Chapter 2
Bond Graph Based Physical Systems Modelling

2.1 Fundamentals

Block diagrams represent signal flows and their processing. In contrast, bond graph modelling starts from the intuitive and physical approach that a dynamic system is composed of subsystems, components, or basic elements that interact by exchanging energy. From this basic description, a first (preliminary) definition of a bond graph can be derived and some important conclusions can be drawn which show that bond graph modelling fundamentally differs from block diagram modelling.

Definition 2.1 (*Undirected bond graph***).** An undirected bond graph is an undirected graph whose vertices denote subsystems, components, or basic elements, while the edges called (power) bonds represent the instantaneous energy transfer between nodes.

Remark 2.1. An undirected bond graph displays the components of a system and their energetic interconnection. Each vertex in a bond graph has a certain number of connection points called *power ports* (cf. Definition 2.3). Bonds connect power ports of two different nodes. Each power port of a vertex must be connected to a power port of another vertex. That is, the number of bonds connected to a bond graph node equals its number of power ports. Power ports are not explicitly marked on a bond graph. The graphical editor of some software packages supporting bond graph modelling can make ports visible on demand.

An instantaneous energy transfer between two power ports means that energy is neither generated, stored, or dissipated in a port to port connection represented by a power bond.

2.1.1 Physical System Structure

In general, the transfer of energy between subsystems is enabled by means of engineering links, e.g., mechanical shafts, electrical wires, hydraulic conduits, hoses,

or glass fibre optics cables. Since subsystems, components and elements are represented by bond graph vertices and their energetic interaction by power bonds, bonds graphs reflect the *physical structure* of a system, the way real engineering system components are connected. As long as bond graphs are constructed according to certain rules and are not simplified, they exhibit a strong topological affinity to the initial schematic of a mechanical system, an electrical circuit, or a cross sectional representation of a hydraulic device. Therefore, topological connections in a system schematic can guide the construction of a bond graph model. This could be achieved by drawing a bond graph directly on top of a schematic. For instance, if two dead volumes in a hydraulic valve are connected by a conduit, their models will be connected by the model of the conduit. The latter most often reduces simply to a power bond. The same strategy is applied to rigid bodies connected by a joint, or to integrated sub-circuits connected by transmission lines. For illustration, Figure 2.1 shows a cross sectional view of the magnetic circuit inside the torque-motor of an electrohydraulic servovalve. The magnetic circuit is composed of an upper and a lower pole-shoe connected by two permanent magnets. The one in the front has been removed as well as the mechanical flexible tube on which the armature is mounted. Although details of bond graph modelling have not yet been introduced, it can be seen how the bond graph is superimposed on the schematic. In block diagram representations, the information about the physical structure of the systems gets lost. They rather represent the *computational* structure, which may change significantly with small model modifications. On the contrary, the structure of a bond graph model is derived from the physical structure of the system and is retained if the computational structure changes.

2.1.2 Physical Systems Modelling

The exchange of energy between subsystems is associated with the exchange of physical quantities, e.g., momentum, mass, electrical charge, or entropy. For these physical quantities and for power conservation, principles should be reflected in a bond graph representation. Since there are no such constraints for information flows, signal processing blocks in block diagrams may represent any functional relation between signals. In bond graphs, however, constitutive relations of nodes must comply with conservation laws from physics. In his short article titled “System Graphing Concepts”, Paynter [28] stresses at the beginning:

> Models of physical systems must be compatible with the conservation of mass, momentum, and energy. Functional models must be compatible with causality, such that a present state depends only on the past states (or: no effect in the absence of cause).

If one assumes that subsystems exchange energy when interacting, then processes must take place in the subsystems by which energy received from a subsystem is either transferred to another subsystem, distributed among others, transformed into other forms, or stored. That is, bond graphs represent physical processes in an engineering system. This is one reason why in bond graph related literature the term

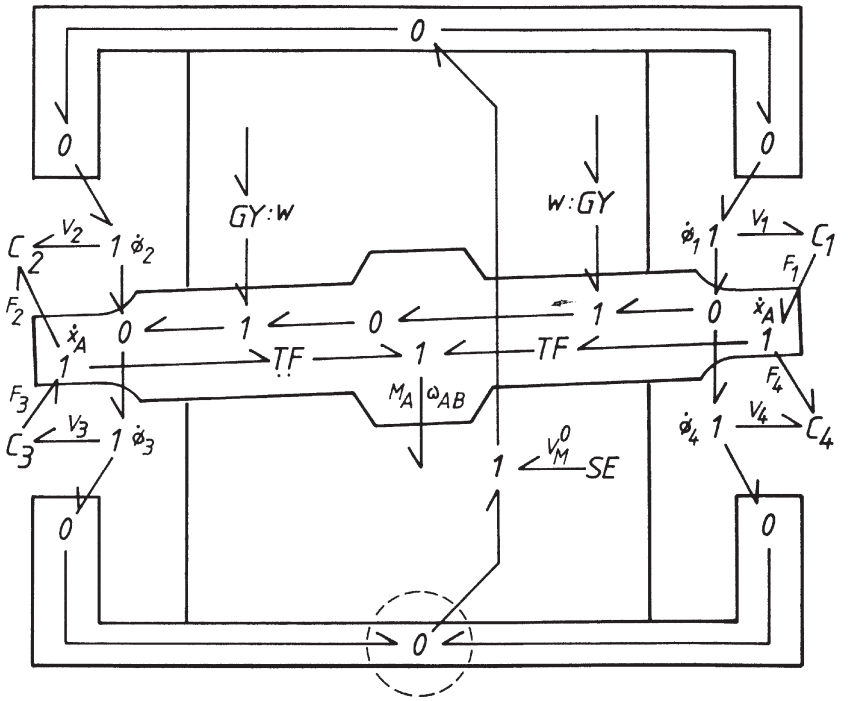


Fig. 2.1 Schematic of the magnetic circuit inside the torque motor of an electrohydraulic valve and a bond graph superimposed on it

physical systems modelling is in use [8]. Since physical processes are continuous with respect to time (and space), bond graphs are particularly suited for modelling time continuous systems. Chapter 7 discusses how the abstraction of a discontinuous description of continuous fast state transitions can be included into the framework of bond graph modelling. In this context, the observation can be made that only during the last 10 years, bond graphs have been increasingly used for models of varying structure. For many years, bond graph modelling was predominantly applied to systems in which energy exchange between subsystems is bound to time invariant real physical links, e.g., mechanical shafts.

2.1.3 Multidisciplinary Engineering Systems

The focus on energy exchange between subsystems and the transformation of energy from one form into another implies that bond graphs, from a conceptual point of view, are particularly suited for modelling *multidisciplinary* engineering systems, or

mechatronic systems in which effects from different energy domains interact with each other. This is an essential feature of bond graphs.

2.1.4 Hierarchical and Recursive Modelling

Like networks, bond graphs support a recursive top-down decomposition of a system into subsystems. That is, models of subsystems are represented by bond graphs until submodels are identified that represent basic physical processes described by equations. In this top-down approach, submodels of subsystems or components are denoted by words or alphanumeric symbols (enclosed by an ellipse). On the lowest hierarchy level the mnemonic code of elements is fixed. It indicates the behaviour of the element with respect to energy or power. As words are used for components or subsystem models, such bond graphs are called *word bond graphs* [21]. In a bottom-up approach, nodes in a word bond graph denote submodels described either by a bond graph or by a set of equations. Word bond graphs are not only a way to clearly represent large models in a hierarchical manner. They are important because they support the first steps of the conceptual modelling phase. Word bond graphs visualise the energy exchange between different parts of a system (subsystems, components, elements) that have been identified without the need for specifying all details of the system parts. At the beginning of the decomposition process, a word is sufficient that *qualitatively* indicates the behaviour of a system part with respect to power processing. Even on the lowest hierarchy level, when nodes describe physical processes and fixed mnemonic codes are used, a 1-port R-element, for instance, *qualitatively* describes dissipation of free energy. With regard to a mathematical description, there must be a relation between two so-called power conjugate variables. The decision on the form of the constitutive equation, however, can be postponed to a later phase of the modelling process.

Definition 2.2 (*Word bond graph***).** A bond graph is called a word bond graph if its vertices represent subsystems or components and are denoted by a word or an alphanumeric symbol.

Remark 2.2. Mnemonic codes in a word bond graph may be enclosed by an ellipse.

As an example, Figure 2.2 shows a word bond graph of a hydrostatic plant. In the early phase of the modelling process, properties of the components are not yet specified.

Definition 2.3 (*Power port***).** The connection points of a bond graph node that enable the energy exchange with other nodes across a power bond are called power ports.

Remark 2.3. Power ports may be viewed as places where energy can enter or leave a subsystem. In this context, the notion of a place is not limited to locations in space. Gawthrop and Smith also use the notion *energy interface* in their book [18]. Power

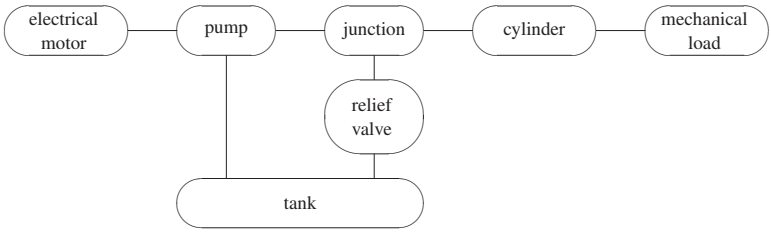


Fig. 2.2 Word bond graph of a hydrostatic plant

ports can be considered as energetic interfaces of an object (subsystem, component, element) to its outside world.

An energy flow between two power ports also involves a transfer of information. If the amount of power in a bond can be neglected with regard to other bonds and if only the flow of information shall be taken into account, which is appropriate if, for instance, a measuring instrument is connected to a system part port, then the port is called a *signal port*. In that case, the bond connecting the signal port of the system part with the instrument signal port can be replaced by an oriented edge as it is used in block diagrams. In that context, the notion of an *activated bond* is also used (cf. Definition 3.3). In the modelling language SIDOPS, ports have several properties. There is not only a distinction between power ports and signal ports, but also with respect to the energy domain. That is, an electrical power port can be only connected directly to an electrical power port of another submodel. In other words, the energy domain is an attribute of a power port among others.

Definition 2.4 (*Multiport*). A bond graph node is called a *multiport* if it has more than one port.

Figure 2.3 depicts an example of a general word bond graph in which S denotes the model of the overall system and S_i $(i = 1, \ldots, 6)$ the model of the i^{th} subsystem. According to definitions 2.1 and 2.4, models S_3, S_4 are one-ports, S_1, S_6 are two-ports, while S_5 is a three-port and S_2 is a four-port.

2.2 Nodes and Edges in Bond Graphs

As mentioned, on the lowest hierarchy level, bond graph models are called elements. They represent basic physical processes in which energy is

- distributed,
- transferred from one power port to another,
- transformed in the same energy domain,
- converted into another energy form, in particular into heat, or
- stored.

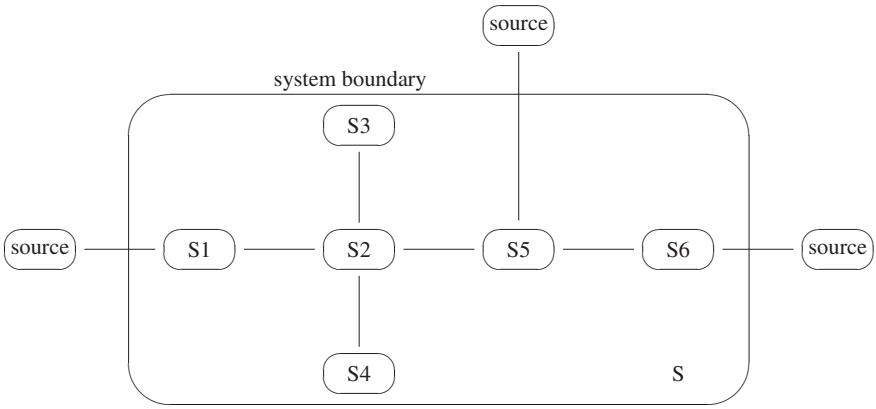


Fig. 2.3 Example of a general word bond graph

As in physical systems modelling based on networks, bond graph modelling also adopts the abstraction of spatially lumped physical properties. That is, mechanical elasticity, for instance, or friction in a fluid may be approximated by assuming that these effects are spatially concentrated in certain locations. Consequently, the above basic physical processes may be located in space and represented by a node in a bond graph. Since these elementary physical processes are encountered in all energy domains, it is reasonable to represent them by means of a unique mnemonic code that indicates the type of the process and that is the same for all energy domains. For instance, the symbol R always denotes irreversible transformation of energy into heat. Spatial concentration of physical properties means that bond graphs represent so-called *lumped parameter models*. The above processes are represented by basic bond graph elements that will be discussed when variables used in bond graphs have been introduced.

