

Chapter 2

The Positive Sequence Model of Symmetrical Lines

2.1 Introduction

In the power frequency steady state regimes of a power line, the positive sequence model is often assumed: this model is almost always acceptable for studies of balanced regimes of overhead lines (even if they are asymmetrical and with earth wires) whereas, in the case of single-core, unarmoured UGCs, it can be accepted only if cross-bonding with phase transpositions is performed in order to both zero the induced current circulation in the sheaths and symmetrize the system.

On the contrary for the Gas Insulated transmission Line the solid-bonding connection is required: the lowest value of the enclosure resistance allows the induced current in each enclosure to be almost equal in magnitude and opposite in angle to the corresponding phase current, so that a very good approximation consists in considering each phase as decoupled from the others (not only capacitively but also inductively), and hence in modelling the line at balanced regimes by means of the positive-sequence circuit.

After these remarks, it appears licit to turn to the classical treatment of the AC uniformly distributed parameter transmission lines.

However, it is worth remembering that also in the aforementioned cases, the determination of equivalent parameters to be introduced in the positive sequence model hold a given degree of uncertainty: the multiconductor analysis together with experimental results can be helpful to deepen, with more accuracy, the knowledge of the phenomena.

2.2 The Transmission Matrix of a Uniform Line

Let us consider a line at power frequency steady state regime as an uniform symmetrical transmission line: Figure 2.1 shows the single-phase model with distributed parameters at positive sequence, where each infinitesimal element dx is qualified by the four parameters (also called primary constants) r , ℓ , g , and c . Their values can

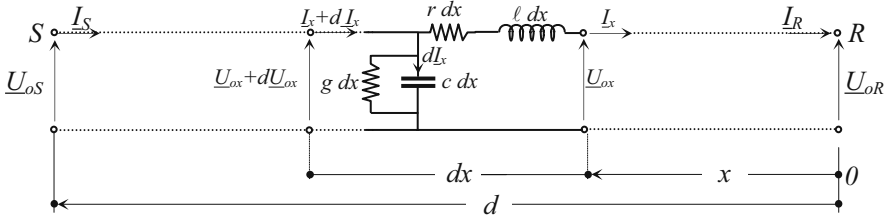


Figure 2.1 Single-phase model at positive sequence of a line

be computed after Sections 2.3.1–2.3.4 in case of cross-bonded cables with phase transpositions and after Sections 2.4.1–2.4.4 in case of solid-bonded GILs. For OHL the formulae are well-known.

The voltage and current references at the sending- and receiving-ends (both of the entire line and of the infinitesimal element) are also defined in Figure 2.1.

In the steady state regime each element of infinitesimal length dx has, at ω angular frequency, a longitudinal impedance and a shunt admittance given by:

$$\begin{aligned}\underline{z}dx &= (r + j\omega\ell)dx, \\ \underline{y}dx &= (g + j\omega c)dx;\end{aligned}\quad (2.1)$$

where $\underline{z}$ is the per unit length (or kilometric) longitudinal impedance expressed in Ω/km and $\underline{y}$ the per unit length (or kilometric) shunt admittance in S/km .

The voltage increase $d\underline{U}_{0x}$ between the two ports of the infinitesimal element dx (located at distance x [km] from the receiving-end R) is the voltage drop on the longitudinal impedance $\underline{z}dx$ so that

$$d\underline{U}_{0x} = \underline{I}_x \underline{z} dx. \quad (2.2)$$

The current variation $d\underline{I}_x$ between the two ports of the infinitesimal element is equal to the current derived from the infinitesimal shunt admittance

$$d\underline{I}_x = [\underline{U}_{0x} + d\underline{U}_{0x}] \cdot \underline{y} \cdot dx. \quad (2.3)$$

By considering that in (2.3) the term of superior order $d\underline{U}_{0x} \cdot \underline{y} dx$ can be neglected, (2.2) and (2.3) can be rewritten in the following form:

$$\frac{d\underline{U}_{0x}}{dx} = \underline{z} \underline{I}_x, \quad (2.4)$$

$$\frac{d\underline{I}_x}{dx} = \underline{y} \underline{U}_{0x}. \quad (2.5)$$

It is worth remembering that the steady state phasors $\underline{U}_{0x}$ and $\underline{I}_x$, rotating with constant angular frequency ω determine in the time domain (with their projections on a reference axis depending upon their phase displacement) the sinusoidal instantaneous values. Consequently it remains only to solve the system of differential equations composed of (2.4) and (2.5) as if the current and voltage phasors were complex functions of the sole spatial variable x .

By $\mathcal{L}$ -transforming ($\mathcal{L} = \text{Laplace}$) the complex functions $\underline{U}_{0x} = \underline{U}(x)$ and $\underline{I}_x = \underline{I}(x)$ of the x real variable and remembering the well-known transform of a function's derivative i.e.

$$\mathcal{L}\left\{\frac{df(x)}{dx}\right\} = s\mathcal{L}\{f(x)\} - f(0)$$

the following system arises

$$\begin{cases} s\mathcal{L}\{\underline{U}_{0x}\} - \underline{U}_{0R} = \underline{z} \cdot \mathcal{L}\{\underline{I}_x\} , \\ s\mathcal{L}\{\underline{I}_x\} - \underline{I}_R = \underline{y} \cdot \mathcal{L}\{\underline{U}_{0x}\} , \end{cases}$$

where (with the abscissa origin at receiving end) the obvious relations $\underline{U}_{0(x=0)} = \underline{U}_{0R}$ and $\underline{I}_{(x=0)} = \underline{I}_R$ have been applied.

