions, it is not possible to achieve full objectivity and thus not essential to seek objectivity.

While correct in basic arguments and dominating in management science, the above described perception is far from completeness. There are classes of individual decision situations where objectivity is needed, because practically all decisions of a given class might influence other people. Such kind of decision situations is dominating in technology creation, because all creation of technological tools assumes impacts on many other people; consider, for example, the issue of constructing a safe bridge or a safe car. Thus, technologists stress objectivity much more than management scientists – while real managers also know well that there are many managerial situations where stressing objectivity is necessary. Technologists also know, since the works of Heisenberg [9] discussed in more detail later, that a full precision of measurement is impossible, thus the concept of a *true state of nature* can be an approximation only and full objectivity is not attainable. However, they interpret this fact quite differently than social scientists, seeing in this fact not a reason to dismiss objectivity, but a constraint to objectivity. We see that different disciplines perceive the issue of objectivity versus subjectivity quite differently and that an interdisciplinary, even philosophical discussion of these concepts is needed; we shall return to such a discussion in the next section.

We must also stress to use here the concept of *ranking* in a wide sense, including the *selection* of one or several best, or worst decision options, or a *classification* of all decision options. All classical approaches of multi-attribute decision analysis – whether presented in [12], or in [24], or in [11] – concentrate on subjective ranking. By this we do not mean *intuitive subjective ranking*, which can be done by any experienced decision maker based on her/his intuition, but *rational subjective ranking*, based on the data relevant for the decision situation – however, using an approximation of personal preferences in aggregating multiple criteria.

And therein is the catch: in many practical situations, if the decision maker wants to have a computerized decision support and rational ranking, she/he does not want to use personal preferences, prefers to have some objective ranking. This is, as suggested above both from social science and technological perspectives, usually because the decision is not only a personal one, but affects many people – and it is often very difficult to achieve an intersubjective rational ranking, accounting for personal preferences of all people involved. We shall discuss in more detail the reasons for the need of objective ranking in the next section.

Decision theory avoided – to some extent, we comment on this issue later – the problem of objective ranking partly because of the general doubt in objectivity

characteristic for the twentieth century. Thus, we recall also some of philosophical foundations and contemporary approaches to the issue of objectivity. While it can be agreed that an absolute objectivity is not attainable, the concept of objectivity can be treated as a goal, a higher-level value, a useful ideal worth striving for; in this sense, we characterize *objective ranking as an approach to ranking that is as objective as possible*.

Several multiple criteria decision analysis approaches are recalled in relation to the problem of objective ranking. Between such possible multiple criteria approaches, the reference point approach seems to be most suited for rational objective ranking, because reference levels needed in this approach can be established – to some extent objectively – statistically from the given data set. Some of the basic assumptions and philosophy of reference point approaches are recalled, stressing their unique concentration on the sovereignty of the subjective decision maker. However, precisely this sovereignty makes it possible also to postulate a proxy, virtual objective decision maker that is motivated only by statistical data. Several approaches to define reference points based on statistical data are outlined. Examples show that such objective ranking can be very useful in many management situations.

2.2 The Need for Objective Ranking and the Issue of Objectivity

Objectivity as a goal and objective ranking are needed not only in technology creation, but also – as we show here – in management. For an individual decision maker, this might mean that she/he needs some independent reasons for ranking, such as a dean cannot rank the laboratories in her/his school fully subjectively, must have some reasonable, objective grounds that can be explained to entire faculty, see one of further examples. For a ranking that expresses the preferences of a group, diverse methods of aggregating group preferences might be considered; but they must be accepted as fair – thus objective in the sense of intersubjective fairness – by the group, and the task of achieving a consensus about the fairness might be difficult. One of acceptable methods of such aggregation might be the specification of a *proxy, virtual decision maker that is as objective as possible, e.g., motivated only by statistical data*.

The need for objective ranking is expressed also in business community by the prevalent practice of hiring external consulting companies to give independent advice, including ranking, to the chief executive officer (CEO) of a company. The CEO obviously could use her/his detailed, tacit knowledge about the company and intuition to select a solution or ranking (either intuitive or rational); but she/he apparently prefers, if the situation is serious enough, not to use personal preferences and to ask for an independent evaluation instead.

There are many other situations where we need ranking, broadly understood thus including also classification and selection of either best or worst options (decisions, alternatives, etc.), performed as objectively as possible. This particularly concerns the task of selecting the worst options, often encountered in management

(some opinions suggest that best management is concentrated on patching the worst observed symptoms); if we have to restructure the worst parts of an organization, we prefer to select them possibly objectively. These obvious needs have been neglected by decision theory that assumed subjectivity of a decision maker because of many reasons, partly paradigmatic, partly related to the anti-positivist and antiscientism turn in the philosophy of twentieth century.

Here we must add some philosophical comments on subjectivity and objectivity. The industrial era episteme – sometimes called not quite precisely positivism or scientism – valued objectivity; today we know that absolute objectivity does not exist. The destruction of this episteme started early, e.g., since Heisenberg [9] has shown that not only a measurement depends on a theory and on instruments, but also the very fact of measurement distorts the measured variable. This was followed by diverse philosophical debates, summarized, e.g., by Van Orman Quine [21] who has shown that the logical empiricism (neo-positivism) is logically inconsistent itself, that all human knowledge “is a man-made fabric that impinges on existence only along the edges”. This means that there is no absolute objectivity; however, this was quite differently interpreted by hard sciences and by technology, which nevertheless tried to remain as objective as possible, and by social sciences which, in some cases, went much further to maintain that all knowledge is subjective – results from a discourse, is constructed, negotiated, relativist, depends on power and money, that the very concept of “*Nature*” is only a construction of our minds, see, e.g., [14]. This has led to a general divergence of the episteme – understood after Michel Foucault as the way of constructing and justifying knowledge, characteristic for a historical era or a cultural sphere, see [41] – of the three different cultural spheres of hard and natural sciences, of technology, and of social sciences and humanities, see [27].