2.3 Bond Graph Variables and Physical Analogies

2.3.1 Power Variables

According to Definition 2.1, bonds in a bond graph represent an instantaneous energy flow, i.e., power between power ports of different bond graph nodes. It is a general observation that in each energy domain, the amount of power transferred equals the product of two physical quantities. Thus, contrary to block diagrams, in bond graphs two power conjugated variables are assigned to each edge. They are called *effort* and *flow* and are denoted by the letters e and f.

$$\text{Power} \;=\; \text{Effort} \times \text{Flow}$$

Fig. 2.4 Historical convention of annotating a bond with power variables effort and flow

This representation of power between two ports by the product of two variables, however, is not a physical law, but a convenience. The amount of power transferred can also be decomposed differently by means of so-called wave-power variables [27]. However, it is remarkable that two variables are sufficient for describing the power across a connection.

Regarding the annotation of bonds with two power conjugated variables, it is a historical convention to write the effort variable on the left side of a vertical bond and the flow variable on its right side. For horizontal bonds, the effort is written above and the flow below the edge (Figure 2.4). None of the two power conjugated variables is discriminated in its role against the other. They are only characterised by the fact that their product equals the energy flow $\dot{E}(t)$ between two ports at a given time t.

2.3.2 Analogies

As both power variables play an equal role for each energy domain, it must be decided which of them shall be the effort, while the other becomes the flow. As a result, two different analogies between mechanical and electrical systems have emerged in the literature.

One option is to let forces in mechanical systems be the efforts. Consequently, translational velocities become flows. At the same time, voltages in electrical systems may be considered efforts and currents as flows. This force-voltage analogy is known as direct or classical analogy (Table 2.1).

The other equally valid option is to denote velocities and voltages as efforts and forces and currents as flows. This velocity-voltage assignment is called *dual* or *mobility analogy* (Table 2.2). It was proposed by Firestone [17] in 1933, while the force-voltage analogy has been known for more than 100 years. This may justify the notion of a classical analogy.

Table 2.1 Classical force-voltage analogy

Force	$\triangleq$	Effort	$\triangleq$	Voltage
Velocity	$\triangleq$	Flow	$\triangleq$	Current

Table 2.2 Mobility (Firestone) analogy

Velocity	$\triangleq$	Effort	$\triangleq$	Voltage
Force	$\triangleq$	Flow	$\triangleq$	Current

The mobility analogy is suggested by the observation that Kirchhoff's current law (The sum of all currents into an electrical node equals zero.) is equivalent to D'Alembert's principle in mechanics. The latter says that for a system of n mass points, the sum of inertial forces and imposed forces is equal to zero. The mobility analogy also appears reasonable from a measurement point of view. For the measurement of an electrical voltage and of mechanical velocities, two points in space are needed, while for measuring electrical currents and forces, one point in space is sufficient. The measurement point of view has led to the two notions *across*- and *through*-variable. The mobility analogy is commonly used when non-electrical subsystems are represented by a generalised network or an iconic diagram.

An essential feature of the mobility analogy is that it conserves the interconnection structure of a system when mechanical power variables are replaced by electrical ones and vice versa. That is, a parallel connection of two elements remains a parallel connection. The same holds for series connections. If the network representations of systems from different energy domains have the same structure, then these systems are called structurally analogue [37].

Another consequence of the mobility analogy is that a mechanical inertia corresponds to an electrical capacitor. This can be seen by comparing the equation for the momentum of a point mass with that for the charge of an electrical capacitor. Assuming that the initial values of momentum and charge vanish, both equations read:

$$\int_0^t F(\tau)d\tau = m \times v(t) \tag{2.1}$$

$$\int_0^t i(\tau)d\tau = C \times u(t)\ . \tag{2.2}$$

Similarly, one can show that a mechanical spring corresponds to an electrical inductance.

In the community of bond graph modellers, it has become common to use the direct analogy. With regard to basic elements, this analogy entails that a mechanical inertia corresponds to an electrical inductance. This can be seen by comparing Newton's third law to Faraday's law.

$$F = \frac{dp}{dt} \ = \ m \times \frac{dv}{dt} \tag{2.3}$$

$$u_L = L \times \frac{di}{dt} \tag{2.4}$$

Likewise, a mechanical spring corresponds to an electrical capacitor. If F_{sp} denotes the spring force, k, the stiffness of a spring with a linear characteristic, and u_C, the voltage drop across a capacitor, then the electrical capacitance parameter, C, corresponds to the compliance, $1/k$, of the spring.

$$F_{sp}(t) = k \times \int_0^t v(\tau)d\tau \; + \; F_{sp}(t=0) \tag{2.5}$$

$$u_C(t) = \frac{1}{C} \times \int_0^t i(\tau)d\tau \; + \; u_C((t=0) \tag{2.6}$$

It is obvious that a mechanical dashpot corresponds to an electrical resistor. The constitutive equation of an electrical resistor algebraically relates the voltage drop u across its terminal to the current i through the two pin element. The linear case is given by Ohm's law

$$u \; = \; R \times i \; , \tag{2.7}$$

where R denotes the resistance. The constitutive equation of an ideal dashpot is an algebraic relation between the damping force F_D and the velocity v

$$F_D \; = \; b \times v \; , \tag{2.8}$$

where b is the dashpot constant. As the classical analogy relates a force to a voltage and a velocity to a current, the two elements correspond to each other.

Contrary to the mobility analogy, the classical analogy does not preserve the model structure. That is, when mechanical power variables are exchanged by those of the electrical domain, a parallel connection of mechanical elements becomes a series connection of corresponding electrical elements. For instance, if a mechanical spring and a damper are connected in parallel (Figure 2.5), then the force acting in each of the two hinge points (on the wall and on the moving body) is obviously the sum of the spring force and the damper force.

If electrical currents are assigned to mechanical forces, the electrical analogue is a parallel connection of an inductance and a resistor. If however, voltages are assigned to forces, then according to Kirchhoff's voltage law, the electrical analogue is a series connection of a capacitor and a resistor (cf. Figure 2.6).

$$F_{sp} \; + \; F_d \; + \; m\frac{dv}{dt} = F(t) \tag{2.9}$$

Table 2.3 Correspondence of mechanical and electrical energy stores according to the classical analogy

mechanical inertia	$\triangleq$ electrical inductance
mechanical (rotational) spring	$\triangleq$ electrical capacitor
mechanical dashpot	$\triangleq$ electrical resistor

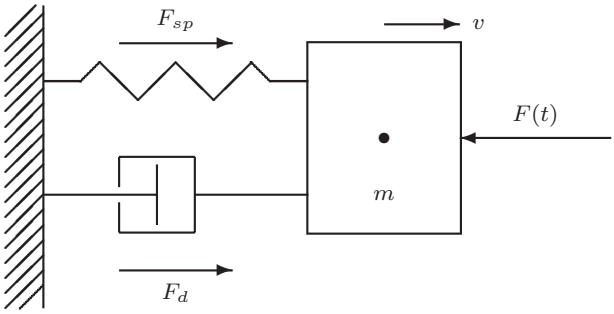


Fig. 2.5 Mechanical parallel connection

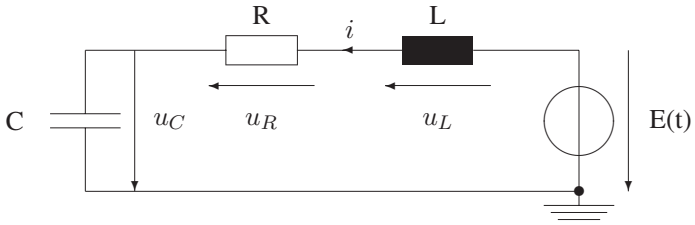


Fig. 2.6 Corresponding electrical series connection according to the classical analogy

$$u_C + u_R + L\frac{di}{dt} = E(t) \tag{2.10}$$

With regard to the derivation of a mathematical model from the bond graph, it does not matter which analogy has been chosen. As the classical analogy is common in bond graph related literature, this convention is also adopted throughout this monograph. Regarding the mobility analogy, see [16, 25, 37].

Table 2.4 shows which physical quantities are commonly chosen as effort and flow variables in the different energy domains. Regarding the electromagnetic energy domain, the horizontal line separating electrical quantities from magnetic ones does not go from left to right because the magnetic domain is not an energy domain independent from the electrical domain. The reader may notice that it is not the magnetic flux that can be chosen as a flow variable, but the magnetic flux rate. The product of magnetomotive force and magnetic flux is not the magnetic power transferred between two points in a magnetic material. The magnetic flux becomes conceivable by looking at paths built by the spatial orientations of tiny magnets inside the magnetic material. The flow is the variation of their orientations with time.

Finally, taking into account that for generalised networks or iconic diagram representations, the mobility analogy is commonly used, while in bond graph modelling, the classical analogy is most widely adopted, across- and through variables in networks can be related directly to efforts and flows in bond graphs, as shown in Table 2.5.

Table 2.4 Bond graph variables used in the various energy domains

Energy domain	Effort e	Flow f	Generalised momentum p	Generalised displacement q
Translational mechanics	Force F [N]	Velocity v [m/s]	Momentum p [Ns]	Displacement x [m]
Rotational mechanics	Angular moment M [Nm]	Angular velocity ω [rad/s]	Angular momentum p_ω [Nms]	Angle θ [rad]
Electro-	Voltage u [V]	Current i [A]	Linkage flux λ [Vs]	Charge q [As]
magnetic domain	Magnetomotive force V [A]	Magnetic flux rate $\dot{\Phi}$ [Wb/s]	–	Magnetic flux Φ [Wb]
Hydraulic domain	Total pressure p [N/m^2]	Volume flow Q [m^3/s]	Pressure momentum p_p [N/m^2 s]	Volume V_c [m^3]
Thermo-dynamic	Temperature T [K]	Entropy flow $\dot{S}$ [J/K/s]	–	Entropy S [J/K]
Chemical domain	Chemical potential μ [J/mole]	Molar flow $\dot{N}$ [mole/s]	–	Molar mass N [mole]

Table 2.5 Relation between across- and through variables in networks and efforts and flows in bond graphs

Energy domain	Effort	Flow
Mechanical domains	Through variable	Across variable
Non-mechanical domains	Across variable	Through variable

2.3.3 Energy Variables

Table. 2.4 introduces two additional physical quantities used in bond graph modelling. They are called *generalised momentum* (p) and *generalised displacement* (q) and are obtained by integration of the power variables with respect to time.

$$p(t) = p(t_0) + \int_0^t e(\tau)d\tau \tag{2.11}$$

$$q(t) = q(t_0) + \int_0^t f(\tau)d\tau \tag{2.12}$$

It has been a convention since the beginning of bond graph modelling to use the notions generalised momentum and generalised displacement. However, these terms are not fully convincing since their roots are obviously in mechanical engineering. This may be considered inappropriate in other engineering disciplines. In electrical engineering, for instance, the charge, q, of a capacitor is the integral with respect to time of the current (flow). However, it is rather unusual to consider the electrical charge a generalised displacement, while in mechanical engineering displacements, in general, are not denoted by the letter q. The additional physical quantities introduced in Table 2.4 are called *energy variables* since they quantify the energy transferred in a time period and accumulated in an ideal energy store (cf. Section 2.5.3, Equations 2.38 and 2.46).