Rearranging and reordering, it has

$$\begin{cases} s\mathcal{L}\{\underline{U}_{0x}\} - \underline{z}\mathcal{L}\{\underline{I}_x\} = \underline{U}_{0R} , \\ -\underline{y}\mathcal{L}\{\underline{U}_{0x}\} + s\mathcal{L}\{\underline{I}_x\} = \underline{I}_R , \end{cases}$$

from which it is immediate to obtain (by means of Cramer's rule)

$$\mathcal{L}\{\underline{U}_{0x}\} = \frac{s\underline{U}_{0R} + \underline{z}\underline{I}_R}{s^2 - \underline{z} \cdot \underline{y}} , \quad (2.6)$$

$$\mathcal{L}\{\underline{I}_x\} = \frac{s\underline{I}_R + \underline{y}\underline{U}_{0R}}{s^2 - \underline{z} \cdot \underline{y}} . \quad (2.7)$$

If the new quantities $\underline{k}$ and $\underline{Z}_c$ are defined

$$\underline{k} = \sqrt{\underline{z} \cdot \underline{y}} , \quad \underline{Z}_c = \sqrt{\frac{\underline{z}}{\underline{y}}} ,$$

so that:

$$\underline{z} = \underline{k} \underline{Z}_c , \quad \underline{y} = \frac{\underline{k}}{\underline{Z}_c} ,$$

it is possible to express (2.6) and (2.7) in their final form:

$$\mathcal{L}\{\underline{U}_{0x}\} = \frac{s\underline{U}_{0R} + \underline{k} \underline{Z}_c \underline{I}_R}{s^2 - \underline{k}^2} , \quad (2.8)$$

$$\mathcal{L}\{\underline{I}_x\} = \frac{s\underline{I}_R + \frac{\underline{k}}{\underline{Z}_c} \underline{U}_{0R}}{s^2 - \underline{k}^2} . \quad (2.9)$$

By implementing the inverse Laplace transform of them, the current and voltage phasors as a function of x are expressed by

$$\begin{cases} \underline{U}_{0x} = \underline{A}_x \cdot \underline{U}_{0R} + \underline{B}_x \cdot \underline{I}_R , \\ \underline{I}_x = \underline{C}_x \cdot \underline{U}_{0R} + \underline{D}_x \cdot \underline{I}_R , \end{cases} \quad (2.10)$$

$$(2.11)$$

where

$$\begin{aligned} \underline{A}_x &= \cosh(\underline{k}x) ; & \underline{B}_x &= \underline{Z}_c \cdot \sinh(\underline{k}x) ; \\ \underline{C}_x &= [\sinh(\underline{k}x)]/\underline{Z}_c ; & \underline{D}_x &= \cosh(\underline{k}x) , \end{aligned} \quad (2.12)$$

x	[km]	distance to the receiving end of the line;
$\underline{k} = \sqrt{\underline{z} \cdot \underline{y}}$	[km ⁻¹]	propagation factor;
$\underline{Z}_c = \sqrt{\frac{\underline{z}}{\underline{y}}}$	[Ω]	characteristic or natural impedance.

If the hyperbolic functions are expressed as exponential ones (by using the Euler–Lambert relations) it is possible to obtain the transmission formulae (solution of the telegraphist's equations) which were first written in 1876 by O. Heaviside [1] and then used, rearranged and divulged by Arnold [2], Rössler [3], Steinmetz [4] et al.

If $x = d$, it yields

$$\underline{U}_{0(x=d)} = \underline{U}_{0S} \quad \text{and} \quad \underline{I}_{x=d} = \underline{I}_S$$

so that

$$\underline{U}_{0S} = \underline{A} \cdot \underline{U}_{0R} + \underline{B} \cdot \underline{I}_R , \quad (2.13)$$

$$\underline{I}_S = \underline{C} \cdot \underline{U}_{0R} + \underline{D} \cdot \underline{I}_R , \quad (2.14)$$

where

$$\begin{aligned} \underline{A} &= \cosh(\underline{k}d) ; & \underline{B} &= \underline{Z}_c \cdot \sinh(\underline{k}d) ; \\ \underline{C} &= [\sinh(\underline{k}d)]/\underline{Z}_c ; & \underline{D} &= \cosh(\underline{k}d) . \end{aligned} \quad (2.15)$$

The single-phase model of the line (of length d) can therefore be considered as a two-port network which is characterized by means of $\underline{A}$, $\underline{B}$, $\underline{C}$, $\underline{D}$ parameters, also called as transmission hybrid parameters since they have different units (and different circuital meanings) as highlighted in any power system analysis book [5, 6].

Since the line has uniformly distributed parameters, the two-port network results symmetric so that:

$$\underline{A} = \underline{D} .$$

Equations 2.13 and 2.14 can be rearranged in the concise elegant matrix form, where $\underline{M}$ is often named transmission matrix,

$$\underbrace{\begin{bmatrix} \underline{U}_{0S} \\ \underline{I}_S \end{bmatrix}}_{\underline{w}_S} = \underbrace{\begin{bmatrix} \underline{A} & \underline{B} \\ \underline{C} & \underline{D} \end{bmatrix}}_{\underline{M}} \cdot \underbrace{\begin{bmatrix} \underline{U}_{0R} \\ \underline{I}_R \end{bmatrix}}_{\underline{w}_R} \quad (2.16)$$

which allows obtaining the hybrid vector $\underline{w}_S$ at sending-end once fixed the hybrid vector $\underline{w}_R$ at receiving-end.

The relations (2.15) allow verifying that

$$\det \underline{\underline{M}} = \underline{\underline{A}} \underline{\underline{D}} - \underline{\underline{B}} \underline{\underline{C}} = [\cosh(kd)]^2 - [\sinh(kd)]^2 = 1, \quad (2.17)$$

so confirming that the two-port network constituted by the line obviously satisfies the reciprocity principle.

2.3 Computation of Single-Core Cable Kilometric Parameters

As already stated and remarked in Sections 2.1 and 2.2, the computation of AC primary parameters is referred to a cross-bonded (with phase transpositions) single-core unarmoured cable.

2.3.1 Computation of r (Cable)

It is important to reaffirm that with the hypothesized perfect cross-bonding the induced sheath currents are zeroed and that, as is well-known, the alternating current Joule losses in the phase conductor are (due to the skin and proximity effects) higher than those in direct current.

This fact brings to consider for the phase conductor a kilometric resistance r_{ac} greater than that in direct current $r_{dc\vartheta^\circ}$ as shown in [7](IEC 60287):

$$r_{ac} = r_{dc\vartheta^\circ} \cdot (1 + y_s + y_p) \quad [\Omega/\text{km}], \quad (2.18)$$

where:

- $y_s \Rightarrow$ skin effect factor;
- $y_p \Rightarrow$ proximity effect factor;
- $r_{dc\vartheta^\circ} \Rightarrow$ DC resistance computed at the maximum operating conductor temperature ϑ°
- $r_{ac} \Rightarrow$ kilometric cable resistance at maximum operating temperature ϑ° .