Full objectivity is obviously – after Heisenberg and Quine – not attainable; but in many situations we must try to be as much objective as possible. This concerns not only technology that cannot advance without trying to be objective and, in fact, pursues Popperian falsificationism [20] in everyday practice when submitting technological artifacts to destructive tests in order to increase their reliability – while postmodern social sciences ridicule falsificationism as an utopian description how science develops. However, objectivity is needed also – as indicated above – in management.

In order to show that the postmodern episteme is not the only possible one, we present here another description of the relation of human knowledge to nature [32]. First, from a technological perspective we do not accept the assumption of post-modern philosophy that “*Nature*” is only a construction of our minds and has only local character. Of course, the word *nature* refers both to the construction of our minds and to something more – to some persisting, universal (to some degree) aspects of the world surrounding us. People are not alone in the world; in addition to other people, there exists another part of reality, that of nature, although part of this reality has been converted by people to form human-made, mostly technological systems. There are aspects of reality that are local and multiple, there are aspects that are more or less universal. To some of our colleagues who believe that there is no universe, only a *multiverse*, we propose the following *hard wall test*: we position

ourselves against a hard wall, close our eyes and try to convince ourselves that there is no wall before us or that it is not hard. If we do not succeed in convincing ourselves, it means that there is no multi-verse, because nature apparently has some universal aspects. If we succeed in convincing ourselves, we can try to verify or falsify this conviction by running ahead with closed eyes.

Second, the general relation of human knowledge to reality might be described as follows. People, motivated by curiosity and aided by intuition and emotions, observe reality and formulate hypotheses about properties of nature, of other people, of human relations; they also construct tools that help them to deal with nature (such as cars) or with other people (such as telephones); together, we call all this knowledge. People test and evaluate the knowledge constructed by them by applying it to reality: perform destructive tests of tools, devise critical empirical tests of theories concerning nature, apply and evaluate theories concerning social and economic relations; in general, we can consider this as a generalized principle of falsification, broader than defined by Popper even in his later works [20].

Such a process can be represented as a general spiral of evolutionary knowledge creation, see Fig. 2.1. We observe reality (either in nature or in society) and its changes, compare our observations with human heritage in knowledge (the transition *Observation*). Then our intuitive and emotive knowledge helps us to generate new hypotheses (*Enlightenment*) or to create new tools; we apply them to existing reality (*Application*), usually with the goal of achieving some changes, modifications of reality (*Modification*); we observe them again.

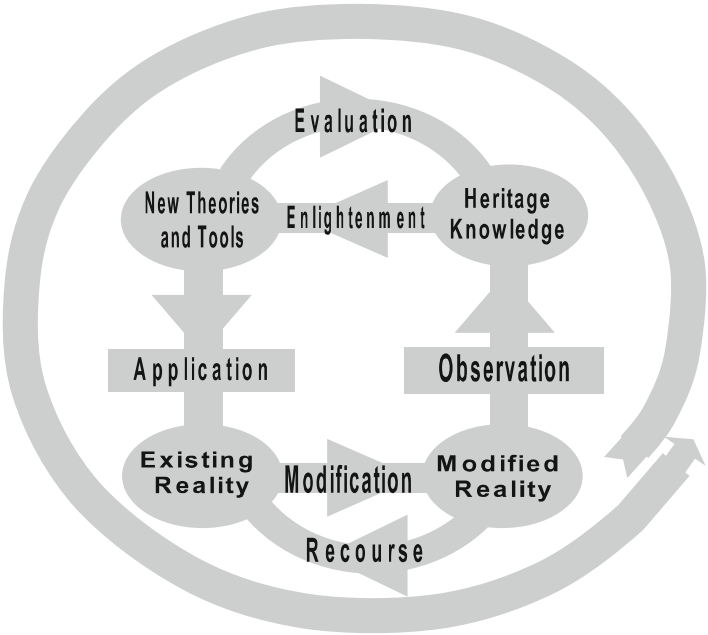


Fig. 2.1 The general OEAM spiral of evolutionary knowledge creation

It is important, however, to note that many other transitions enhance this spiral. First is the natural evolution in time: modified reality becomes existing reality through *Recourse*. Second is the evolutionary selection of tested knowledge: most new knowledge might be somehow recorded, but only the positively tested knowledge, resilient to falsification attempts, remains an important part of human heritage (*Evaluation*); this can be interpreted as an objectifying, stabilizing feedback. Naturally, there might be also other transitions between the nodes indicated in the spiral model, but the transitions indicated in Fig. 2.1 are the most essential ones.

Thus, nature is not only the effect of construction of knowledge by people, nor is it only the cause of knowledge: it is both cause and effect in a positive feedback loop, where more knowledge results in more modifications of nature and more modifications result in more knowledge. As in most positive feedback loops, the overall result is an avalanche-like growth; and this avalanche-like growth, if unchecked by stabilizing feedbacks, beside tremendous opportunities creates also diverse dangers, usually not immediately perceived but lurking in the future. Thus, the importance of selecting knowledge that is as objective as possible relates also to the fact that avalanche-like growth creates diverse threats: we must leave to our children best possible knowledge in order to prepare them for dealing with unknown future.