2.4 Orientation of Power Bonds

First, the energy exchange between power ports of submodels can be represented by non-directed edges in a bond graph. If, however, equations are given for all basic submodels, a sign convention is needed for the derivation of a set of equations from a bond graph in which variables are consistent with respect to their sign. This is achieved in bond graph modelling by adding a so-called *half arrow* to each bond indicating the positive *reference direction* of the energy flow. A half arrow is chosen to distinguish between energy flows and signal flows. The latter are commonly represented by edges with a full arrow. This orientation of a bond graph edge does not represent the actual direction of the energy flow which can vary with time. Rather it means a time-invariant reference direction which coincides with the energy direction at time t, if for the amount of power, P, holds: $P(t) = e(t) \times f(t) > 0$. Aside from energy sources, the reference direction of power is assumed positive if a bond connected to a power port of a submodel is oriented toward the power port. That is, if $P(t) = e(t) \times f(t) > 0$, energy flows towards the port. If, in addition, $f(t) > 0$, then the flow is directed towards the port at time t. This convention is motivated by the fact that energy stores and resistors consume energy whereas energy stores give up the accumulated energy at a later time, resistors irreversibly convert it into heat. For a bond connected to a power port of an energy source, the positive reference direction of the power is oriented away from the port. That is, if $P(t) = e(t) \times f(t) > 0$, energy flows out of the source corresponding to the fact that sources usually supply energy to a system. These considerations of a positive reference direction for energy flows allow for a refinement of Definition 2.1 and the introduction of directed bond graphs.

Definition 2.5 (*Directed bond graph***).** A bond graph is called a directed bond graph if a half arrow has been added to each bond indicating the positive reference direction of the energy flow across the bond.

By convention, the half arrow is added on that side of the bond where the flow variable is annotated (cf. Figure 2.7). In the case the inclination of a bond is not a multiple of $90°$, the question as to where to mark the half arrow doesn't has not a unique answer. It depends on how the bond is virtually rotated into a vertical or horizontal position. In such cases, the convention is adopted that the flow variable is always on that side of a bond where the half arrow has been attached [6].

2.5 Basic Bond Graph Elements and Power Port Orientations

The fundamental physical processes, already mentioned in Section 2.2, suggest the introduction of the following classes of basic multiport elements used for an idealised description of physical processes.

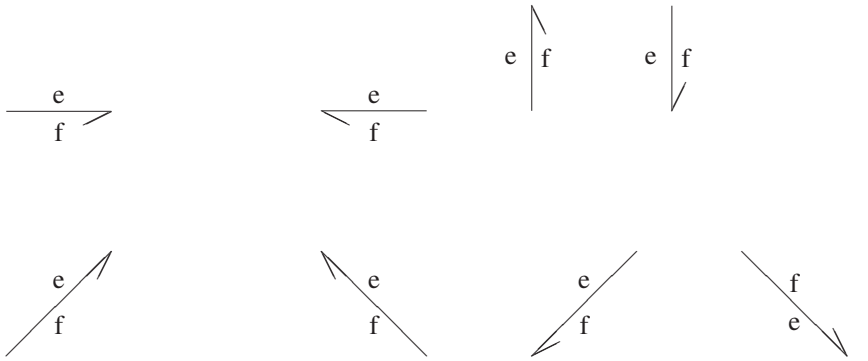


Fig. 2.7 Conventions for adding the half arrow to power bonds

- Energy sources and sinks (= negative energy sources),
- Energy stores,
- Dissipators converting energy irreversibly into heat,
- Power couplers and transducers, and
- Power nodes that instantaneously distribute power.

Energy sources deliver energy into a system, whereas sinks consume energy flowing out of the system. As depicted in Figure 2.3, sources and sinks do not belong to a system. They rather represent boundary conditions of a system embedded into the surroundings. In other words, sources and sinks model the impact of the surrounding on a system. In [10], Cellier points out:

> A source as drawn in the circuit example above is actually a non-physical element. Power cannot be generated, only transported and converted. However, a "system" never denotes the whole of the universe. It denotes a piece of the universe. Sources are interfaces between the system and the universe around it.

Definition 2.6 (*Junction strucure*). A bond graph in which bonds connect only nodes that instantaneously transfer or distribute power (without energy storage or conversion into heat), is called *Junction Structure* (JS).

The above classification indicates that basic elements are an idealised description of physical processes. That is, only *one* effect is represented, while other simultaneous effects are not taken into account. If several effects are involved in a physical process, different elements will have to be composed in a bond graph submodel representing the process. Besides the 0-junction and the 1-junction that are introduced in the next section, bond graph elements are similar to those used in generalised networks. This is not surprising since generalised networks also start from basic physical processes.

If the elements of a bond graph of a system model S are combined into submodels according to the classification discussed above, and if all power bonds between

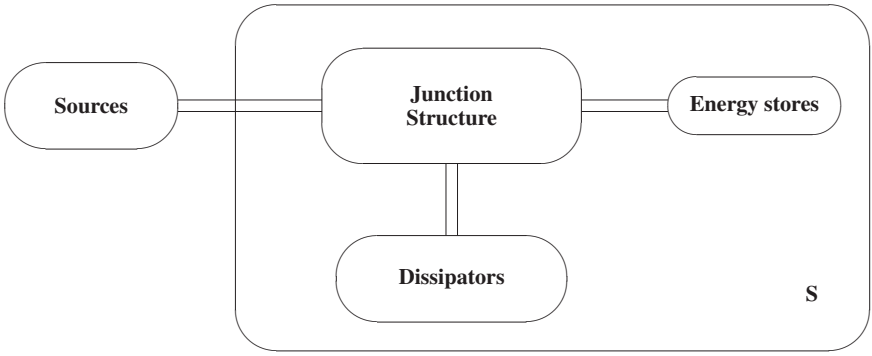


Fig. 2.8 General structure of bond graphs

the submodels introduced this way are represented by two parallel edges (undirected multibonds), then all bond graphs exhibit the general structure depicted in Figure 2.8. An essential feature of bond graphs is that the structure of the interconnections is a separate model part. The junction structure contains multiport elements that distribute or transfer power without storage or conversion into heat. As will be considered subsequently, the junction structure includes a type of an interconnection node that has no equivalent in networks. The reason is that networks are *terminal* oriented, whereas bond graphs are power port oriented.

In the following, the above classes of bond graph multiport elements will be discussed. Before starting with the bond graph elements that compose the general junction structure, the notion of *constitutive equations*, often used in the following section, is introduced. The constitutive equations describe the behaviour at the ports of a multiport element by relating power port variables. In the case of a 1-port element, its characteristic is a graphical representation of its constitutive equation.

2.5.1 Power Conserving Junctions

An ideal node that instantaneously distributes energy without storing it or converting it into heat must comply with the principle of *power conservation*. If P_{in} denotes the power entering the node and P_{out} the power leaving the node, then

$$P_{in} \, - \, P_{out} \, = \, 0 \ . \tag{2.13}$$

If such a node has $n \geq 3$ power ports and if one assumes without loss of generality that power enters at port 1 while leaving the node at all others ports simultaneously, then power conservation reads

$$e_1 f_1 \, - \, e_2 f_2 \, - \ldots - \, e_n f_n \, = \, 0 \, . \tag{2.14}$$

Zero Junctions

As has been pointed out by F. Cellier in [11], among all possibilities to comply with the power balance, the simplest ones are to assume either equal efforts or equal flows. Assuming that all efforts are equal, the constitutive equations of an element are obtained that Paynter termed 0-junction (*zero junction*). In bond graphs, its type is denoted by the symbol 0.

Definition 2.7 (*0-junction*). A 0-junction is a multiport element defined by the following equations

$$e_1 = e_2 = \ldots = e_n \tag{2.15a}$$

$$f_1 = f_2 + \ldots + f_n \, . \tag{2.15b}$$

According to Equation 2.15a the element is also called *common effort junction*. Equation 2.15b has given rise to the notion of a *flow junction*.

Equations 2.15a and 2.15b are well known in electrical engineering. Let $e_1, e_2, \ldots, e_n$ denote the voltage drops across the n ports of a subnetwork and $f_1, f_2, \ldots, f_n$ the port currents, then Equation 2.15b is just Kirchhoff's current law. Thus, parallel connections in electrical networks can be represented by a 0-junction in a corresponding bond graph (cf. Figure 2.9).

If in Figure 2.9 node 2 is the common ground node, then the 0-junction just represents the voltage of node 1 with respect to ground. In hydraulics, a pipe tee junction can be represented by a 0-junction if dynamic pressures can be neglected with regard to hydrostatic pressures. The reader may notice that the number of ports of a 0-junction is not fixed.

According to what has been said regarding to the orientation of bonds and according to Equation 2.14, bond 1 must point toward the 0-junction while all other edges are directed away from the node (Figure 2.10).

Equation 2.15b indicates that power reference directions correspond to the sign of the flow variables. That is, if a power bond points towards a 0-junction, it is

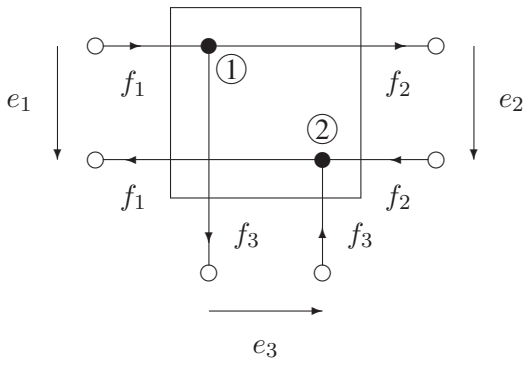


Fig. 2.9 Example of a 0-junction in an electrical network

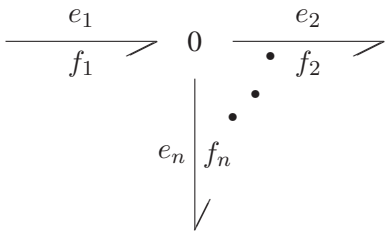


Fig. 2.10 Orientation of the bonds connected to a 0-junction according to Equation 2.14

assumed that its flow variable has positive sign, otherwise its sign is negative. If in a bond graph of a *mechanical* system one of the power ports of a 0-junction is connected to a power port of an energy store (spring) or to the port of a dissipator (dash-pot) by means of bond 2 then one of the remaining bonds connected to the 0-junction must point towards the junction while another bond must be directed away from the junction (cf. Figure 2.41). This rule reflects that physical connections between two bodies become effective if there is a relative velocity between them. It is the *difference* of velocities, not a sum of velocities, that gives rise to an action. On the other hand, a real physical link between two bodies, e.g., a spring, or a dashpot impose one and the same force, or a moment on both bodies. Consequently, in bond graphs of mechanical systems, 0-junctions are used for describing links.

It should be stressed that Equation 2.15b is not an assignment statement. It may be solved for any of the flow variables as needed. Which of the flow variables is a dependent one depends on the element ports, or submodels ports the 0-junction is connected to.

One Junctions

Let us now assume that all flow variables are equal in Equation 2.14. Thus, the sum of all efforts must vanish.

$$f_1 = f_2 = \ldots = f_n \tag{2.16a}$$

$$e_1 - e_2 - \ldots - e_n = 0 \tag{2.16b}$$

Equations 2.16a and 2.16b characterise an element that apparently is the dual to the 0-junction. For this reason, Paynter called it the 1-junction (one junction). In bond graphs, it is denoted by the symbol 1. According to its constitutive equations, it is also known as *common flow junction*, or as *effort junction*.

Definition 2.8 (*1-junction*). A 1-junction is a multiport element, for which power port variables comply with Equations 2.16a and 2.16b.

Equations 2.16a and 2.16b are also well known in electrical engineering. If the flow variables denote the current through elements with two terminals connected in series and if the effort variables are the voltage drops across the elements, then

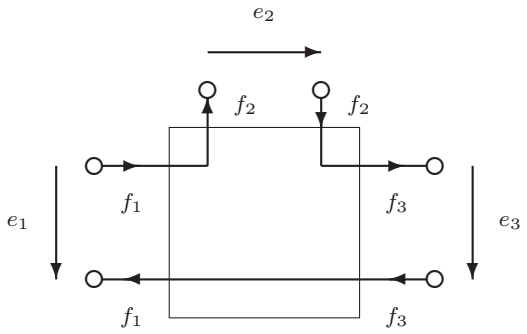


Fig. 2.11 Example of a 1-junction in an electrical network

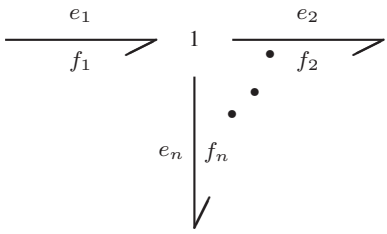


Fig. 2.12 Multiport 1-junction with power bond orientations according to Equation 2.14

Equation 2.16b is just Kirchhoff's voltage law (Figure 2.11). Figure 2.12 depicts a multiport 1-junction with power bond orientations according to Equation 2.14.

In contrast to networks, bond graphs use an additional node to allow for series interconnection of elements in non-mechanical systems. There is a need for such a node since in bond graphs power ports are to be connected, whereas in networks, terminals are connected.