Table 2.1 Permissible conductor temperatures for HV and EHV cables [8]

Cable insulating material	Maximum operating temperature [$^\circ\text{C}$]	Short circuit temperature [$^\circ\text{C}$]
Impregnated paper	85–90	160–180
Low-density polyethylene – LDPE	70	150
High-density polyethylene – HDPE	80	180
Cross-linked polyethylene – XLPE	90	250
Ethylene Propylen Rubber – EPR	90	250

Table 2.2 Electrical resistivity at 20 °C and temperature coefficients of materials used in cable construction [7]

Material used in	Resistivity $\rho_{20^\circ} \Omega \text{ m at } 20^\circ \text{C}$	Temperature coefficient α
Conductors		
Copper	1.7241×10^{-8}	3.93×10^{-3}
Aluminium	2.8264×10^{-8}	4.03×10^{-3}
Sheaths and armours		
Lead or lead alloy	21.4×10^{-8}	4.0×10^{-3}
Steel	13.8×10^{-8}	4.5×10^{-3}
Bronze	3.5×10^{-8}	3.0×10^{-3}
Stainless steel	70×10^{-8}	Negligible
Aluminium	2.84×10^{-8}	4.03×10^{-3}

Usually it is conservative to assume that ϑ° is the maximum operating conductor temperature [8] compatible with the insulating materials (see Table 2.1). It should be noted that XLPE and EPR are the most interesting dielectrics.

For the computation of $r_{dc\vartheta^\circ}$ the following well-known formula can be used starting from the kilometric resistance r_{20° at 20 °C

$$r_{dc\vartheta^\circ} = r_{20^\circ} \cdot [1 + \alpha \cdot (\vartheta^\circ - 20^\circ)] \quad [\Omega/\text{km}] , \quad (2.19)$$

where:

$\alpha \Rightarrow$ temperature coefficient $1/^\circ\text{C}$;

$\vartheta^\circ \Rightarrow$ maximum operating conductor temperature.

With regard to the resistivities and temperature coefficients of the metals used in cable construction, Table 2.2 sums up the values [7].

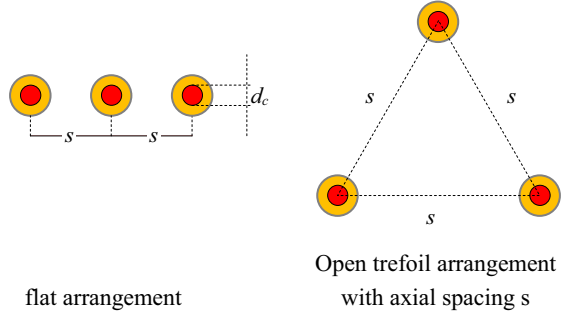
The computation of y_s and y_p must be performed after the detailed formulae of IEC 60287 [7].

With regard to the large cross section conductors, it is worth remembering a segmental conductor design (Milliken H. Electrical Cable. Patent Publication 1933, US 1904162), which is commonly referred to as a Milliken conductor: it has insulated segments consisted of layers of wires (more or less insulated each other) stranded around the segment “centre” aiming at reducing the magnitude of skin and proximity effects [9, 10].

Nowadays, cross sections up to 2500 mm^2 are customarily used and this topic continues to be reviewed [11] and developed so that any formula must be used with due caution (possibly supported by experimental validations of manufacturer).

2.3.2 Computation of ℓ (Cable)

In the hypothesis that the current distribution is uniform in the whole cross-section of the phase conductor and that the sheath currents are zeroed by the cross-bonding,

Figure 2.2 Single-core cable laying arrangements

the kilometric inductance (to be given to each phase) can be computed by means of

$$\ell = 0.2 \ln \left(\frac{K \cdot s}{GMR} \right) \quad [\text{mH/km}] , \quad (2.20)$$

where:

$s \Rightarrow$ axial spacing (mm); (see Figure 2.2);

$K \Rightarrow 1$ for trefoil arrangement;

$K \Rightarrow 1.26$ for flat laying (this value can be easily confirmed by means of the Geometrical Mean Distance among conductor centres

$GMD = \sqrt[3]{s_{12} \cdot s_{23} \cdot s_{13}} = \sqrt[3]{s \cdot s \cdot 2s} = 1.26 \cdot s$ when the cross-bonding is performed with phase transpositions);

$GMR \Rightarrow$ geometrical mean radius (mm) of the phase conductor.

Usually in EHV XLPE-cables the conductor is hollow Milliken type; for GMR , see [12], Vol. II, page 155.

It is worth noting that the axial spacing can be reduced to a minimum (i.e. touching trefoil) but with a decreased ampacity.

2.3.3 Computation of c (Cable)

The values of the single-core cable kilometric capacitance c (between the phase conductor and the metallic sheath) can be computed by considering the cylindrical symmetry, so that (with $\varepsilon_0 = 8.8542 \cdot 10^{-12} \text{F/m}$)

$$(c) = \frac{2\pi \varepsilon_0 \varepsilon_r}{\ln \left(\frac{d_0}{d_1} \right)} \quad [\text{F/m}] \Rightarrow c = \frac{\varepsilon_r}{18 \ln \left(\frac{d_0}{d_1} \right)} \quad [\mu\text{F/km}] , \quad (2.21)$$

where:

- $\varepsilon_r \Rightarrow$ relative dielectric constant of the insulating material;
- $d_0 \Rightarrow$ outer diameter of insulating medium excluding semi conductive screen or layer;
- $d_1 \Rightarrow$ conductor diameter including semi conductive screen or layer, if any.

2.3.4 Computation of g (Cable)

The shunt conductance g (S/km) is the parameter taking into account the active power “dielectric” losses $g \cdot U_0^2$ (per kilometre) given for each phase by the electric field in the insulating material [10, 13].

It is worth highlighting that the circuital element given to it (conductive shunting of capacity as in Figure 2.1) allows evaluating correctly the dielectric losses, but it not constitutes a realistic modelling in order to qualify the complex physical phenomena which are the fundamental cause and are referable to a kind of hysteresis in the dielectric polarization given by the alternating electric field at angular frequency ω : only in a very small part the losses are referable to a proper Joule effect due to imperfect insulation, also present in DC.

The shunt conductance can be computed with

$$g = b \cdot \tan \delta , \quad (2.22)$$

where

- $\tan \delta \Rightarrow$ loss factor of insulating material (see Table 2.3),
- $b = \omega c$ [S/km] $\Rightarrow$ the per unit length capacitive susceptance of the cable,

so highlighting active power losses proportional to capacitive power absorbed along the cable and hence proportional to the frequency and to the square of the voltage.

The total (in the three phases) dielectric losses are given by

$$w_{\text{dTOT}} = 3 \cdot g \cdot U_0^2 = g \cdot U_n^2 \quad [\text{W/km}]$$

where U_n [V] is the rated phase-to-phase voltage (see Table 6.4).