This description of a spiral-like, evolutionary character of knowledge creation presented in Fig. 2.1 was proposed first in [31] as consistent with our technological cognitive horizon, and different than presented in [10] from a position of an economic cognitive horizon; we are aware that there are many theories and schools of thought concerning philosophy of life and development of science, but we present this description as an extension of one of them. It is an extension of the concept of *objective knowledge* as presented in [20] which, however, admits relativistic interpretations. It only postulates objectivity as a higher level value, similar to justice: both absolute justice and absolute objectivity might be not attainable, but are important, worth striving for, particularly if we take into account uncertainty about future (see also [22]). This description is, however, concentrating not on individual knowledge creation, but on the evolutionary value of well-tested, as objectively as possible, knowledge for human societies and for humanity as a whole, including future generations.

2.3 Basic Formulations and Assumptions

We turn now to the main subject of this paper. We assume that we have a decision problem with n criteria, indexed by $i = 1, \dots, n$ (also denoted by $i \in I$), and m alternative decisions called also alternatives, indexed by $j = 1, \dots, m$ or $j = A, B, \dots, H$ (also denoted by $j \in J$). The corresponding criteria values are denoted by q_{ij} ; we assume that all are maximized or converted to maximized variables. The maximal values $\max_{j \in J} q_{ij} = q_i^{\text{up}}$ are called upper bounds for criteria and are often equivalent to the components of so called ideal or utopia point $\mathbf{q}^{\text{uto}} = \mathbf{q}^{\text{up}} = (q_1^{\text{up}}, \dots, q_i^{\text{up}}, \dots, q_n^{\text{up}})$ – except for cases when they were

established a priori as a measurement scale, see further comments. The minimal values $\min_{j \in J} q_{ij} = q_i^{\text{lo}}$ are called lower bounds and, generally, are not equivalent to the components of so called nadir point $\mathbf{q}^{\text{nad}} \approx \mathbf{q}^{\text{lo}} = (q_1^{\text{lo}}, \dots, q_i^{\text{lo}}, \dots, q_n^{\text{lo}})$; the nadir point \mathbf{q}^{nad} is defined similarly as the lower bound point \mathbf{q}^{lo} , but with minimization restricted to Pareto optimal or efficient or nondominated alternatives, see, e.g., [3]. An alternative $j^* \in J$ is Pareto optimal (Pareto-nondominated or shortly nondominated, also called efficient), if there is no other alternative $j \in J$ that dominates j^* , that is, if we denote $\mathbf{q}_j = (q_{1j}, \dots, q_{ij}, \dots, q_{nj})$, there is no $j \in J$ such that $\mathbf{q}_j \geq \mathbf{q}_{j^*}$, $\mathbf{q}_j \neq \mathbf{q}_{j^*}$.

While there is an extensive literature how to select the best alternative (usually between nondominated ones) or to rank or classify all alternatives in response to the preferences of a decision maker, this literature usually makes several tacit assumptions:

1. A standard and usually undisputed assumption is that there is a decision maker (DM) that does not mind to reveal her/his preferences – either a priori, before the computer system proposes her/his supposedly best decision (in this case, we should actually not speak about decision support, only about decision automation), or interactively, exchanging information with a computerized decision support system (in this case, truly supporting decisions). In group decision making, it is often assumed that the group members do not mind discussing their preferences. However, highly political decision makers might intuitively (using their experience in political negotiations) refuse to discuss their preferences, and do not have time for a long interaction with the decision support system. Moreover, as discussed above, there are also many rational reasons why a decision maker might want to obtain an advice on the best decision or ranking of decisions that is as objective as possible, thus independent from her/his preferences, particularly if the final decision will be highly political, or there is actually a large group of decision makers or stakeholders in the decision situation.
2. Another standard and usually undisputed assumption is that there is an analyst (AN) that knows well decision theory and practice, interacts with decision makers on the correct definition and modeling of the decision situation, thus influences, e.g., the choice of criteria, further programs or fine-tunes the decision support system, etc. (even if the role of the analyst might be hidden just by an assumed approach used for constructing the decision support system). However, the role of an analyst is essential even if it should not be dominant; for example, the choice of criteria might be a result of a political process, and even if the analyst would know the extensive literature how to select criteria reasonably from decision theoretical point of view, she/he has just to accept even unreasonable criteria.

In further discussions, we assume that there are decision makers and analysts, but their roles should be interpreted more broadly than usually.

2.4 Why Classical Approaches Are Not Applicable in This Case

We discuss here two classes of methods taught usually – for historical reasons – as “the basic approach” to multiple criteria decision making. The first of them is the weighted sum aggregation of criteria: determining by diverse approaches, between which the AHP [24] is one of the most widely known, weighting coefficients w_i for all $i \in I$, with the additional requirement on the scaling of weighting coefficients that $\sum_{i \in I} w_i = 1$, and then using them to aggregate all criteria by a weighted sum:

$$\sigma_{jsum} = \sum_{i \in I} w_i q_{ij}. \quad (2.1)$$

We use the aggregated values σ_{jsum} to select the best alternative (maximizing σ_{jsum} between $j \in J$) or to rank alternatives (ordering them from the largest to the lowest value of σ_{jsum}). Such an aggregation might be sometimes necessary, but it has several limitations, particularly for the problem of objective ranking. The most serious between them are the following:

1. The weighted sum is based on a tacit (unstated) assumption that a compensatory trade-off analysis is applicable to all criteria: a worsening of the value of one criterion might be compensated by the improvement of the value of another one. While often encountered in economic applications, this compensatory character of criteria is usually not encountered in interdisciplinary applications.
2. Changes of weighting coefficients in interactive decision processes with more than two criteria often lead to counter-intuitive changes of criteria values [69] explained below.
3. The linear aggregation of preferences expressed by the weighted sum tends to promote decisions with unbalanced criteria, as illustrated by the Korhonen paradox quoted below; in order to accommodate the natural human preference for balanced solutions, a nonlinear aggregation is necessary.
4. In the weighted sum approach, it is not easy to propose a way of defining weighting coefficients that are as objective as possible (except if all criteria have the same importance and we assume simply equal weighting coefficients).