If in bond graphs of non-mechanical systems one bond of a 1-junction is oriented towards to the port of an adjacent energy store or a resistor, then one of its other bonds must point towards the 1-junction while another edge must be directed away from the 1-junction. This rule takes into account that it is a voltage drop across an electrical element with two terminals that gives rise to the current through the element and not the sum of electrical voltages with respect to ground. Likewise, it is a pressure difference across a hydraulic line and not the sum of pressures that is related to a fluid flow through the line. As a consequence of this rule, half arrows pointing to and away from the 1-junction also indicate the flow, e.q. electrical current, volume flow, or mass flow through the element from the higher to the lower potential. In bond graphs of mechanical systems, 1-junctions indicate the (angular) velocity at a point and at the same time, the sum of all forces (moments) acting on that point (D'Alembert's principle).

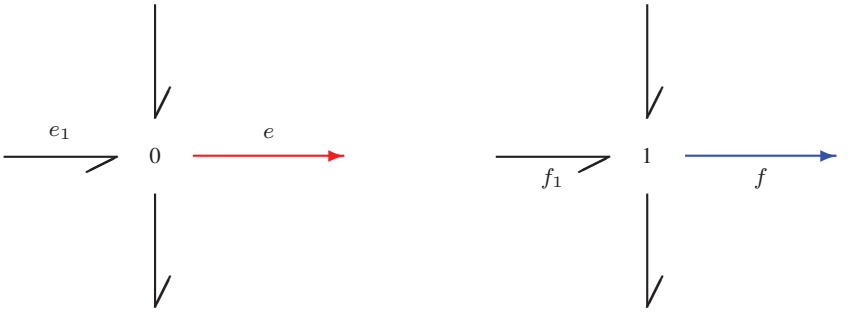


Fig. 2.13 Signal arrows taking up the *information* of the effort of a 0-junction or of the flow of 1-junction

Since both types of junction indicate a power variable that is common for all adjacent bonds, they may have one signal port in addition to their power ports. From this signal port, the *information* of the common power variable can be taken up and fed into a block diagram for signal processing. Ideal measuring of the common power variable does not affect the power balance of the junction and can be represented by attaching a signal arrow to the junction pointing away from it (Figure 2.13). Clearly, the information taken up at the signal port must be provided at one and only one of the power ports (cf. Section 3.2 on computational causalities, Figure 3.5).

The introduction of 0- and 1-junctions gives rise to further definitions.

Definition 2.9 (*Internal bond*). A bond is called an internal bond if it connects a 0- or 1-junction to another 0- or 1-junction.

Definition 2.10 (*Simple junction structure*). A bond graph is called a simple junction structure, or Kirchhoff junction structure if each node is either a 0- or a 1-junction.

Definition 2.11 (*External bond*). A bond is called an external bond if it connects a 0- or 1-junction to a power port of an element that does not belong to the simple junction structure.

2.5.2 Ideal Power Couplers and Power Transducers

The assumption that a power coupler or power transducer neither stores energy nor converts it irreversibly into heat means that the constitutive equations of such a device must comply with the principle of power conservation. First, two-port elements are considered. The general case of n-port elements will be dealt with in Chapter 8

in the context of multibond graphs. In the case of a two-port element, power conservation means

$$e_1 f_1 \;=\; e_2 f_2 \;. \tag{2.17}$$

Transformers

Assuming a constraint between the two efforts

$$e_1(t) = \; m \times e_2(t) \;, \tag{2.18}$$

where m is a non-negative real parameter and by substituting Equation 2.18 into the power balance yields

$$f_2(t) = \; m \times f_1(t) \tag{2.19}$$

for the flow variables. Another possible constraint between the two efforts is

$$k \times e_1 = e_2 \;. \tag{2.20}$$

That is, both parameters, $m, k \in \mathbb{R}$, $m, k \geq 0$, are constrained by the relationship $m = 1/k$.

If the two efforts are considered to be the voltages across the ports of an electrical two-port element and the flows are the currents flowing into and out of the ports, then Equations 2.18 and 2.19 describe an ideal electrical transformer. These observations have lead to a bond graph element called transformer. In bond graphs, it is denoted by the symbol TF.

Definition 2.12 (*Two-port transformer***).** A two-port transformer is an element with constitutive Equations 2.18 and 2.19. Its modulus m may be a constant of non-negative real value, a function of some other power variable or conserved physical quantity, or a function of time.

In bond graphs, the symbol TF may be annotated by the modulus m separated from the symbol TF by a colon. If the modulus is not constant, the symbol TF is prefixed by the letter M (Figure 2.14). In that case, the transformer is called a *modulated* transformer. Moreover, modulation may be emphasised by connecting the output of a signal processing block and the transformer node via a signal arrow. In that case, the two-port transformer becomes a 3-port element with two power ports and an additional signal port. Thus, the graphical representation of a model becomes a combination of a bond graph and a block diagram. In a slider crank mechanism for instance, the angular velocity ω is transformed into the translational velocity v_p of the piston. The transformer modulus is a (complicated) function of the angle ϕ that is derived from the geometric constraints (cf. Figure 4.6).

Since an ideal two-port transformer does not store energy, it is appropriate to have one bond pointing towards the element while the other one is directed away from the

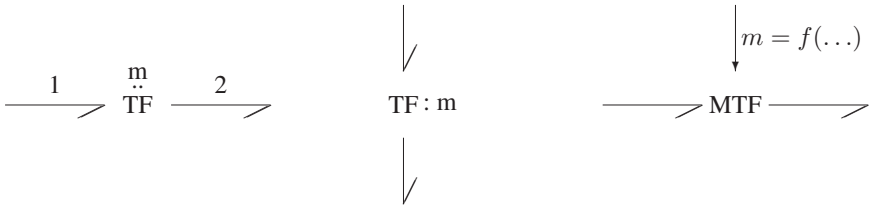


Fig. 2.14 Representation of two-port transformers

elements. This convention supports the view that energy flows *through* the element[1]. The transformer modulus is defined unambiguously because the power variables e_1 and f_1 are always associated with the bond pointing *towards* the transformer [9].

Example: Mechanical Gear Pair

In mechanics, an ideal transformer can be used to capture the main function of a gear pair (Figure 2.15). Both gears have the same tangential velocity v_t.

$$r_1 \times \omega_1 \; = \; v_t \; = r_2 \times \omega_2 \tag{2.21}$$

Substitution of this kinematic relationship into the power balance yields for the moments

$$r_2 \times M_1 \; = \; r_1 \times M_2 \; . \tag{2.22}$$

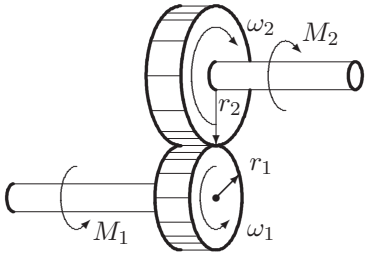


Fig. 2.15 Gear pair

[1] If an inward orientation of the bonds is adopted, then a minus sign would result in Equation 2.19 for the flows due to the rule that power is assumed positive if the bond is directed toward the element. $e_1 f_1 + e_2 f_2 = e_2(m f_1 + f_2) = 0$

Example: Hydraulic Cylinder

In hydraulics or pneumatics, the core function of a cylinder can be represented by an ideal transformer. If the piston has cross section areas A_1 and A_2 and a translational velocity v, and if Q_1 and Q_2 denote the inlet and the outlet volume flow, then the velocity of an *incompressible* fluid reads

$$\frac{Q_1}{A_1} = v = \frac{Q_2}{A_2} . \tag{2.23}$$

If it is assumed that hydraulic power is approximately the product of hydrostatic pressure times volume flow, then the power balance for the hydrostatic pressures yields

$$p_1 A_1 = p_2 A_2 . \tag{2.24}$$

Another device that may be approximately described by an ideal transformer is a hydraulic flow pump. In this case, the transformer represents the instantaneous transformation of mechanical power into hydraulic power. Losses and storage effects in real pumps are accounted for by further bond graph elements.

Gyrators

If Faraday's law is applied to a conductor of length l moving at velocity v in a magnetic field of magnetic flux density B, then there is a relation between the electrical effort, the voltage u across the conductor, and the mechanical flow, the velocity v

$$u = (B \times l) \times v . \tag{2.25}$$

Substitution of this relation into the power balance yields a relationship for the power conjugated variables, the Lorentz force, F, acting on the conductor and the current, i, through the conductor

$$F = (B \times l) \times i . \tag{2.26}$$

Both relations assume that the vectors $\mathbf{B}, \mathbf{i}, \mathbf{v}$ are perpendicular to each other. In contrast to the constitutive equations of a two-port transformer, these equations relate efforts to flows. Moreover, they describe a power conversion between electrical and mechanical energy.

Definition 2.13 (*Two-port gyrator*). An ideal two-port gyrator is a power conservative element defined by the two constitutive relations

$$e_1 = r \times f_2 \tag{2.27a}$$
$$e_2 = r \times f_1 . \tag{2.27b}$$

The parameter, $r \in \mathbb{R}$, $r > 0$, is called gyrator ratio.

In bond graphs, gyrators are denoted by the symbol GY. The gyrator ratio r may be attached to the symbol separated from the symbol by a colon. If the ratio is not

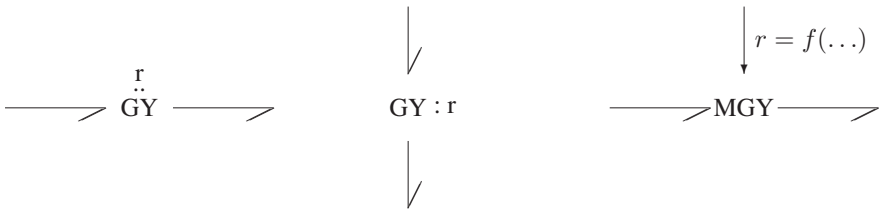


Fig. 2.16 Gyrator and modulated gyrator

a constant, the symbol MGY is used and the gyrator is called a *modulated* gyrator (Figure 2.16).

As to the orientation of the two adjacent bonds, a through direction is adopted like for transformers. The gyrator ratio is unambiguously defined by relating the effort of the bond pointing towards the two-port gyrator to the flow of the other bond pointing away from the node. Consequently, unlike the transformer modulus, the gyrator ratio always has a physical dimension. Since the constitutive equations of a gyrator are symmetric, it does not matter which of the two bonds has an inward orientation. A gyrator is mostly used to approximately describe transducers that transform energy from one form into another. Examples are electrical DC motors, electrodynamic loudspeakers, mass accelerometers, or centrifugal pumps.

Example: Energy Conversion in an Electrical Coil

A phenomenon that can be represented by a gyrator with *constant* ratio is the conversion of electrical energy into magnetic energy that happens in an electrical coil wound on a magnetic core. The voltage u across the terminals of a coil with n turns is related to the rate of the magnetic flux Φ according to Faraday's Law

$$u = n \times \frac{d\Phi}{dt} \,. \tag{2.28}$$

Its substitution into the power balance

$$u \times i = V \times \frac{d\Phi}{dt} \tag{2.29}$$

yields for the magnetomotive force V setting up the magnetic field

$$V = n \times i \,. \tag{2.30}$$

Both equations can be represented by a gyrator (Figure 2.17).

Storage effects, e.g., the self-inductance of the coil, storage of magnetic energy in the ferromagnetic material, losses, e.g., due to eddy currents in the magnetic core are to be accounted for separately.

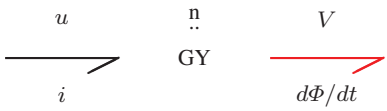


Fig. 2.17 Conversion between electrical and magnetic power in a coil

Example: DC Motor

The power conservative conversion of electrical energy into mechanical energy in a DC motor can be described by a *modulated* gyrator (MGY). Its ratio is a nonlinear function of the field current due to saturation. Further effects, e.g., the self-inductance of the field and armature coils, the mechanical inertia of the rotor, electrical resistances and mechanical bearing losses will have to be accounted for by additional bond graph elements.