The dielectric losses of HV and EHV cables ought to be always accounted for whilst they can be neglected for low and medium voltage since they are very small in comparison with the Joule losses in the longitudinal phase resistance.

In any case, the IEC standard 60287-1-1 states that the dielectric losses should be taken into account for values of the phase-to-ground voltage U_0 equal to or greater than those of the Table 2.4.

Table 2.3 Values of relative permittivity ϵ_r and loss factor $\tan \delta$ at 50–60 Hz (see Section 2.3.4), for different insulating materials after [7]

Type of cable	ϵ_r	$\tan \delta$
<i>Cables insulated with impregnated paper</i>		
Solid-type, fully impregnated, preimpregnated or mass-impregnated nondraining	4	0.01
Oil-filled, self-contained		
up to $U_0 = 36$ kV	3.6	0.0035
up to $U_0 = 87$ kV	3.6	0.0033
up to $U_0 = 160$ kV	3.5	0.0030
up to $U_0 = 220$ kV	3.5	0.0028
Oil-pressure, pipe-type	3.7	0.0045
Internal gas-pressure	3.4	0.0045
External gas-pressure	3.6	0.0040
<i>Cables with other types of insulation</i>		
Butyl rubber	4	0.050
EPR – up to and including 18/30 kV	3	0.020
EPR – above 18/30 kV	3	0.005
PVC	8	0.1
PE (HD and LD)	2.3	0.001
XLPE up to and including 18/30(36) kV – unfilled	2.5	0.004
XLPE above 18/30(36) kV – unfilled	2.5	0.001
XLPE above 18/30(36) kV – filled	3	0.005
Paper–polypropylene–paper (PPL) $\geq 63/110$ kV	2.8	0.0014

Table 2.4 Values of phase-to-ground voltage above (or equal to) which the dielectric losses should be considered [7]

Type of cable	U_0 [kV]
<i>Cables insulated with impregnated paper</i>	
Solid-type	38
Oil-filled and gas-pressure	63.5
<i>Cables with other types of insulation</i>	
Butyl rubber	18
EPR	63.5
PVC	6
PE (HD and LD)	127
XLPE (unfilled)	127
XLPE (filled)	63.5

2.4 Computation of GIL Kilometric Parameters

It has been already premised in Section 2.1 that the single-phase model at the positive sequence of a GIL with solid-bonding connection represents a good approximation: if the enclosures of a GIL (which are always very low resistive) are solid-bonded, they allow the circulation of induced currents almost equal (in magnitude)

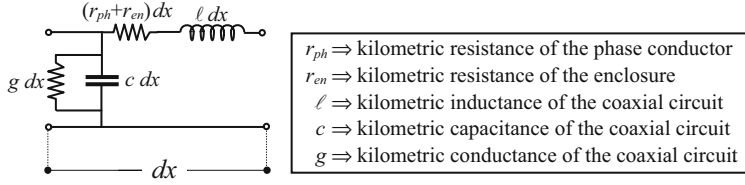


Figure 2.3 Single-phase model of a GIL element dx at positive sequence

to those of phases and opposite in angle [14–17], so that they can be considered as return paths of each current in simple single phase circuits electromagnetically decoupled, having parameters computable on the basis of typical coaxial configurations (see Figure 2.3).

2.4.1 Computation of GIL Apparent Kilometric Resistance r

It is plain that, with the aforementioned hypothesis for the phase and enclosure current phasors, the total Joule losses must be computed by fixing (in the model of Figure 2.3) for the phase conductor an apparent kilometric resistance r equal to the sum of phase resistance r_{ph} and enclosure resistance r_{en}

$$r = r_{ph} + r_{en} \quad [\Omega/\text{km}] . \quad (2.23)$$

It is worth mentioning that the thicknesses of the pipes and the used spacing render negligible the skin and proximity effects at power frequency [16]: so that r_{ph} and r_{en} can be evaluated in DC. The value of phase resistance r_{ph} is given by:

$$r_{ph} = \frac{\rho_{ph}}{S_{ph}} \quad [\Omega/\text{km}] , \quad (2.24)$$

where ρ_{ph} is expressed in $[\Omega \text{ mm}^2/\text{km}]$ and the cross-sectional area is

$$S_{ph} = \pi \cdot (R_2^2 - R_1^2) \quad [\text{mm}^2] \quad (2.25)$$

with R_2 and R_1 the outer and inner radii of phase conductor expressed in millimetres (see Figure 2.4).

Data concerning the involved materials and the resistivities can be obtained by Tables 3.4 and 3.5 or by more precise “manufacturer’s” information.

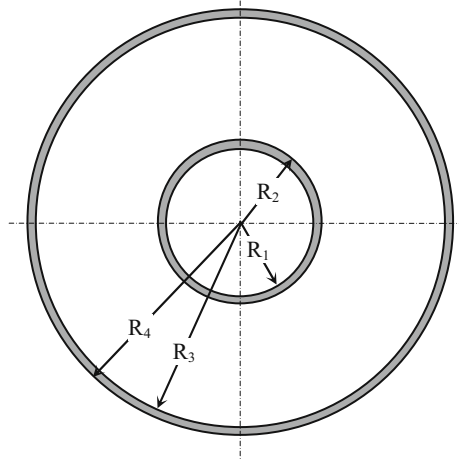
Analogously for the computation of r_{en} :

$$r_{en} = \frac{\rho_{en}}{S_{en}} \quad [\Omega/\text{km}] , \quad (2.26)$$

$$S_{en} = \pi \cdot (R_4^2 - R_3^2) \quad [\text{mm}^2] , \quad (2.27)$$

where R_4 and R_3 are the outer and inner radii of the enclosure expressed in mm.

Figure 2.4 One phase and corresponding enclosure of a GIL



The resistivities ρ_{ph} and ρ_{en} of material ($\Omega \text{ mm}^2/\text{km}$) must be evaluated at the operating temperature: the maximum allowable ones are stated in IEC 61640 [18].

2.4.2 Computation of GIL Kilometric Inductance ℓ

By considering the typical coaxial structure of a GIL (see sketch in Figure 2.4) and by adopting the above-mentioned solid-bonding technique, if it is assumed that opposite currents with magnitude I flow in the phase conductor and in the enclosure with uniform distribution (no skin and proximity effects), it is possible to compute the kilometric inductance as the sum of three terms $\ell = \ell_a + \ell_b + \ell_c$ (the first represents the phase internal inductance, the second the inductance due to the magnetic field between phase and enclosure, the third the enclosure internal inductance); it is also simple to ascertain that the value of ℓ_b is prevalent among the others and that, for the assumed hypothesis, the magnetic field external to the enclosure is zeroed.