The Korhonen paradox can be illustrated by the following example. Suppose we select a product and consider two criteria: quality and cost, while using an assessment scale 0–10 points for both criteria (0 points for cost means very expensive, 10 points means very cheap products). Suppose we have three alternative decisions. Alternative A has 10 points for quality, 0 points for cost. Alternative B has 0 points for quality, 10 points for cost. Alternative C has 4.5 points for quality and 4.5 points for cost. It is easy to prove that when using a weighted sum for ranking the alternatives, alternative C will be never ranked first – no matter what weighting coefficients we use. Thus, weighted sum indeed tends to promote decisions with unbalanced criteria; in order to obtain a balanced solution (the first rank for alternative product C), we have either to use additional constraints or a nonlinear aggregation scheme.

Educated that weighting coefficients methods are basic, the legislators in Poland introduced a public tender law. This law requires that any institution preparing a tender using public money should publish beforehand all criteria of ranking the offers and all weighting coefficients used to aggregate the criteria. This legal innovation backfired: while the law was intended to make public tenders more transparent and accountable, the practical outcome was opposite because of effects similar to the Korhonen paradox. Organizers of the tenders soon discovered that they are forced either to select the offer that is cheapest and worst in quality or the best in quality but most expensive one. In order to counteract, they either limited the solution space drastically by diverse side constraints (which is difficult but consistent with the spirit of the law) or added additional poorly defined criteria such as the degree of satisfaction (which is simple and legal but fully inconsistent with the spirit of the law, since it makes the tender less transparent and opens hidden door for graft).

The example of counter-intuitive effects of changing weighting coefficients given by Nakayama [16] is simple: suppose $n = 3$ and the criteria values for many alternatives are densely (or continuously) spread over the positive part of the surface of a sphere, $q_1^2 + q_2^2 + q_3^2 = 1$. Suppose we select first $w_1 = w_2 = w_3 = 0.3333$, which results in the best alternative with criteria values $q_1 = q_2 = q_3 = 0.577$. Suppose we want next to increase the values of q_1 strongly and of q_2 slightly, while agreeing to decrease q_3 ; what modifications of weighting coefficients would do the job? If we choose $w_1 = 0.55$, $w_2 = 0.35$ and $w_3 = 0.1$, the result will be a strong increase of $q_1 = 0.8338$ accompanied by a decrease of both $q_2 = 0.5306$ and $q_3 = 0.1516$; in order to increase q_1 strongly and q_2 slightly we must increase w_2 almost as strongly as w_1 . If we have more criteria, it might be sometimes very difficult to choose a change of weighting coefficients resulting in a desired change of criteria values.

Both such theoretical examples and recent practical experience presented above show that we should be very careful when using weighted sum aggregation. In short summary, a linear weighted sum aggregation is simple mathematically but too simplistic in representing typical human preferences that are usually nonlinear; using this simplistic approach resulted in practice in adverse and unforeseen side-effects. For objective ranking, weighted sum aggregation is not applicable, except in the most simplest case of equal weighting coefficients.

Thus, we should rather look for nonlinear approximations of the preferences of decision makers. There are many highly developed methods of the elicitation of nonlinear utility or value functions, see, e.g., [11, 12]. However, these classical methods are not directly applicable for objective ranking, because they are developed precisely in order to express the subjectivity of the decision maker. As noted above, in decisions involving political processes such elicitations of utility or value functions might be not applicable because of several reasons:

1. Politically minded decision makers might be adverse to a disclosure and detailed specifications of their preferences.
2. Such elicitations of utility or value functions require a large number of pairwise comparisons of alternatives, done in the form of questions addressed to the decision maker and her/his answers; this number is nonlinearly growing with the number of criteria.

For these and other reasons, we should further look for more ad hoc and rough nonlinear approximations of preferences of decision makers, which do not require much time nor a detailed specification or identification of preferences. However, it is not obvious how to define the grounds of an objective selection or ranking. In multiple criteria optimization, one of similar issues was to propose compromise solutions, see, e.g., [5, 34, 36]; however, such solutions might depend too strongly on the assumed metric of the distance from the utopia or ideal point. In [28] it is proposed to define objective selection and ranking as dependent only on a given set of data, agreed upon to be relevant for the decision situation (generally, for any selected *data information system*, see [19]), and independent of any more detailed specification of personal preferences than that given by defining criteria and the partial order in criteria space. The specification of criteria and their partial order (whether to minimize, or maximize them) can be also easily be agreed upon, be objective in the sense of intersubjective fairness.

It is also not obvious how an objective selection and ranking might be achieved, because almost all the tradition of aggregation of multiple criteria concentrated on rational subjective aggregation of preferences and thus subjective selection and ranking. While we could try, in the sense of intersubjective fairness, identify group utility functions or group weighting coefficients, both these concepts are too abstract to be reasonably debated by an average group (imagine a stockholder meeting trying to define their aggregate utility function under uncertainty). Thus, neither of these approaches is easily adaptable for rational objective selection or ranking. The approach that can be easily adapted for rational objective selection and ranking, also classification, is reference point approach as described below, because reference levels needed in this approach can be either defined subjectively by the decision maker, or established objectively statistically from the given data set.