Although only bond graph elements belonging to the junction structure have been introduced so far, Figure 2.19 depicts an entire bond graph model of a shunt motor shown in Figure 2.18. The core bond graph element in this model is the modulated gyrator (MGY) representing ideal instantaneous lossless conversion from electrical into mechanical energy. One of its two constitutive equations relates the current, i_a, through the armature of the motor to the torque, M, driving the mechanical load. The second equation takes into account that the angular velocity, ω, of the load causes an induced voltage u_a.

$$M = \Psi \times i_a \tag{2.31a}$$

$$u_a = \Psi \times \omega \tag{2.31b}$$

The 0-junction represents the voltage, E, delivered by the voltage source. Simultaneously, it indicates that the current, i, through the voltage source is the sum of the current, i_a, through the armature and the current, i_f, through the field winding.

$$i = i_a + i_f \tag{2.32}$$

The left-hand side 1-junction represents the current i_a and the sum of all voltage drops along mesh II.

$$-E + u_{R_a} + u_{L_a} + u_a = 0 \tag{2.33}$$

The upper 1-junction indicates the current, i_f, through the field winding and that the voltage, E, of the voltage source is the voltage across the resistor of the field winding according to the schematic of the motor.

Finally, the right-hand side 1-junction represents the angular velocity, ω, and, simultaneously, the sum of all torques acting on the flywheel with the moment of inertia J_m.

$$M + M_{\text{load}} - M_R - J_m\dot{\omega} = 0 \tag{2.34}$$

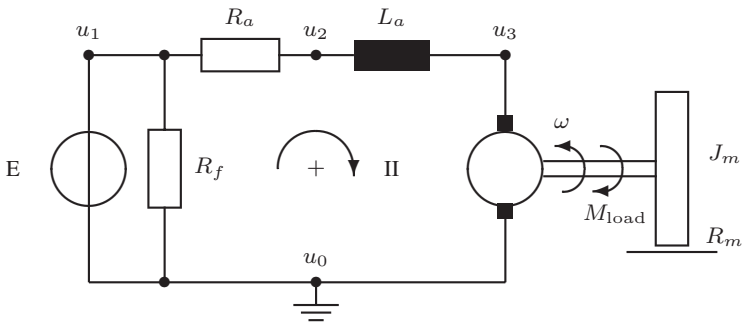


Fig. 2.18 Shunt motor

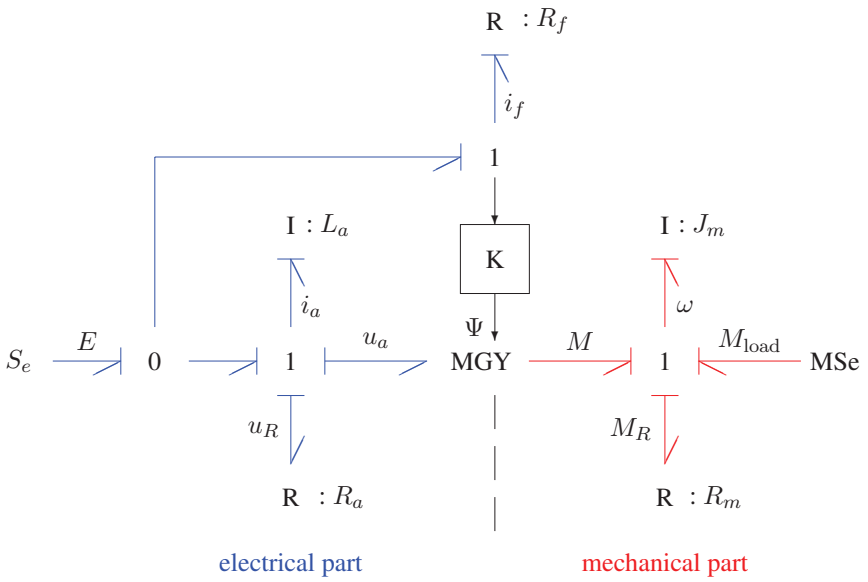


Fig. 2.19 Bond graph model of a shunt motor

The further bond graph elements, the energy storing elements denoted by the symbol I, the resistors (R elements) and the effort sources (Se) can be related to the elements in the schematic of the motor. They will be introduced in subsequent Sections 2.5.3, 2.5.4 and 2.5.6. The perpendicular strokes at the bonds in Figure 2.19 will be introduced later in Section 3.2. The bond graph model of the shunt motor is used in Chapter 11 as a reference in order to illustrate various aspects.

Now, having introduced the bond graph nodes of type 0,1, TF, GY, a formal definition of the notion *General Junction Structure* can be given.

Definition 2.14 (*General junction structure***).** A bond graph with nodes of type 0,1, (M)TF, (M)GY is called General Junction Structure (GJS) [31].

A special case is the so-called *weighted* junction structure.

Definition 2.15 (*Weighted junction structure***).** A bond graph with nodes of type 0,1, (M)TF is called a Weighted Junction Structure (WJS) [31].

Remark 2.4. Since the general junction structure is a bond graph of which all nodes are power conservative, it is a power conservative multiport.

Definition 2.16 (*Environmental elements***).** All elements that do not belong to the general junction structure are called *environmental elements* [31, 36].

2.5.3 Energy Storage Elements

Like in generalised networks [25], in bond graphs, also two types of energy stores are used to describe energy storage in all energy domains except in the magnetic domain, the thermodynamic and in the chemical domain. In bond graphs, these two types of stores are designated by the symbols C and I. Although energy storage elements are multiport elements in the general case, in the following, the discussion will first consider 1-port energy storage elements. This limitation is justified since for many real problems, bond graph models can be developed that only use 1-port energy stores. After the discussion of further aspects of bond graph modelling in the subsequent chapters, we will come back to multiport energy stores. These are also called energy storage fields in the context of modelling rigid multibody systems (see Chapter 8).

Definition 2.17 (*1-port C energy store***).** An ideal 1-port element of type C is defined by a one-to-one function $\Phi_C : \mathbb{R} \rightarrow \mathbb{R}$ relating the effort variable, e, of the power port to the generalised displacement q

$$q(t) = \Phi_C(e(t)) \,, \tag{2.35}$$

where $t \in \mathbb{R}$ and $t \geq 0$. The function Φ_C must have a unique single valued inverse Φ_C^{-1}

(cf. [22]). According to its definition (see Equation 2.12), the rate of the generalised displacement equals the flow variable of the power port

$$\dot{q} = f \,. \tag{2.36}$$

Remark 2.5. This definition of a C element is independent of the form in which its constitutive equation is written. Due to the existence of a single valued inverse Φ_C^{-1}, the constitutive equation could also be written in the form $e = \Phi_C^{-1}(q)$.

The existence of a single valued inverse Φ_C^{-1} is required to ensure that the constitutive Equation 2.35 can be solved for the effort variable, if needed. It is not acceptable that the solution of a set of coupled equations depends on how elements are interconnected. An element should be independent of the surroundings in which it is embedded. For instance, if two point masses are connected via two springs in series, the position of the interconnection point of both springs is used in the constitutive equations of both springs. That is, the springs cannot act independently of each other. In order to eliminate the position of the point connecting both springs and to replace the two springs by a single one, imposing a force F on both point masses requires the existence of a single valued inverse function for one of the two constitutive relations.

From the non-uniqueness of the inverse relation of an element, Beaman and Rosenberg conclude that multiple physical effects have been accounted for improperly by the same element. They call such elements *composite* [2]. The requirement of constitutive relations having a unique inverse does not mean a limitation to modelling. Since non-unique inverse relations can cause problems, both authors suggest to avoid composite elements and to try to capture effects in separate elements that have a single valued inverse characteristic.

The relationship defining an ideal C energy store is well known from modelling mechanical springs, torsion springs, electrical capacitors, or volumes that store a compressible fluid. For instance, if q designates the charge and u the voltage drop across the terminals of an electrical capacitor, its constitutive relations reads

$$q \;=\; C \times u \;. \tag{2.37}$$

The capacitance C may depend on the voltage drop u. Following the example of the electrical capacitor, the bond graph element is called a (1-port) capacitor.

If an initially empty capacitor is assumed and if the direction towards the element is chosen as a positive reference direction of the energy flow, then the energy stored at time instant t is

$$\begin{aligned} E(t) &= \int_0^t e(\tau) \times f(\tau)\, d\tau \\ &= \int_0^q \Phi_C^{-1}(\tilde{q}) d\tilde{q} \;. \end{aligned} \tag{2.38}$$

Equation 2.38 indicates that the integral of the flow with respect to time, the conserved quantity, q, can be considered to be a so-called *energy variable*. It is a measure for the stored energy.

In the case of a linear characteristic

$$q = C \times e \;, \tag{2.39}$$

Fig. 2.20 1-port C element

the capacitance, C, may be added to the symbol C indicating the element type. The parameter C is separated from the symbol of the element by a colon (Figure 2.20).

The half arrow of the adjacent bond should point towards the element meaning that the energy store is accumulating energy. If the half arrow is added such that it points away from the element, this might be compensated by a negative parameter. This, however, is a source for potential sign errors if variables and parameters are not used consistently. Moreover, real devices are usually characterised by positive parameters. A mechanical spring, for instance, has a positive stiffness parameter or an electrical capacitor has a positive capacitance. For an electrical capacitor the parameter of the bond graph element equals the capacitance. In the case of a mechanical spring, it equals the inverse of the stiffness parameter. For a hydraulic fluid store, the parameter of the bond graph C element is the ratio of the bulk modulus of the fluid (and the wall of the container) and the container volume.

In the case of a linear characteristic, Equation 2.38 takes the well known form

$$E(t) \;=\; \frac{q(t)^2}{2C} \;. \tag{2.40}$$

If x designates the elongation of a linear spring of stiffness k, then its stored potential energy is expressed as

$$E_{pot}(t) \;=\; \frac{1}{2} k x^2(t) \;. \tag{2.41}$$

For nonlinear characteristics, we will have to distinguish between the stored energy and the so-called co-energy E^*

$$E^*(e) \;=\; \int_0^e \Phi_C(\tilde{e}) d\tilde{e} \;. \tag{2.42}$$

There is a relation between both energies

$$E^* \;=\; e \times q \;-\; E \;. \tag{2.43}$$

The co-energy can be represented by the area above the characteristic $e = \Phi_C^{-1}(q)$.

Definition 2.18 (*1-port I energy store*). An ideal 1-port element of type I is defined by a one-to-one function $\Phi_I : \mathbb{R} \rightarrow \mathbb{R}$ relating the flow variable, f, of the power port to the generalised momentum p

$$p(t) \;=\; \Phi_I(f(t)) \;. \tag{2.44}$$

The function Φ_I must have a unique single valued inverse Φ_I^{-1}

(cf. [22]). According to its definition (see Equation 2.11), the rate of the generalised momentum equals the effort variable of the power port

$$\dot{p} = e \, . \tag{2.45}$$

An element defined by Equations 2.44 and 2.45 is called an *inertia*, or an I element in bond graph methodology. In non-relativistic mechanics, Equation 2.44 is a linear relation. Its parameter designated by the letter I may be the mass or the moment of inertia of a rigid body, whereas in electrical engineering, Equation 2.44 relates the flux linkage λ and the current i. In the latter case, Equation 2.11 is a nonlinear relation in general. That is, in mechanics, a bond graph I element represents the storage of kinetic energy of a rigid body. In electrical engineering, the I element accounts for the storage of magnetic energy in a coil.

If we assume that the initial momentum vanishes, that is, the I energy store is empty at $t = 0$, then the amount of stored energy is

$$E = \int_0^p \Phi_I^{-1}(\tilde{p}) d\tilde{p} \, . \tag{2.46}$$

In hydraulics, the use of a generalised momentum is uncommon. However, Newton's third law can be applied to an incompressible fluid in a volume of length l. Assuming a 1-dimensional flow and by replacing mechanical quantities by their corresponding hydraulic ones in the two equations

$$p = m \times v \tag{2.47}$$

$$\dot{p} = F \tag{2.48}$$

yields

$$\Delta p = I \times \dot{Q} \tag{2.49}$$

and

$$I = \int_0^l \frac{\varrho}{A(x)} dx \, . \tag{2.50}$$

In these equations, Δp designates the pressure drop across the length l and $A(x)$, the cross section area of the volume at the position x ($0 \leq x \leq l$). That is, the parameter I represents the inertia of a 1-dimensional incompressible fluid in a volume of length l. As can be seen, the smaller the cross section area of a pipe, the higher the inertia of the fluid.

As to the orientation of the bond connected to a port of an I element and regarding the annotation of the parameter, the same rules hold as for a C store (Figure 2.21).