Since (see Figure 2.4) R_1 , R_2 are the phase inner and outer radii respectively and R_3 , R_4 the enclosure inner and outer radii respectively, it is easy, by integrating the magnetic field between R_2 and R_3 , to express the inductance ℓ_b through

$$\ell_b = \frac{\mu_0}{2\pi} \ln \frac{R_3}{R_2} \quad [\text{H/m}] , \quad (2.28)$$

where:

$$\mu_0 \Rightarrow \text{magnetic permeability of the free space} = 4\pi \times 10^{-7} \text{ H/m.}$$

In the usual installations, it is $\ln \left(\frac{R_3}{R_2} \right) \cong 1$ in order to obtain, as is well-known, an optimal configuration of the electric field.

In order to obtain the expression of ℓ_a it is convenient to premise that (in accordance with electromagnetic theory) the following relation can be written

$$en_a = \int [(\mu_0 H^2)/2] dv = \ell_a I^2/2 ; \quad (2.29)$$

it correlates the magnetic energy en_a , expressed by the integral computed in the whole volume of the phase conductor (of 1 m length), with the corresponding quantity $\ell_a I^2/2$.

Once computed en_a , it yields immediately

$$\ell_a = 2en_a/I^2 . \quad (2.30)$$

It is sufficient to observe that the magnitude of H_x in the integration interval R_1 – R_2 can be written (since the circular symmetry and the hypothesis of uniform current density σ_{ph} in the phase) by means of

$$H_x = \frac{\sigma_{ph} \cdot \pi \cdot (x^2 - R_1^2)}{2\pi x} ; \quad \sigma_{ph} = \frac{I}{\pi (R_2^2 - R_1^2)} \quad (R_1 \leq x \leq R_2) .$$

Analogously, the computation of ℓ_c can be performed by writing for the magnitude of H_x in the integration interval R_3 – R_4 the two relations (note that the current $-I$ flows in the enclosure)

$$H_x = \frac{I}{2\pi x} + \frac{\sigma_{en} \cdot \pi \cdot (x^2 - R_3^2)}{2\pi x} ; \quad \sigma_{en} = -\frac{I}{\pi (R_4^2 - R_3^2)} \quad (R_3 \leq x \leq R_4) .$$

By developing the above-mentioned procedure, the following formulae (see also [19, 20]) can be achieved

$$\ell_a = \frac{\mu_0}{2 \cdot \pi} \cdot \left[\frac{R_1^4}{(R_2^2 - R_1^2)^2} \ln \left(\frac{R_2}{R_1} \right) + \frac{(R_2^2 - 3R_1^2)}{4(R_2^2 - R_1^2)} \right] \quad (\text{H/m}) , \quad (2.31)$$

$$\ell_c = \frac{\mu_0}{2 \cdot \pi} \cdot \left[\frac{R_4^4}{(R_4^2 - R_3^2)^2} \ln \left(\frac{R_4}{R_3} \right) + \frac{(R_3^2 - 3R_4^2)}{4(R_4^2 - R_3^2)} \right] \quad (\text{H/m}) . \quad (2.32)$$

In order to have some orders of magnitude, it seems useful this example. Once the following values are assumed:

$$R_1 = 80 \text{ mm} , \quad R_2 = 90 \text{ mm} , \quad R_3 = 250 \text{ mm} , \quad R_4 = 260 \text{ mm} ,$$

by means of (2.31), (2.28) and (2.32) and by converting from H/m to mH/km the following values yield:

$$\begin{aligned} \ell_a &= 0.0074 , \quad \ell_b = 0.20433 , \quad \ell_c = 0.0026 \text{ mH/km} , \\ \ell &= \ell_a + \ell_b + \ell_c = 0.2144 \text{ mH/km} , \end{aligned}$$

where it appears that ℓ_b is the most important one and, as first approach, it always gives sufficient accuracy.

2.4.3 The Computation of GIL Kilometric Capacitance c

The value of capacitance can be easily computed considering the cylindrical symmetry and that the permittivity of insulating gas mixture is $\varepsilon_0 = 8.8542 \times 10^{-12}$ F/m (as for the free space):

$$(c) = \frac{2\pi\varepsilon_0}{\ln\left(\frac{R_3}{R_2}\right)} \quad [\text{F/m}] \quad \Rightarrow \quad c = \frac{1}{18 \cdot \ln\left(\frac{R_3}{R_2}\right)} \quad [\mu\text{F/km}] . \quad (2.33)$$

2.4.4 Computation of GIL Kilometric Shunt Conductance g

It is worth noting that the dielectric used in the GIL (pure pressurized SF₆ or a mixture of it with N₂, at about 7 bar for $U = 380$ kV) has always a very low loss factor and gives losses that result absolutely negligible also if compared with the small Joule longitudinal losses due to the low apparent resistance r expressed by (2.23).

Also in precise economic evaluations of losses, it is always assumed $g = 0$.

2.5 Some Other Matrix Relations Deriving from the Fundamental One

In the transmission equations (2.13) and (2.14), there are four variables: the voltage and current phasors at sending and receiving ends.

When two of the four variables are fixed, the remaining ones are univocally determined.

Consequently, in this paragraph, different ways of using the transmission equations are highlighted in order to both give the reader a test-bed for getting some practice and have suitable formulae for Chapters 3 and 4.

As a first example (2.34), as much typical as (2.16), can be derived by rearranging (2.13) and (2.14): it expresses the vector $\underline{w}_R$ (hybrid), composed of the voltage and current at receiving-end, as a function of the vector $\underline{w}_S$ at sending-end:

$$\underbrace{\begin{bmatrix} U_{0R} \\ I_R \end{bmatrix}}_{\underline{w}_R} = \underbrace{\begin{bmatrix} D & -B \\ -C & A \end{bmatrix}}_{\underline{M}_a} \cdot \underbrace{\begin{bmatrix} U_{0S} \\ I_S \end{bmatrix}}_{\underline{w}_S}; \quad \underline{M}_a = \underline{M}^{-1} . \quad (2.34)$$

It is easy to demonstrate that $\underline{M}_a$ is the inverse of $\underline{M}$ (or that the product of $\underline{M}$ and $\underline{M}_a$ is the identity matrix), by remembering that $\underline{A}\underline{D} - \underline{B}\underline{C} = 1$ in virtue of the reciprocity principle (see (2.17)).