2.5 Reference Point Approaches for Objective Ranking

A rough approximation of decision maker preferences is provided by reference point approaches. In these approaches, we note that:

1. The preferences of decision maker can be approximated using several degrees of specificity, and the reference point approaches assume that this specification should be as general as possible, since a more detailed specification violates the sovereign right of a decision maker to change her/his mind.
2. The most general specification of preferences contains a selection of outcomes of a model of decision situation that are chosen by the decision maker (or analyst) to measure the quality of decisions, called criteria (quality measures, quality indicators) or sometimes objectives (values of objective functions) and denoted here by q_i , $i \in I$. This specification is accompanied by defining a partial order in the space of criteria – simply asking the decision maker which criteria should be maximized and which minimized, while another option, stabilizing some criteria around given reference levels, is also possible in reference point approaches, see

[30]. Here we consider – in order to simplify presentation – the simplest case when all criteria are maximized.

3. The second level of specificity in reference point approaches is assumed to consist of specification of reference points – generally, desired levels of criteria. These reference points might be interval-type, double, including aspiration levels, denoted here by a_i (levels of criteria values that the decision maker would like to achieve) and reservation levels r_i (levels of criteria values that should be achieved according to the decision maker). Specification of reference levels is treated as an alternative to trade off or weighting coefficient information that leads usually to linear representation of preferences and unbalanced decisions as discussed above, although some reference point approaches – see, e.g., [16, 23] – combine reference levels with trade-off information.
4. While the detailed specification of preferences might include full or gradual identification of utility or value functions, as shortly indicated above, this is avoided in reference point approaches that stress learning instead of value identification – according to the reference point philosophy, the decision maker should learn during the interaction with a decision support system, hence her/his preferences might change in the decision making process and she/he has full, sovereign right or even necessity to be inconsistent.
5. Thus, instead of a nonlinear value function, reference point approaches approximate the preferences of the decision maker by a nonlinear *achievement function* which is an ad hoc, easily adaptable nonlinear approximation of the value function of decision maker consistent with the information contained in criteria specification, their partial order and the position of reference point (or points) between the lower and upper bounds for criteria. As opposed to goal programming, similar in approach to reference point methods but using distance concepts instead of achievement functions, the latter functions preserve strict monotonicity with respect to the partial order in criteria space – because they are not equivalent to distances, see later comments.
6. The particular form of this nonlinear approximation of value function is determined essentially by max–min terms that favor solutions with balanced deviations from reference points and express the Rawlsian principle of justice (concentrating the attention on worst off members of society or on issues worst provided for, see [22]; these terms are slightly corrected by regularizing terms, resulting in nondomination (Pareto optimality) of alternatives that maximize achievement functions. It can be shown [26] that such achievement functions have the property of *full controllability*, independently of convexity assumptions. This means that, also for discrete decision problems, any nondominated (Pareto optimal) alternative can be selected by the decision maker when modifying reference points and maximizing the achievement function; this provides for the full sovereignty of the decision maker.

While there are many variants of reference point approaches, see [15, 23], we concentrate here on a reference point approach that requires the specification of interval-type reference, that is, two reference levels (aspiration and reservation) for

each criterion. After this specification, the approach uses a nonlinear aggregation of criteria by an achievement function that is performed in two steps:

1. We first count achievements for each individual criterion or satisfaction with its values by transforming it (strictly monotonically and piece-wise linearly), e.g., in the case of maximized criteria as shown in Eq. 2.2. For problems with a continuous (nonempty interior) set of options, for an easy transformation to a linear programming problem, such a function needs additional specific parameters selected to assure the concavity of this function, see [10]. In a discrete decision problem, however, we do not necessarily need concavity and can choose these coefficients to have a reasonable interpretation of the values of the *partial (or individual) achievement function*:

$$\sigma_i(q_i, a_i, r_i) = \begin{cases} \alpha (q_i - q_i^{\text{lc}}) / (r_i - q_i^{\text{lo}}) & \text{if } q_i^{\text{lo}} \leq q_i < r_i, \\ \alpha + (\beta - \alpha) (q_i - r_i) / (a_i - r_i) & \text{if } r_i \leq q_i < a_i, \\ \beta + (10 - \beta) (q_i - a_i) / (q_i^{\text{up}} - a_i) & \text{if } a_i \leq q_i \leq q_i^{\text{up}}. \end{cases} \quad (2.2)$$

Since the range of $[0; 10]$ points is often used for eliciting expert opinions about subjectively evaluated criteria or achievements, we adopted this range in Eq. 2.2 for the values of a partial achievement function $\sigma_i(q_i, a_i, r_i)$. The parameters α and β , $0 < \alpha < \beta < 10$, in this case denote correspondingly the values of the partial achievement function for $q_i = r_i$ and for $q_i = a_i$. The value $\sigma_{ij} = \sigma_i(q_{ij}, a_i, r_i)$ of this achievement function for a given alternative $j \in J$ signifies the satisfaction level with the criterion value for this alternative. Thus, the above transformation assigns satisfaction levels from 0 to α (say, $\alpha = 3$) for criterion values between q_i^{lo} and r_i , from α to β (say, $\beta = 7$) for criterion values between r_i and a_i , from β to 10 for criterion values between a_i and q_i^{up} .