Table 2.6 lists some examples of devices and physical effects in various energy domains that can be approximately described by a C energy store or by an I energy store. The missing entries indicate that there are no inertia elements in the mag-

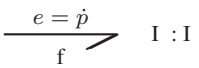


Fig. 2.21 1-port I element

Table 2.6 Capacitor and inertia in various energy domains

Energy domain	C store	I store
Translational mechanics	Spring	Rigid body
Rotational mechanics	Torsion spring	Flywheel
Electro-	Capacitor	Coil
magnetic domain	Ferromagnetic material	—
Hydraulics	Fluid compressibility	Fluid inertia
Thermodynamics	Lump of material	—

netic and in the thermal domain, as it is well known. This is also the reason why in Table 2.4, generalised momenta for these energy domains are not given.[2]

[2] Occasionally, it has been argued in the literature that a thermal inertia would violate the second law of thermodynamics. In order to prove this statement, a *hypothetical* thermal inertia with the constitutive equation

$$\frac{d\dot{S}(t)}{dt} = \frac{1}{I} \times (\Delta T)(t)$$

is assumed. Apparently, such a relation implies a constant entropy flow, $\dot{S} = \text{const.} = \dot{\mathrm{S}}(\mathrm{t} = \mathrm{t}_0)$ in the case of a vanishing temperature difference $\Delta T = 0$. It has been argued that this would contradict the second law of thermodynamics. Consequently, thermal inertia could not exist. However, since this proof, constructing a contradiction does not provide a constant > 0 and the latter could be zero as well. In this case, there would be no contradiction and nothing has been proven.

The Thermal Capacitor

In Table 2.6, a lump of material is considered a thermal capacitor. The constitutive equation of such an energy storage element is

$$S = \Phi_S(T) \tag{2.51}$$

relating entropy[3], S, to temperature, T, can be derived in the following manner. First, we assume that the expansion of the volume can be neglected when the lump of material under consideration is heated. That is, the mechanical work it performs can be neglected. Otherwise, an additional port would be needed at which mechanical energy may enter or leave. From physics, it is known that the thermal capacitance is defined as the stored amount of heat, Q, divided by the temperature increase ΔT. Differentiation of the equation

$$Q = C \times \Delta T \tag{2.52}$$

with respect to time and observing that $\dot{Q} = T\,\dot{S}$ results in

$$T\,\dot{S} = C\,\dot{T}\,. \tag{2.53}$$

Integration with respect to time and defining $T_0 := T(t = t_0)$, $S_0 := S(t = t_0)$ gives the required constitutive relation

$$S - S_0 = C\,\ln\frac{T}{T_0}\,, \tag{2.54}$$

relating the effort T to the displacement S.

Equivalent Representation of the I Store

The introduction of a second type of an energy store indicates that this type is simply obtained by interchanging the role of effort and flow. For this reason, the I energy store is called *dual* to the C energy store. An I element can be replaced by combining a C energy store with a gyrator of unity ratio. Such a gyrator has been termed *symplectic gyrator* [5]. This equivalence can be seen by taking the following steps. First, Equation 2.27b of the gyrator is differentiated with respect to time. Then, $\dot{e}_2$ is replaced by f_2 using the constitutive relation of a C store. Finally, f_2 is replaced by e_1 using Equation 2.27b of the gyrator.

$$\dot{f}_1 = \frac{1}{r}\dot{e}_2 = \frac{1}{r}\frac{1}{C}f_2 = \frac{1}{r}\frac{1}{C}\frac{1}{r}e_1$$

[3] Around 1855, Clausius introduced entropy by the equation

$$dS = \frac{dQ}{T}\,,$$

where dQ denotes the change of heat.

Fig. 2.22 Equivalence of an I energy store to a gyrator–C energy store combination

$$= \frac{1}{r^2 C} e_1 = \frac{1}{I} e_1 \,, \tag{2.55}$$

where $I := r^2 C$ (Figure 2.22).

The equivalent representation of an I energy store by means of a symplectic gyrator and a C energy store could be used to give up the introduction of a second store in order to overcome the discrepancy that the second type of an energy store does not exist in all energy domains. For this reason, P. Breedveld introduced a general uniform bond graph concept that uses only the C type store. First, he called such bond graphs *thermodynamic* bond graphs [3]. Later, he changed the term into *Generalised Bond Graphs* [4]. Most bond graph modellers, however, prefer to keep two types of stores for convenience, although this is not fully satisfying with regard to a general uniform theory. In this book, we will follow the long lasting tradition of using two types of energy stores.

Are There Controlled Energy Stores?

In contrast to ideal power conservative couplers or transducers, energy storage elements cannot be controlled by a signal (Figure 2.23) because this would violate the principle of energy conservation.

Consider, for instance, a capacitor with movable plates. For such devices, the voltage e across the two terminals not only depends on the charge q of the plates, but also on their distance x. If $C(x)$ designates the capacitance, then

$$e(t) = C^{-1}(x(t)) \times q(t) \,. \tag{2.56}$$

Consequently, the stored energy is a function of the charge q and the distance x of the plates

$$E(q, x) = \frac{q^2}{2C(x)} \,. \tag{2.57}$$

Thus, a change of the stored energy is

Fig. 2.23 Non-existing signal controlled energy C store

$$P(t) \;=\; \dot{E}(t) \;=\; \frac{\partial E}{\partial q}\dot{q}(t) \;+\; \frac{\partial E}{\partial x}\dot{x}(t)\;. \tag{2.58}$$

If a *signal* controlled energy store of variable capacitance is assumed, then the second term in that sum must vanish because the energy flow associated with a controlling signal is neglected. For this reason, there is a distinction between power ports and signal ports in bond graphs. This, however, means that the stored energy and hence the voltage e cannot be a function of the modulating signal. Thus, modulated stores cannot exist [2].

In other words, if the stored energy, E, is a function of q and x, then Equation 2.58 must hold. However, if $P(t) = e(t) \times \dot{q}(t)$, then the second term in Equation 2.58 must be zero. As $\dot{x} \neq 0$, the factor $\partial E/\partial x$ must vanish. This means that

$$\frac{\partial}{\partial q}\left(\frac{\partial E}{\partial x}\right) \;=\; 0\;. \tag{2.59}$$

On the other hand, as E is assumed to be a function of q *and* x,

$$\frac{\partial}{\partial x}\left(\frac{\partial E}{\partial q}\right) \;\neq\; 0\;. \tag{2.60}$$

That is, the value of the line integral

$$\int_{\mathscr{C}} \left(\frac{\partial E}{\partial q}\right) dq \;+\; \left(\frac{\partial E}{\partial x}\right) dx \tag{2.61}$$

along the path $\mathscr{C}$ is not independent of the path $\mathscr{C}$, which means that the *modulated* 2-port C element is not energy conservative.

This does not mean that the stored energy cannot depend on a variable distance of the plates. However, it must be taken into account that a change of their distance is combined with an energy flow. Accordingly, a modulated C energy store is to be replaced by a multiport store. Such elements will be considered in Chapter 8.

2.5.4 Dissipators

Since there is no loss of energy, the notion of *free* energy is introduced.

Definition 2.19 (*Free energy*). If E denotes the total energy in a system and E_{th} the thermal energy, then the free energy, E_f, is the difference

$$E_f \;:=\; E \;-\; E_{th} \tag{2.62}$$

(see, for instance, [9]).

The loss of *free* energy due to friction in mechanical and hydraulic systems, due to heat production in electrical circuits, or due to thermal conduction is modelled in bond graphs (like in generalised networks) by means of resistors. In bond graphs,

Fig. 2.24 1-port R element

the type of these elements, which may be multiport elements, is designated by the symbol 'R'. In the following, 1-port resistors will be considered.

Definition 2.20 (*1-port resistor*)**.** A 1-port resistor is defined by one of the two constitutive equations relating the power port variables e and f

$$e(t) = \Phi_R(f(t)) \tag{2.63}$$

or

$$f(t) = \Phi_G(e(t)) \,, \tag{2.64}$$

where $t \in \mathbb{R}$ and $t \geq 0$. Both functions, Φ_R and Φ_G, must be one-to one and have a single valued inverse. Their characteristics must fall into the first and third quadrants of the e-f plane.

In case of a 1-port resistor with a linear characteristic

$$e = r \times f \,, \tag{2.65}$$

the parameter r may be attached to the symbol R denoting the type of the element (Figure 2.24). In the general case of a (non)linear (multiport) resistor or (multiport) energy store, the symbol X (C or R) may be annotated by a string (Figure 2.25). In terms of object-oriented modelling, X : s denotes an instantiation of the class X. The string s is the name of the instantiation that allows one to distinguish it from others. In case of a single element parameter, the string may be the name of that parameter. For more than one parameter, s may be the name of the set of parameters used in the constitutive equations. This is similar to the way in which elements in a circuit diagram are distinguished. There is a standardised graphical symbol for an element of type resistor. Different resistors have different names. The name R1 associated with one of them may be the name of its parameter having a value of $10\ \Omega$, viz., $R1 = 10\ \Omega$. Resistors are also called dissipators, or R elements.

The *free* energy entering an R element is lost in the system. It is dissipated by the element. However, according to the first law of thermodynamics, energy cannot is conserved. In R elements, energy is converted irreversibly into heat. It is appropriate to define the direction towards the element as the positive reference direction of the

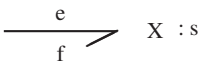


Fig. 2.25 Nonlinear 1-port element of type X (C or R) with s being the name of its parameter set

energy flow. Since true resistors irreversibly convert energy into heat, the reference direction is also the actual direction of the energy flow. Thus, in addition to the constitutive relation of an R element, there is the constraint $e(t) \times f(t) > 0 \quad \forall t > 0$.

Remark 2.6. The requirement of single valued inverse functions Φ_R^{-1} and Φ_G^{-1} shall ensure that the constitutive relation of an R element can be solved for the power variable in the function's argument list if necessary due to the connection of the resistor with other elements. Consider, e.g., an R element and a C energy store both of which are connected to a 0-junction as depicted in Figure 2.26.

Assume that the R element is defined by a nonlinear relation $e_2 = \Phi_R(f_2)$, whereas the characteristic of the C energy store is linear for simplicity. Observing the equations of the 0-junction, we have the following equations

$$e_1 = e_2 = e_3 \tag{2.66a}$$

$$e_2 = \Phi_R(f_2) \tag{2.66b}$$

$$C\,\dot{e}_3 = f_1 - f_2\,. \tag{2.66c}$$

Now, suppose the effort e_1 is to be computed for a given flow f_1. This is only possible if Φ_R has a unique inverse.

If the constitutive relation of an R element does not have a unique inverse relation, then Beaman and Rosenberg assume that the characteristic describes a composite element that captures more than one physical effect [2] and they make the conjecture that with more detailed modelling, such composite elements could be avoided. Despite the problem with the guaranteed uniqueness of the solution, it is common and convenient in electronics to model real devices by resistors with a characteristic that exhibit a negative gradient in some region. This modelling approach is less common in other disciplines. A well known example in mechanics is the approximation of dry friction by a `sign` (signum) function neglecting stiction.

In contrast to stores, R elements may be modulated by a signal.

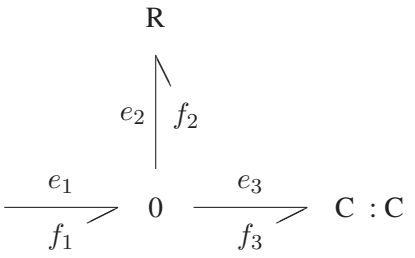


Fig. 2.26 Connection of a nonlinear R element and a linear C store

The RS Element

If isothermal conditions cannot be assumed, e.g., the impact of temperature on the operating point of an electronic circuit, or on the elasticity of the oil in a hydraulic hose must be taken into account, then instead of free energy, total energy including thermal energy must be considered. Since total energy is conserved in a closed system, dissipators become energy transducers converting non-thermal energy into heat. For that reason, a closer look reveals that 1-port R elements have an additional port, i. e., a thermal port. Since the conversion into heat is irreversible, dissipators can be considered to be heat sources in the thermal domain. For that reason, Thoma introduced the symbol 'RS' [34]. It accounts for the loss of free energy at the non-thermal port and the production of entropy at the thermal port (Figure 2.27). The letter 'S' means source and expresses the source character in the thermal domain. Since this special transducer does not store energy, the principle of power conservation must hold.

$$e\,f \;=\; T\,\dot{S} \tag{2.67}$$

According to the second law of thermodynamics, $\dot{S} > 0$. Moreover, since $T > 0$, power conservation implies that the power port variables of a dissipator must hold the constraint $e(t) \times f(t) > 0, \quad \forall t > 0$, as stated previously.