As a second example, if the sending-end voltage phasor and the receiving-end current phasor are fixed, it is easy to obtain the matrix relation (with $\underline{D} \equiv \underline{A}$)

$$\begin{bmatrix} U_{0R} \\ I_S \end{bmatrix} = \underbrace{\begin{bmatrix} 1/\underline{A} & -B/\underline{A} \\ C/\underline{A} & 1/\underline{A} \end{bmatrix}}_{\underline{M}_b} \cdot \begin{bmatrix} U_{0S} \\ I_R \end{bmatrix} \quad (2.35)$$

which allows determining the other two phasors $\underline{U}_{0R}$ and $\underline{I}_S$.

As it will be used in Chapter 3, in order to know the variables along the transmission line, it is rather immediate to achieve from (2.34):

$$\underbrace{\begin{bmatrix} \underline{U}_{0x} \\ \underline{I}_x \end{bmatrix}}_{\underline{w}_x} = \underbrace{\begin{bmatrix} \underline{D}_x & -\underline{B}_x \\ -\underline{C}_x & \underline{A}_x \end{bmatrix}}_{\underline{M}_x} \cdot \underbrace{\begin{bmatrix} \underline{U}_{0S} \\ \underline{I}_S \end{bmatrix}}_{\underline{w}_S} \quad (2.36)$$

which allows calculating the vector $\underline{w}_x$ along the cable (at a distance x from S) having $\underline{A}_x \dots \underline{D}_x$ obvious meaning (once known the vector $\underline{w}_S$).

It will be often sufficient to compute only the values of voltage $\underline{U}_{0T}$ and current $\underline{I}_T$ at line midpoint T with (2.37)

$$\underbrace{\begin{bmatrix} \underline{U}_{0T} \\ \underline{I}_T \end{bmatrix}}_{\underline{w}_T} = \underbrace{\begin{bmatrix} \underline{D}_T & -\underline{B}_T \\ -\underline{C}_T & \underline{A}_T \end{bmatrix}}_{\underline{M}_T} \cdot \underbrace{\begin{bmatrix} \underline{U}_{0S} \\ \underline{I}_S \end{bmatrix}}_{\underline{w}_S} \quad (2.37)$$

by giving x the distance of T from S .

In Chapter 4 other relations, besides those of this chapter, will be developed bringing again reasons to appreciate the matrix algorithms [21, 22].

Furthermore, it is worth remembering the matrix relation:

$$\begin{bmatrix} \underline{U}_{0S} \\ \underline{U}_{0R} \end{bmatrix} = \underbrace{\begin{bmatrix} \underline{A}/\underline{C} & -1/\underline{C} \\ 1/\underline{C} & -\underline{D}/\underline{C} \end{bmatrix}}_{\underline{Z}} \cdot \begin{bmatrix} \underline{I}_S \\ \underline{I}_R \end{bmatrix} \quad (2.38)$$

which involves an impedance matrix $\underline{Z}$ (where for the current $\underline{I}_R$ at port R as well the same phasor references of Figure 2.1 are held).

In order to deduce the matrix relation in (2.38), (2.16) can be made explicit so that, by setting $\underline{I}_R = 0$:

$$\frac{\underline{U}_S}{\underline{I}_S} = \frac{\underline{A}}{\underline{C}} = \underline{Z}_{SS} \quad \text{and} \quad \frac{\underline{U}_R}{\underline{I}_S} = \frac{1}{\underline{C}} = \underline{Z}_{RS}.$$

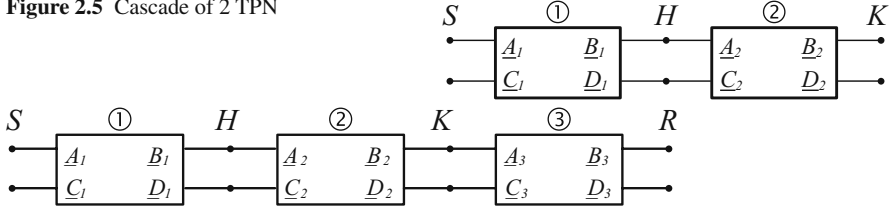
By making (2.34) explicit, with $\underline{I}_S = 0$, it easily yields:

$$\frac{\underline{U}_R}{\underline{I}_R} = -\frac{\underline{D}}{\underline{C}} = \underline{Z}_{RR} \quad \text{and} \quad \frac{\underline{U}_S}{\underline{I}_R} = -\frac{1}{\underline{C}} = \underline{Z}_{SR}.$$

2.6 Cascade Connections of Two Port Networks (TPN)

Let us suppose to have the cascade connections of Figures 2.5 and 2.6.

As is well-known, the resulting transmission matrix is given by the product of the transmission matrices.

Figure 2.5 Cascade of 2 TPN**Figure 2.6** Cascade of 3 TPN

For each section ①, ② and ③ the corresponding transmission matrices can be calculated as already thoroughly explained in the previous sections:

$$\begin{matrix} \begin{matrix} \underline{A}_1 & \underline{B}_1 \\ \underline{C}_1 & \underline{D}_1 \end{matrix} \\ \underline{M}_1 \end{matrix}; \begin{matrix} \begin{matrix} \underline{A}_2 & \underline{B}_2 \\ \underline{C}_2 & \underline{D}_2 \end{matrix} \\ \underline{M}_2 \end{matrix}; \begin{matrix} \begin{matrix} \underline{A}_3 & \underline{B}_3 \\ \underline{C}_3 & \underline{D}_3 \end{matrix} \\ \underline{M}_3 \end{matrix}. \quad (2.39)$$

The equivalent cascade matrices are given (with evident symbols)

$$\underline{M}_{12} = \underline{M}_1 \underline{M}_2 = \begin{matrix} \begin{matrix} \underline{A}_{12} & \underline{B}_{12} \\ \underline{C}_{12} & \underline{D}_{12} \end{matrix} \end{matrix}; \underline{M}_{123} = \underline{M}_1 \underline{M}_2 \underline{M}_3 = \begin{matrix} \begin{matrix} \underline{A}_{123} & \underline{B}_{123} \\ \underline{C}_{123} & \underline{D}_{123} \end{matrix} \end{matrix}. \quad (2.40)$$

For example, by considering the ports in Figure 2.6, the following expressions appear immediately understandable:

$$\begin{aligned} \underline{w}_S &= \underline{M}_1 \cdot \underline{w}_H; & \underline{w}_R &= \underline{M}_3^{-1} \cdot \underline{w}_K; \\ \underline{w}_S &= \underline{M}_{12} \cdot \underline{w}_K; & \underline{w}_H &= \underline{M}_{23} \cdot \underline{w}_R; \\ \underline{w}_S &= \underline{M}_{123} \cdot \underline{w}_R; & \underline{w}_R &= \underline{M}_{123}^{-1} \cdot \underline{w}_S. \end{aligned}$$

It is worth noting that generally even if $\underline{D}_1 \equiv \underline{A}_1$, $\underline{D}_2 \equiv \underline{A}_2$, $\underline{D}_3 \equiv \underline{A}_3$ (see (2.15)), the same is not necessarily valid in a cascade matrix where for example it is (but in particular cases) $\underline{D}_{12} \neq \underline{A}_{12}$ and $\underline{D}_{123} \neq \underline{A}_{123}$.