2. After this transformation of all criteria values, we might use then the following form of the overall achievement function:

$$\sigma(\mathbf{q}, \mathbf{a}, \mathbf{r}) = \min_{i \in I} \sigma_i(q_i, a_i, r_i) + \varepsilon/n \sum_{i \in I} \sigma_i(q_i, a_i, r_i), \quad (2.3)$$

where $\mathbf{q} = (q_1, \dots, q_i, \dots, q_n)$ is the vector of criteria values, $\mathbf{a} = (a_1, \dots, a_i, \dots, a_n)$ and $\mathbf{r} = (r_1, \dots, r_i, \dots, r_n)$ are the vectors of aspiration and reservation levels, while $\varepsilon > 0$ is a small regularizing coefficient. The achievement values $\sigma_j = \sigma(\mathbf{q}_j, \mathbf{a}, \mathbf{r})$ for all $j \in J$ can be used either to select the best alternative, or to order the options in an overall ranking list or classification list, starting with the highest achievement value.

The formulae (2.2), (2.3) do not express the only form of an achievement function; there are many possible forms of such functions as shown in [30]. All of them, however, are not equivalent to a distance: a distance, say, from the aspiration point \mathbf{a} has the value 0 when $\mathbf{q} = \mathbf{a}$ and loses its monotonicity when crossing this point, while the overall achievement function maintains its strict monotonicity as a strictly monotone function of strictly monotone partial

achievement functions. Moreover, all of them have an important property of partial order approximation: their level sets approximate closely the positive cone defining the partial order in criteria space (see [26]). As indicated above, the achievement function has also a very important theoretical property of *controllability*, not possessed by utility functions nor by weighted sums: for sufficiently small values of ε , given any point \mathbf{q}^* in criteria space that is (ε -properly) Pareto-nondominated and corresponds to some alternative decision (such as the alternative C in the Korhonen paradox), we can always choose such reference levels – in fact, it suffices to set aspiration levels equal to the components of \mathbf{q}^* – that the maximum of the achievement function (3) is attained precisely at this point. Conversely, if $\varepsilon > 0$, all maxima of achievement function (2.3) correspond to Pareto-nondominated alternatives – because of the monotonicity of this function with respect to the partial order in the criteria space, mentioned above, similarly as in the case of utility functions and weighted sums, but not in the case of a distance norm used in goal programming, since the norm is not monotone when passing zero. As noted above, precisely the controllability property results in a fully sovereign control of the decision support system by the user.

We turn now to the question how to use reference point approaches for objective ranking. Since an achievement function models a proxy decision maker, it is sufficient to define – as objectively as possible – the corresponding aspiration and reservation levels. Several ways of such definition were listed in [6]: *neutral, statistical, voting*; we shall concentrate here on statistical determination. A statistical determination of reference levels concerns values q_i^{av} that would be used as basic reference levels, a modification of these values to obtain aspiration levels a_i , and another modification of these values to obtain reservation levels r_i ; these might be defined (for the case of maximization of criteria) as follows:

$$q_i^{\text{av}} = \sum_{j \in J} q_{ij} / m; \quad r_i = 0.5 (q_i^{\text{lo}} + q_i^{\text{av}}); \quad a_i = 0.5 (q_i^{\text{up}} + q_i^{\text{av}}). \quad (2.4)$$

Recall that m is just the number of alternative decision options, hence q_i^{av} is just an average criterion value between all alternatives, and aspiration and reservation levels – just averages of these averages and the lower and upper bounds, respectively. However, as shown by examples presented later, there are no essential reasons why we should limit the averaging to the set of alternative options ranked; we could use as well a larger set of data in order to define more adequate (say, historically meaningful) averages, or a smaller set, e.g., only the Pareto-nondominated alternatives.

Thus, we are ready to propose one basic version of an objectified reference point approach for discrete decision alternatives. Here are our advices for the analyst:

1. Accept the criteria and their character (which to maximize, which to minimize) proposed by decision maker(s), but insist on a reasonable definition of their upper and lower bounds.

2. Gather (the evaluation of) all criteria values for all alternative decisions. In the case that some criteria have to be assessed by expert opinions, organize an objectifying process for these assessments (e.g., voting on these assessments as if judging ski-jumping, with deleting extreme assessments or even with using median score, allowing for a dispute and a repeated vote in cases of divergent assessments).
3. Compute the averages of criteria values, the statistically objective reservation and aspiration points as in Eq. 2.4. Assuming $\alpha = 3$ and $\beta = 7$ for all criteria and using the achievement functions as defined by (2.2), (2.3), compute achievement factors σ_j for all alternatives and order alternatives in a decreasing fashion of these factors (say, randomly if $\sigma_j = \sigma_{j'}$ for some j and j' ; we shall suggest in the next section a way of improving such ordering). Use this ordering either for a suggested (objective and neutral) selection of the best alternative, or a classification of alternatives (say, into projects accepted and rejected), or an objective and neutral ranking.
4. Discuss with decision maker(s) the suggested objective and neutral outcome. If she/he wants to modify it, several ways of interaction are possible, starting with subjective modifications of reference levels, or an intersubjective definition of importance factors for every criterion (see [29]).

2.6 Examples

The first example concerns international business management. Suppose an international corporation consists of six divisions A, ..., F. Suppose these units are characterized by diverse data items, such as name, location, number of employees, etc. However, suppose that the CEO of this corporation is really interested in ranking or classification of these divisions taking into account the following attributes used as criteria:

1. Profit (p., in percent of revenue)
2. Market share (m.s., in percent of supplying a specific market sector, e.g., global market for a type of products specific for this division)
3. Internal collaboration (i.t., in percent of revenue coming from supplying other divisions of the corporation)
4. Local social image (l.s.i., meaning public relations and the perception of this division – e.g., of its friendliness to local environment – in the society where it is located, evaluated on a scale 0–100 points)

All these criteria are maximized, improve when increased. An example of decision table of this type is shown in Table 2.1 (with data distorted for privacy reasons), while Pareto-nondominated divisions are distinguished by mark *.