Since the power conversion is unidirectional from non-thermal energy into heat, it cannot be represented by a transformer with constant modulus or by a gyrator with constant ratio. The orientation of bonds connected to the ports of the RS element introduced by Thoma indicates the actual direction of the irreversible energy flow.

From Equation 2.67, it can be seen that the additional constitutive relation of an RS element is always nonlinear, even if the relation between the non-thermal power port variables is linear. Assuming $\Delta e = R \times f$, then power conservation yields for the thermal port

$$\dot{S} \;=\; \frac{R\,f^2}{T}\;. \tag{2.68}$$

Since the absolute temperature T cannot be negative, it follows from the second law of thermodynamics that the resistance parameter must be positive, $R > 0$. Or, the other way round, given a positive resistance, R, a resistor, in fact, produces entropy in accordance with the second law of thermodynamics.

Transfer of the generated heat may take place either via thermal conduction, convection, or radiation.

Fig. 2.27 Extension of a 1-port R element according to Thoma

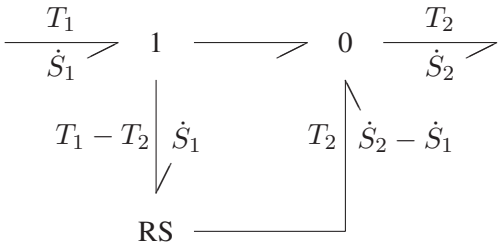


Fig. 2.28 Bond graph model of heat conduction using the RS element (Thoma 1975)

Example: Thermal Conduction

Thermal conduction between two points with absolute temperatures T_1 and T_2, and $T_1 > T_2 > 0$ may be modelled by means of an RS element as depicted in Figure 2.28 [33].

For the RS element in Figure 2.28 power conservation reads

$$(T_1 - T_2)\,\dot{S}_1 = T_2\,(\dot{S}_2 - \dot{S}_1)\,. \tag{2.69}$$

With $\dot{Q}$ denoting the heat flow from bond 1 to bond 2, it follows

$$\dot{Q} = T_1\,\dot{S}_1 = T_2\,\dot{S}_2\,. \tag{2.70}$$

That is, the model for heat transfer from point 1 to point 2 is power conservative. From Equation 2.69, it follows that

$$\dot{S}_2 - \dot{S}_1 > 0 \tag{2.71}$$

because the heat flow into the RS element is positive, $(T_1 - T_2) \times \dot{S}_1 > 0$, and temperature $T_2 > 0$. That is, at the point where the heat flow leaves the lump, entropy is higher than the entropy at the point where the heat flow enters the lump. Thus, there is an entropy flow from the higher to the lower temperature and entropy increases in accordance with the second law of thermodynamics.

Heat conduction is also described by Fourier's law in the form

$$\dot{Q} = K\,(T_1 - T_2)\,. \tag{2.72}$$

K denotes a constant that depends on the thermal conductivity and the geometry of the heat conductor. Fourier's law and Equation 2.70 for power conservation yield for the two entropy flows

$$\dot{S}_1 = K\,\frac{T_1 - T_2}{T_1} \tag{2.73a}$$

$$\dot{S}_2 = K\,\frac{T_1 - T_2}{T_2}\,. \tag{2.73b}$$

For the irreversible entropy production during heat transfer, it follows

$$\dot{S}_2 - \dot{S}_1 \;=\; K\,\frac{(T_1 - T_2)^2}{T_1 T_2} > 0\;. \tag{2.74}$$

Thus, the constitutive equations of the RS element read:

$$\dot{S}_1 = K\,\frac{T_1 - T_2}{T_1} \tag{2.75a}$$

$$\dot{S}_2 - \dot{S}_1 = K\,\frac{(T_1 - T_2)^2}{T_1 T_2}\;. \tag{2.75b}$$

Remark 2.7. For an RS element, relating a non-thermal energy domain to the thermal domain, energy that disappears at the non-thermal port *reappears* at the thermal port. In the case of thermal conduction, the amount of heat entering into the R-port remains in the thermal domain. It is fed back to the system via the source port of the RS element. Hence, the flow of produced entropy must be added to a junction in the model. In Figure 2.28, this is done by adding it to the outward power port of the model. Without affecting the port behaviour of the heat conduction model, the flow of produced entropy can also be added to the model's inward power port. In the alternative model, the locations of the 1- and the 0-junction are interchanged.

The heat conduction model in Figure 2.28 assumes $T_1 > T_2$. Index 1 denotes the inward power port and index 2 the outward power port. In the case $T_2 > T_1$, heat conduction is represented by the slightly modified bond graph model in Figure 2.29 [33].

Instead of the RS element, a modulated transformer may also be used for modelling heat conduction as depicted in Figure 2.30. The transformer expresses power conservation (Equation 2.70). The variable transformer modulus results from Equations 2.73a and 2.73b. Finally, the modulation of the transformer correctly reflects the production of entropy.

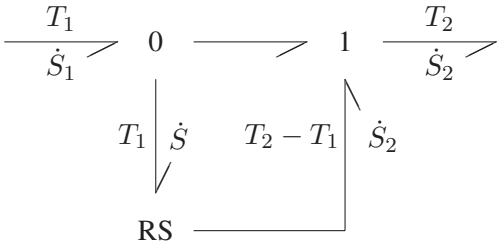


Fig. 2.29 Heat conduction model using an RS element in case $T_1 < T_2$

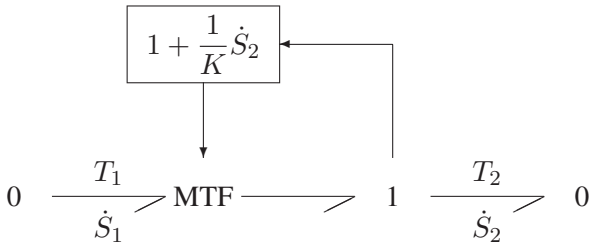


Fig. 2.30 Modelling heat conduction by means of a modulated transformer

$$\dot{S}_2 - \dot{S}_1 = \frac{1}{K}\,\dot{S}_1\,\dot{S}_2 \geq 0 \tag{2.76}$$

Like the bond graph of Figure 2.29, its alternative of Figure 2.29 assumes that $T_1 > T_2$. In case $T_2 > T_1$, the modulus is determined by $\dot{S}_2$ instead of $\dot{S}_1$.

Example: Electrical Current and Entropy Flow in an Electrical Conductor

Let us return to the irreversible conversion of non-thermal energy into heat and consider an electrical conductor of resistance R. The conductor gets heated while an electrical current is flowing. Thus, there is a flow of electrical energy and of entropy as well. The entropy flow can be accounted for by replacing the common electrical 1-port resistor by the submodel depicted in Figure 2.31. The result is an extended bond graph that uniformly accounts for the electrical as well as for the thermal energy flow.

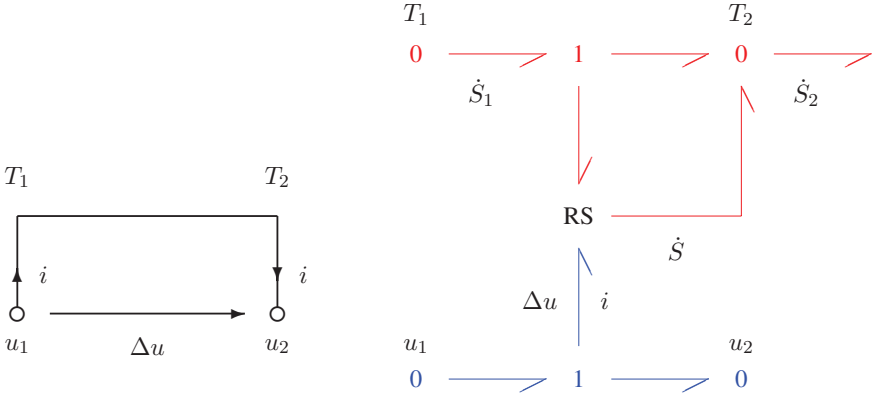


Fig. 2.31 Flow of electrical energy and of entropy in an electrical conductor

A closer look reveals that in an electrical conductor not only electrical energy flow and heat conduction take place. The material also stores thermal energy as illustrated in Table 2.6. The storage of thermal energy can be taken into account by adding a thermal capacitor to the 0-junction. That is, thermal energy storage distributed over the volume of the conductor has been lumped into the point with the lower temperature T_2 in this one-dimensional model.

In conclusion, the R element representing the loss of *free* energy in the non-thermal energy domains is to be replaced by the RS element, if thermal effects must be taken into account. Finally, a 'dissipator' does not exist in the thermal domain. The thermal energy leaving at the resistive port of an RS element reenters the system at its S-port.

2.5.5 Memristors

As early as 1961, in his class notes for M.I.T. course 2.751 [27], H. Paynter represented the possible functional relationships between the key variables, $\mathbf{e}, \mathbf{f}, \mathbf{p}, \mathbf{q}$ by the so-called *tetrahedron of state*. In this graph, the vertices are associated with the power and the energy variables of a given system while the edges denote functional relationships between them (Figure 2.32).

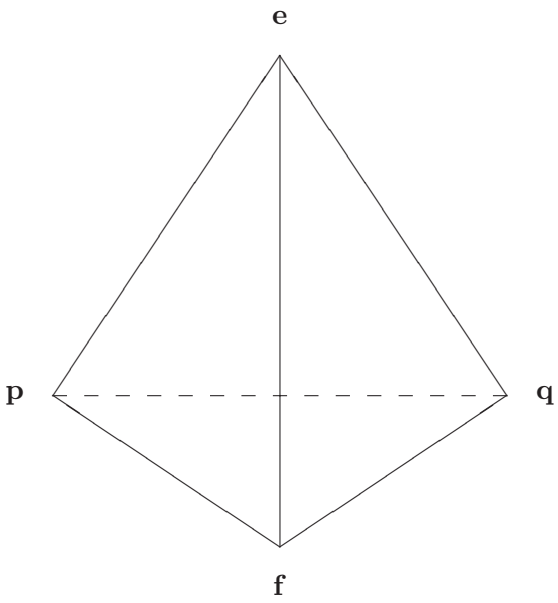


Fig. 2.32 Tetrahedron of state (H. M. Paynter, 1961)

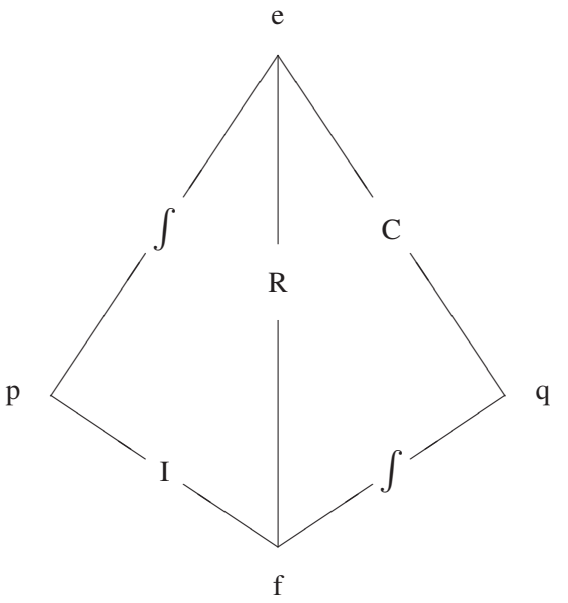


Fig. 2.33 Tetrahedron of state for the three 1-port elements R, C, and I (Karnopp, Margolis, Rosenberg, 2005)

In order to point out the nature of the relationships, other authors have added two integral operators and the basic elements R, C and I, as shown in Figure 2.33 [23].

Interestingly, in his diagram, Paynter has connected the vertices for the energy variables p and q by a dashed line indicating a possible functional relation between the energy variables. The symmetry in the tetrahedron of state may suggest the existence of another basic element besides the two types of energy stores and resistors. In 1971, L. O. Chua postulated the existence of a fourth basic two-terminal circuit element besides the capacitor, inductor and resistor that is characterised by such a relationship between flux linkage and electrical charge. He termed that element *memristor* (memory resistor) and presented an active circuit realisation containing many circuit elements [12]. Inspired by this introduction of a fourth basic circuit element, Oster and Auslander proposed the memristor as a new bond graph element. The abstract of their 1972 article in the Journal of Dynamic Systems, Measurement and Control [26] reads:

> The "memristor", firstly defined by L. Chua for electrical circuits, is proposed as a new bond graph element on an equal footing with R,L, & C, and having some unique modelling capabilities for nonlinear systems.