In any case, as long as non-reciprocal two port circuits are introduced (e.g. a phase shifter two-port), relations such as $\underline{A}_{123} \cdot \underline{D}_{123} - \underline{B}_{123} \cdot \underline{C}_{123} = 1$ remain still valid.

2.7 Parallel Connection of Equal Two Port Circuits Thermally and Electrically Decoupled

In case of a transmission link of a double circuit of equal cables in electric parallel (a situation that can be useful to gain a great power capability and enhance the reliability at the same time), the equivalent matrix can be easily deduced on the

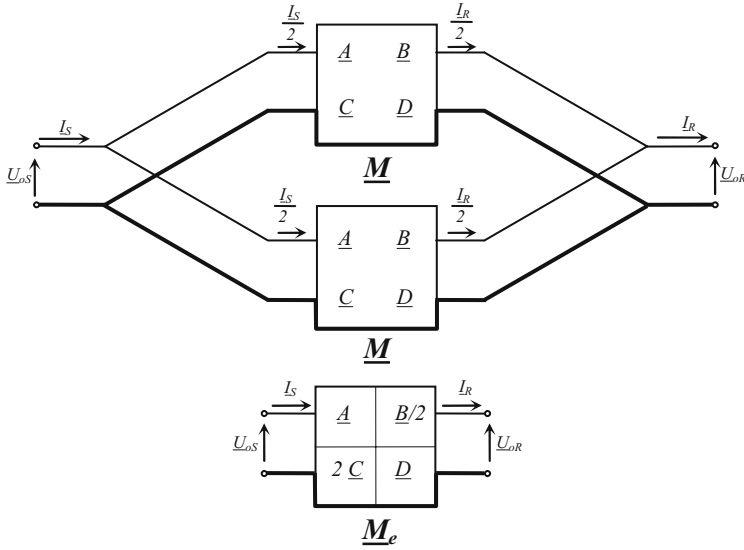


Figure 2.7 The parallel of two, perfectly decoupled, equal circuits

basis of the obvious equations in (2.41) valid for each of the two circuits (which are supposed to be thermally and electrically decoupled) with equal sharing of the currents shown in the scheme of Figure 2.7. Of course, it is true if the port voltages have the same reference (thick links in the sketch) as it is always valid in the positive and negative sequence models.

Therefore it is possible to determine the equations in (2.42) and hence the equivalent transmission matrix $\underline{M}_e$ of the double circuit, once computed the single circuit matrix $\underline{M}$,

$$\begin{cases} \underline{U}_{0S} = (\underline{A}) \cdot \underline{U}_{0R} + (\underline{B}) \cdot \frac{\underline{I}_R}{2}, \\ \frac{\underline{I}_S}{2} = (\underline{C}) \cdot \underline{U}_{0R} + (\underline{D}) \cdot \frac{\underline{I}_R}{2}, \end{cases} \quad (2.41)$$

$$\begin{cases} \underline{U}_{0S} = (\underline{A}) \cdot \underline{U}_{0R} + \left(\frac{\underline{B}}{2}\right) \cdot \underline{I}_R, \\ \underline{I}_S = (2\underline{C}) \cdot \underline{U}_{0R} + (\underline{D}) \cdot \underline{I}_R. \end{cases} \quad (2.42)$$

$$\begin{bmatrix} \underline{A} & \underline{B} \\ \underline{C} & \underline{D} \end{bmatrix}; \quad \begin{bmatrix} \underline{A} & \underline{B}/2 \\ 2\underline{C} & \underline{D} \end{bmatrix}.$$

$\underline{M}$ $\underline{M}_e$

It seems interesting to note that the same result can be achieved (following a more intuitive procedure) by giving the double circuit the kilometric parameters $\underline{z}/2$ and $2\underline{y}$ (since $\underline{z}$ and $\underline{y}$ are those of the single circuit) so having the same propagation factor and the halving of the characteristic impedance.

2.8 The Shunt Reactive Compensation

It is known that the kilometric cable capacitive susceptance has often a so high value to determine (chiefly with great lengths, voltages and cross-sections) steady state and transient regimes which could become very severe or even unacceptable (as it will be shown in Chapters 3 and 4), unless a suitable shunt compensation is adopted.

Usually the cable link modelling must be completed with the insertion of the necessary single-phase shunt reactors at the two cable ends and in case (depending upon the link length) also in some other intermediate locations (lumped compensation).

By considering for example the circuital scheme of Figure 2.8, the steady state regime at angular frequency ω of a short link compensated only at the ends can be studied by means of the matrix cascade $\underline{M}_\xi \cdot \underline{M} \cdot \underline{M}_\xi$, being that $\underline{M}$ is the cable transmission matrix and $\underline{M}_\xi$ is the transmission matrix which qualifies the behaviour of a shunt reactor with inductive susceptance equal to $\underline{Y}_\xi$ (as it will be shown in Section 2.8.2); it remains a useful basis also for the switching transients studies, since the subtransient regime at the same angular frequency ω (e.g. Sections 3.7 and 4.7) has a precise “indicial” role for the transient evaluations.