The CEO obviously could propose an intuitive, subjective ranking of these divisions – and this ranking might be even better than a rational one resulting from Table 2.1, if the CEO knows all these divisions in minute detail. However,

Table 2.1 Data for an example on international business management (Empl. = employees)

Division	Name	Location	Empl.	q_1 : p.	q_2 : m.s.	q_3 : i.t.	q_4 : l.s.i.
A	Alpha	USA	250	11%	8%	10%	40
B*	Beta	Brasilia	750	23%	40%	34%	60
C*	Gamma	China	450	16%	50%	45%	70
D*	Delta	Dubai	150	35%	20%	20%	44
E*	Epsilon	C. Europe	350	18%	30%	20%	80
F	Fi	France	220	12%	8%	9%	30

when preparing a discussion with her/his stockholders, (s)he might prefer to ask a consulting firm for an objective ranking.

Thus, we first illustrate the issue of objective ranking and statistical determination of reservation and aspiration levels. The principle that all criteria improve when increasing is easy to agree upon; similarly, the stockholders would easily accept the principle that the details of ranking should be determined mostly by the data contained in Table 2.1 and not by any personal preferences. The question how to statistically define reservations and aspirations is actually technical, but interesting for illustration. There are no essential reasons why we should limit the averaging to the set of alternatives ranked; we could use as well a larger set of data in order to define more adequate (say, historically meaningful) averages, or a smaller set – for example, only the Pareto-nondominated alternatives denoted by * in Table 2.1 – in order to define, say, more demanding averages and aspirations. For the data from Table 2.1, we can thus present two variants of objective ranking: A – based on averages of data from this table; B – based on averages from Pareto optimal options – see Table 2.2. We use here the achievement function from Eq. 2.3 with $\varepsilon = 0.4(n = 4)$.

We do not observe changes of ranking and classification when shifting from average A to more demanding B aspirations and reservations; this is confirmed by other applications and shows that objective ranking gives – at least, on the examples considered – rather robust results. Generally, we might expect rank reversals, although usually not very significant, when shifting to more demanding aspirations. This is, however, a natural phenomenon: average aspirations favor standard though good solutions, truly interesting solutions result from demanding aspirations. Note that we did not change the estimates of the lower and upper bounds and thus measurement ranges when averaging over Pareto-nondominated solutions; although the lower bounds for Pareto-nondominated alternatives (so called nadir point) are in this case different than the lower bounds for all alternatives, a change of ranges would mean a change of measurement units and should be avoided, see also [11].

The second example concerns knowledge management at a university. It illustrates a management application where the worst ranked options are the most interesting, because they indicate the need of a corrective action. Objective ranking was actually motivated originally by this specific application when evaluating scientific creativity conditions in a Japanese research university, JAIST, see [25]. The evaluation was based on survey results. The survey included 48 questions with diverse answers and over 140 respondents with diverse characteristics: school

Table 2.2 An example of objective ranking and classification for the data from Table 2.1

Criterion	q1	q2	q3	q4			
Upper bound	35%	50%	45%	80			
Lower bound	11%	8%	9%	30			
Reference A							
(average)	19.2%	26%	23%	54			
Aspiration A	27.1%	38%	34%	67			
Reservation A	15.1%	17%	16%	42			
Reference B							
(Pareto average)	23%	35.0%	29.7%	63.5			
Aspiration B	29%	42.5%	37.4%	71.7			
Reservation B	17%	17%	19.4%	46.7			
Ranking A: Division	σ_1	σ_2	σ_3	σ_4	σ	Rank	Class
A	0.00	0.00	0.37	2.50	0.29	5	III
B	5.63	7.50	7.00	5.88	8.23	1	I
C	3.30	10.0	10.0	7.62	6.39	2	II
D	10.0	3.57	3.89	3.32	5.40	4	II
E	3.97	5.48	3.89	10.0	6.30	3	II
F	0.73	0.00	0.00	0.00	0.07	6	III
Ranking B: Division							
A	0.00	0.00	0.29	1.80	0.21	5	III
B	5.00	6.61	6.24	5.13	7.30	1	I
C	2.50	10.0	10.0	6.73	5.42	2	II
D	10.0	3.47	3.13	2.51	4.42	4	II
E	3.33	5.04	3.13	10.0	5.28	3	II
F	0.50	0.00	0.00	0.00	0.05	6	III

attachment (JAIST consists of three schools), nationality (Japanese or foreign – the latter constitute over 10% of young researchers at JAIST), research position (master students, doctoral students, research associates, etc.). In total, the data base was not very large, but large enough to create computational problems.

The questions were of three types. The first type was assessment questions, assessing the situation between students and at the university; the most critical questions of this type might be selected as those that correspond to worst responses. The second type was important questions, assessing importance of a given subject; the most important questions might be considered as those that correspond to best responses. For those two types of questions, responders were required to tick appropriate responses in the scale *vg* (*very good*), *g* (*good*), *a* (*average*), *b* (*bad*), *vb* (*very bad*) – sometimes in an inverted scale if the questions were negatively formulated. The third type was controlling questions, testing the answers to the first two types by indirect questioning revealing responder attitudes or asking for a detailed explanation.