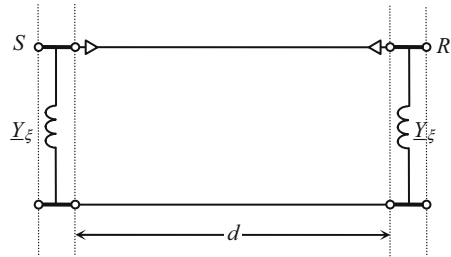


Figure 2.8 Lumped shunt compensation of short UGC ($d < 20\text{--}25$ km)

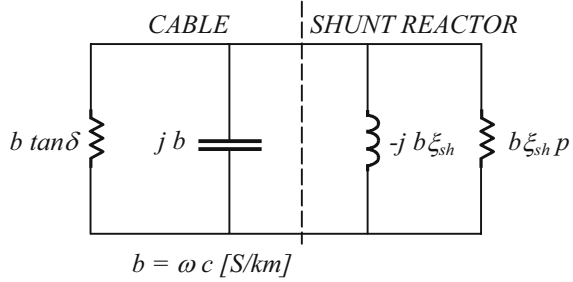
By considering that the lumped shunt reactors (located at short mutual distances $< 20\text{--}25$ km) can be numerous and can render the diagrams and the analytical relations heavy, it appears to be more efficient and meaningful, as a first approach, to imagine alternatively a uniformly distributed shunt compensation (see Section 2.8.1) in a way consistent with the uniform distribution of the cable capacitance: it allows easily computing (and with sufficient accuracy) the necessary shunt compensation degree and having an equivalent transmission matrix which, even if ideal, is extremely useful.

2.8.1 The Uniformly Distributed Compensation

The compensation of the uniformly distributed cable capacitive susceptance would theoretically require the insertion of uniformly distributed inductive susceptance.

Since this is hard to imagine constructively, it is possible to think to insert, in each phase and at each kilometre, a reactor of inductive susceptance of finite value

Figure 2.9 Kilometric shunt branch in the hypothesis of uniformly distributed shunt compensation



equal to $-j \xi_{sh} \omega c = -j \xi_{sh} b$ (see Figure 2.9), which compensates at each kilometre a fraction ξ_{sh} of the cable capacitive susceptance, bringing to compute the resulting kilometric susceptance $\underline{y}$ after (2.43)

$$\underline{y} = b \cdot \tan \delta + b \cdot \xi_{sh} \cdot p + j b \cdot (1 - \xi_{sh}) ; \quad (2.43)$$

where the reactor “loss factor” p (which allows considering the reactor power losses, always very low) has been introduced.

In case of absence of compensation it has $\xi_{sh} = 0$, and (2.43) returns the usual one:

$$\underline{y} = b \cdot (\tan \delta + j) . \quad (2.44)$$

Therefore, once fixed the shunt compensation degree ξ_{sh} and evaluated the suitable kilometric admittance after (2.43), it is possible to continue directly, by means of the already detailed relations, with the computation of all the new transmission parameters ($\underline{k}$, $\underline{Z}_c$, and so on) up to the uniformly compensated link transmission matrix.

2.8.2 The Lumped Compensation

The transmission matrix obtained for a cable link uniformly compensated (as described in Section 2.8.1) allows developing (as it will be shown) quick procedures for a precise choice of the compensation degree ξ_{sh} : once fixed such value the link planning must be completed with an accurate configuration of the corresponding lumped compensation and with the check of it by means of a specific circuital analysis.

In the example of Figure 2.10, it is evident that the analysis regards the cascade of the three matrices $\underline{M}_\xi \cdot \underline{M} \cdot \underline{M}_\xi$: it will be sufficient to form each matrix $\underline{M}_\xi$ (regarding a reactor of inductive admittance $\underline{Y}_\xi$) after (2.45):

$$\underline{M}_\xi = \begin{bmatrix} 1 & 0 \\ \underline{Y}_\xi & 1 \end{bmatrix} \quad (2.45)$$

which has an immediate confirmation in the circuit and in the obvious relations of Figure 2.11.

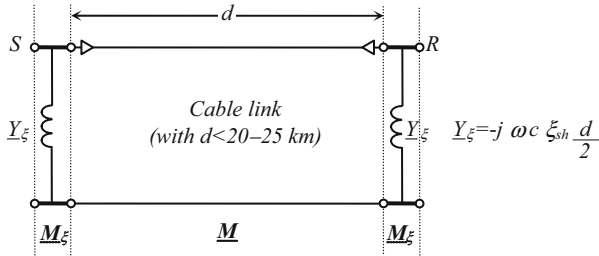


Figure 2.10 Cable link compensated at both ends

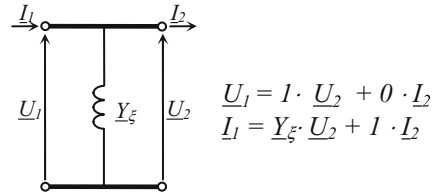


Figure 2.11 Shunt reactor circuit

If the cable to be compensated has a length greater than 20–25 km it is suitable to install shunt reactors also in n intermediate sections, respecting the limits shown in Figure 2.12.

The admittance value are subsequently modified in $Y'_ξ = 2Y_ξ = -j ω c ξ_{sh} d / n$; it is almost trivial to observe that each cable section of length d/n must be modelled by its transmission matrix without compensation.

Once formed the cascade of the different two port circuits, the regime at angular frequency $ω$ at each port can be analysed by making suitable use of the matricial algorithms already described in this chapter and of the procedures which will be exposed in the Chapters 3 and 4.

Hence it will be possible not only to ascertain the effectiveness of the hypothesis of distributed compensation, but also to evaluate in detail the effects, more or less positive, due to possible alternative schemes.

Usually the reactor percentage losses range between 0.1% and 0.3%.

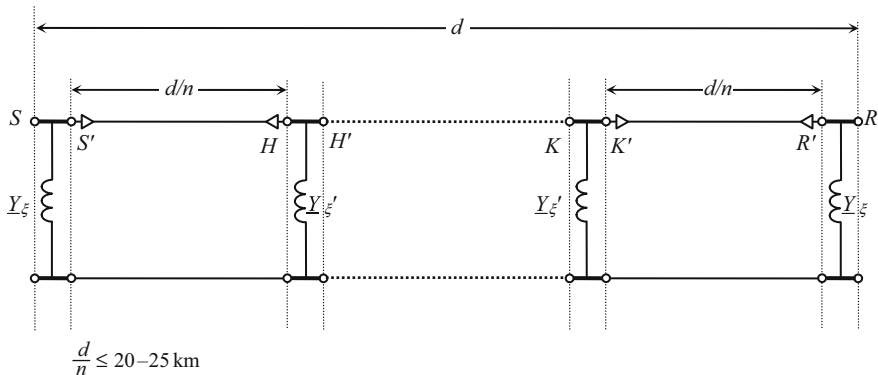


Figure 2.12 Lumped compensation of a cable link

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