Answers to all questions of first two types were evaluated on a common scale, as a percentage distribution (histogram) of answers *vg* – *g* – *a* – *b* – *vb*. It is good if

there are many answers specifying positive evaluations *very good* and *good*, and if there are only few answers specifying negative evaluations *bad* and *very bad*. The interpretation of the evaluation *average* was *almost bad*; if we want most answers to be *very good* and *good*, we admit only a few answers to be *average*. Therefore, in this case $I = G \cup B$, $G = \{vg, g\}$, $B = \{a, b, vb\}$; the statistical distributions (percentage histograms) of answers were interpreted in the sense of multiple criteria optimization, with $i \in G = \{vg, g\}$ counted as positive outcomes (quality indicators that should be maximized) and $i \in B = \{a, b, vb\}$ counted as negative outcomes (quality indicators to be minimized).

A reference point approach (similar as described here, only using single reference point r) was proposed for this particular case of ranking probability distributions; other approaches are usually more complicated (see, e.g., [18]). However, when the dean of the School of Knowledge Science in JAIST, himself a well-known specialist in multiple criteria decision support, was asked to define his preferences or preferred aspiration levels, the reality of the managerial situation overcome his theoretical background: he responded “in this case, I want the ranking to be as objective as possible – I must discuss the results with the deans of other schools and with all professors”. This was the origin of reflection on objective versus subjective rational ranking.

Thus, a statistical average of the percentages of answers in the entire data set was taken as the reference distribution or profile. Since it was realized that such a reference profile might result in good but standard answers, some artificial reference distributions were also constructed as more demanding than the average one; averages over Pareto optimal options were not computed because of the complexity of the data set.

The reference distribution called *Average* above (r_D) represents the actual average of percentages of answers for all questions (of the first and second type) and all responders. This distribution might be taken as the basic one, because it results from the experimental data and might be considered as independent from the preferences of the decision maker, thus resulting in a ranking of questions that is as objective as possible – although, theoretically, average aspirations result only in average, not necessarily interesting answers (actually, this theoretical conclusion was later confirmed in practice). Truly interesting results might correspond to more demanding aspirations, hence beside the average distribution we postulated synthetic users and considered three more demanding ones, which were characterized by the types of neutral reference distributions. The one called *Regular* (r_A) was almost linearly decreasing; the one called *Stepwise* (r_C) was almost uniform for positive and for negative outcomes; while the one called *Demanding* (r_B) was almost hyperbolically decreasing and actually the most demanding (Table 2.3).

The detailed results of the survey were not only very interesting theoretically, but also very useful for university management, see [25]. It was found that seven questions of the first (assessment) type ranked as worst practically did not depend on the variants of reference distributions and ranking, on the schools or on the characteristics of respondents; thus, the objective ranking gave robust results as to the problems that required most urgent intervention by the university management. The

Table 2.3 Four different types of reference profile distributions

Name	Symbol	vg (%)	g (%)	a (%)	b (%)	vb (%)
Regular	r_A	36	28	20	12	4
Demanding	r_B	48	26	14	8	4
Stepwise	r_C	42	42	7	5	4
Average	r_D	21	38	22	14	5

best ranked questions of the second (importance) type were more changeable, only three of them consistently were ranked among the best ones in diverse ranking profiles. Moreover, a rank reversal phenomenon was observed: if the average reference distribution was used, best ranked were questions of rather obvious type, more interesting results were obtained when using more demanding reference profile. This rank reversal, however, influenced more the best ranked questions than worst ranked questions, more significant for university management.

In [25], the more demanding reference distributions or profiles were constructed by an arbitrary modification of the statistical average reference profile. However, can we construct them more objectively? The answer is positive, as shown in the preceding example, provided we have a good algorithm for finding all Pareto optimal (nondominated) options in a complex data set. In classical approaches, Pareto optimal points in complex data sets are found by envelope analysis using appropriate linear and mixed integer programming formulations. However, envelope analysis results only in the envelope – a convex hull of Pareto optimal points, while discrete alternative problems are known to possess many Pareto optimal points in the interior of the convex hull. Thus, EMO algorithms are a natural candidate to resolve the problem of a sufficiently fine approximation of the Pareto set (that can have many elements for complex data sets) to estimate well the averages of criteria values over Pareto set. Naturally, because of the discrete character of the problem, the genetic variant of the evolutionary algorithms should be also considered, see, e.g., [2].

2.7 Conclusions and Further Research

While absolute objectivity is known not to be attainable, postmodern sociology of science is wrong in reducing scientific objectivity to power and money: *we must transfer knowledge that is as objective as possible to future generations, because only this way we can help them in facing uncertain future*. In the same sense, we use *the concept of objective ranking* as such a ranking (including classification and selection of decision options) that is *not absolutely objective, but as objective as possible*.

We define here *objective ranking* as *dependent only on a given set of data, relevant for the decision situation, and independent of any more detailed specification of personal preferences than that given by defining criteria and the partial order in criterion space*.

Rational objective ranking can be based on reference point approach, because reference levels needed in this approach can be established statistically from the given data set.

Examples show that such objective ranking can be very useful in many management situations. A technical problem of finding objective but demanding reference levels can be solved by averaging over Pareto set and using, e.g., EMO algorithms for this purpose.

The concept of objective ranking opens many avenues of possible future research, such as: the use of equitable aggregation, see [13, 17] and the use of ordered weighted averaging (OWA, see [36]), both in objective ranking; possible extensions of rough set theory [8, 19] for objective ranking; multiobjective comparison of empirical statistical profiles [7], and many other possibilities.

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