In [26], the authors call the integral of the effort with respect to time, $p = \int_0^t e(\tau)d\tau$, impulse and distinguish between a charge controlled memristor with a constitutive relation

$$p = G(q) \tag{2.77}$$

and an impulse controlled memristor with a constitutive relation of the form

$$q = F(p) \,, \tag{2.78}$$

where F, G are functions $\mathbb{R} \to \mathbb{R}$ with trajectories in the first and third quadrant. Differentiation of these two forms of constitutive relations with respect to time results in the two equations

$$e = M(q) \times f \tag{2.79a}$$
$$f = W(p) \times e \,, \tag{2.79b}$$

where $M(q)$ is called incremental *memristance* and $W(p)$ incremental *memductance*. The equations obtained after differentiation with respect to time show that a memristor turns into an ordinary resistor in case of a linear constitutive relation. Nevertheless, the memristor is a peculiar element. It is dissipative, but at the same time, it is a dynamic element requiring the specification of an initial value. Oster and Auslander considered a tapered dashpot and an electrochemical system with two oppositely charged membranes that can be modelled by using a memristor. In [13], Chua shows that the electrical behaviour with a hysteresis loop in the voltage-current plane observed from a two-terminal nanowire device driven by a low frequency periodic voltage signal can be explained by using a memristor.

The view that there are physical phenomena that justify the introduction of a memristor to be added to the small set of fundamental bond graph elements has not been shared by most members of the bond graph community. Even in the fourth edition of their renowned textbook [23], Karnopp, Margolis and Rosenberg note that "no element will relate p and q". This statement is annotated by the following footnote

> One can, in fact, define an element corresponding to the hidden edge, the "memristor". While interesting and occasionally useful, memristors can be represented in terms of other elements to be introduced later, so the memristor will not be considered to be a basic element. ...

However, in May 2008, Strukov and his colleagues from a Hewlett-Packard laboratory in Palo Alto, California, USA reported in a communication published in the journal nature that they have been able to build an integrated nanoscale circuit device that behaves like a memristor [32]. The device is composed of a 5 nm titanium dioxide film with two layers of different resistivities that are connected to wire electrodes. One layer has a slight depletion of oxygen atoms which results in a lower resistance in comparison to the non-depleted layer. An electric field applied to the device lets the oxygen vacancies serving as charge carriers pass in one direction. As a result, the boundary between the two layers moves and by this way, the resistance of the device changes. This discovery by Stanley Williams and his team at HP Labs has received considerable attention worldwide. It opens up possibilities for various practical applications and gives rise for further research. As a memristor is a dynamic element that is described by an algebraic relation between effort and flow and an additional differential equation, it will affect standard state space modelling.

The dimension of the state space is not only determined by the storage elements, but also by memristors. An extension of the so-called port-based Hamiltonian formulation framework [35] including memristors is considered by Jeltsema and Maks in a paper presented at the Mathmod 2009 conference [19].

2.5.6 Ideal Energy Sources and Sinks

Having considered passive bond graph elements, in the following, we will briefly address sources. As mentioned, they are not part of a system model itself. Rather, they describe boundary conditions of the system, or in other words, the impact of the environment in which the system is embedded. Like in physical systems modelling by means of generalised networks, in bond graph modelling, there are also two types of sources. They are designated by the symbol 'S' (source). Their type is indicated either by a subscript 'e' or 'f' depending on whether the source imposes an effort or a flow on the system (Figure 2.34). Often, the letter characterising the type of a source is not a subscript. That is, the notations Se and Sf are also used. For sources, it is appropriate to assume the outward orientation as the positive reference direction of the energy flow.

Sources may provide a power variable that is either constant or time dependent. Moreover, sources may be controlled by a signal. In this case, the symbol 'S' is prefixed by the letter 'M' as shown by the example of a modulated effort source on the right-hand side of Figure 2.34. Furthermore, as resistors, energy stores and all other bond graph elements, the symbol of an element of type source may be annotated by a string that distinguishes the source from other sources of the same type. The string may also be a constant parameter. This is convenient if, for instance, the source represents a vanishing boundary condition, or if the source provides a power variable of constant value (Figure 2.35).

Examples of Sources

With good approximation, gravity near the surface of the earth may be modelled by a constant effort source independent of the coordinates of the place where gravity is effective. Gravity acting on a rigid body of mass m can be represented as shown in Figure 2.35.

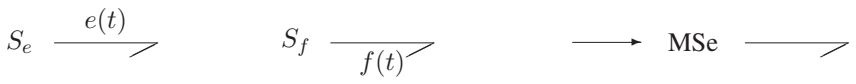


Fig. 2.34 Ideal sources in bond graphs

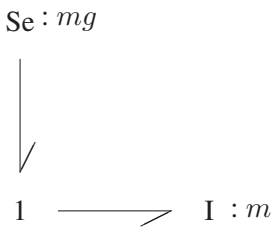


Fig. 2.35 Bond graph fragment accounting for gravity acting on a rigid body

If only the current is of interest that an amplifier feeds into an electrohydraulic servovalve then the amplifier may be captured by a flow source. Furthermore, isentropic boundary conditions of a system may be represented by a flow source imposing a vanishing entropy flow ($dS/dt = 0$).

Another example in which boundary conditions are to be represented by sources is a submodel of a rigid body with two hinge points. The model has two ports for translational velocities and two ports for angular velocities to the outside world. If the body as part of a manipulator is mounted on a basis that does not move, then the two ports of the hinge connected to the bed need to be connected to flow sources that provide zero values.

Controlled Sources

In modelling electronic circuits, it is appropriate and convenient to use controlled sources. The core of a functional model that captures the terminal behaviour of an operational amplifier, for instance, is often a voltage controlled voltage source. Furthermore, if the dynamic behaviour of an actuator in a controlled engineering system is not relevant, it may be represented by a source that is controlled by a signal provided by a signal processor. The latter serves as a controller. A D/A converter converts the signal into an analog signal of low power level, while it is the role of an actuator to provide an effort or a flow at a sufficient power level that affects the system behaviour. Therefore, controlled sources are also available in bond graph modelling. If a source is controlled by a signal, then this is often highlighted by a letter 'M' preceding the source symbol (cf. Figure 2.34).

However, one should be aware of potential risks. Controlled sources can be used to represent relations that do not comply with the principle of energy conservation. In [20], Karnopp and Rosenberg stress

> ... at once very powerful and at the same time hard to discipline, because the nature of the signal input that sets the modules appears to be quite arbitrary. It is possible to represent virtually any system using these modulated 2-ports and other elements in a variety of ways if the moduli are allowed to be dynamically related to system variables. When much of the dynamics of the system is put into signals setting the moduli, the organizational structure which bond graph techniques usually bring to the study of physical system dynamics may be obscured.

This statement is also recalled by Beaman and Rosenberg in [2]. In [1], Beaman and Breedveld conclude

> Although the models with controlled sources are functionally correct, these active sources can violate the energetic basis of bond graphs. Hence, they should be avoided in physical models, whenever possible.

In other words, if given relations are essentially represented by means of controlled sources, then the virtue of bond graphs over block diagrams expressing the physical structure gets lost. Moreover, in bond graphs, the use of controlled sources in accordance with the principle of energy conservation, as required by Paynter for physical system models, is not evident. As mentioned in Section 1.4.1, in block diagrams, all kinds of functional relations may be represented that are not necessarily consistent conservation laws from physics. This is also possible in bond graphs if controlled sources are used. If, however, bond graphs are meant to be more than merely a graphical representation of equations, then they should at least comply with the principle of energy conservation as they represent the energy exchange between subsystems.

From power continuity of the RS element (Equation 2.67, it can be seen that the flow of entropy provided at the S-port of the element is not independent of the temperature T. Moreover, it depends on the power fed into the element from the non-thermal side. Hence, from the thermal side, the RS element may be considered a controlled source [18].

Energy Sinks

The environment of a system may be considered intuitively to be a reservoir of infinite capacity. Independent of the amount of entropy flow it receives from the system, the ambient temperature remains constant. A similar observation holds for a hydraulic return reservoir. Independent of the amount of returning hydraulic volume flow, the pressure in the tank remains constant (at atmospheric level). That is, the environment imposes a boundary condition on the system and at the same time it receives an energy flow that does not affect its impact on the system. Both aspects suggest the use of a *sink*, i.e., a source element with a positive reference direction of the energy flow *towards* the element. Isothermal boundary conditions, e.g., may be represented by an effort sink that imposes a constant temperature on the system independent of the amount of entropy it receives from the system. This view does not exclude the possibility that a sink may operate temporarily as a source. Likewise, sources may operate temporarily as sinks.

According to Equations 2.73a and 2.73b, heat conduction between two points in space may be represented by a doubly modulated entropy source (Figure 2.36). This is an alternative to the modulated transformer representation introduced in the previous section (Figure 2.30). While the element is power conservative, it produces entropy. If the reference direction in Figure 2.30 corresponds with the actual direction of energy flow, i. e., there is a heat flow from T_1 to T_2, then the source receives an entropy flow at the left port while it provides a higher amount of entropy flow at

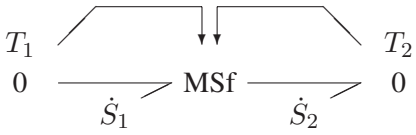


Fig. 2.36 Representation of heat conduction between two temperatures by a doubly modulated entropy flow source

its right port. If the actual direction of the energy flow is opposite to the reference direction, then it becomes a negative source at the port where the adjacent bond is pointing outward and a negative sink at the other port.

2.5.7 Sensors

For the control of engineering systems, sensors are clearly necessary to measure aspects of the system response, e.g., velocities and displacements in mechanical systems. Sensors, in general, perform a conversion of a non-electrical signal of low power into an electrical one, which is converted into a digital signal and fed into a controller via a feedback loop. A characteristic feature of sensors is that they sense a signal without affecting the system. The amount of power they take out of the system is very small and can be neglected. If the dynamic behaviour of a sensor can also be neglected, then the device can be modelled by an energy sink that provides a zero effort or a zero flow.

According to Table 2.7, an effort sensor can be represented by a zero flow sink and a flow sensor can be modelled by a zero effort sink. Table 2.7 also includes alternative representations of ideal sensors that are particularly popular in the community of bond graph modellers in France. The symbol for the type of element clearly indicates which power variable is measured and the signal arrow points out that the conjugate power variable vanishes. In order to distinguish between detectors representing real sensors of measurable variables and fictitious (virtual) detectors of non-measurable variables, a star, $*$, is added as a superscript to the latter (De^* and Df^*) [14].

Table 2.7 Representation of sensors in bond graphs

$0:$ Sf $\overset{e}{\underset{f=0}{\lessdot}}$ 0	Effort sensor, zero flow source	De $\longleftarrow$ 0	Effort detector
$0:$ Se $\overset{e=0}{\underset{f}{\lessdot}}$ 1	Flow sensor, zero effort source	Df $\longleftarrow$ 1	Flow detector

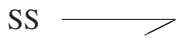


Fig. 2.37 Source-sensor element (Gawthrop and Smith, 1996)

Fig. 2.38 Connection of passive 1-port elements

When a source imposes a power variable on a system, then, at the same time, the conjugate power variable is the feedback of the system onto the source. Hence, the source can be a sensor of the conjugate power. Therefore, Gawthrop and Smith introduced a so-called *source-sensor* element, SS, that combines a source with a sensor. This element can be used for representing sources as well as sensors and has proven particularly useful in so-called bicausal bond graphs used for system inversion (cf. Sections 6.3 and 6.7). The SS element, shown in Figure 2.37, can represent either an effort source and simultaneously a flow sensor, or it can be a flow source combined with an effort sensor.

The two detector elements (De and Df) can be considered special cases of this SS element. An effort detector corresponds to a zero flow source-effort sensor, while a flow detector is a zero effort-flow sensor (cf. Section 6.7 on bicausal bond graphs, Table 6.2).

These considerations of sources close the introduction of basic ideal bond graph elements. The top-down decomposition of an initial word bond graph model comes to an end when incompletely specified multiports representing submodels have been replaced recursively until bond graphs of submodels are only composed of basic bond graph elements. We shall call a bond graph model at the bottom of the model hierarchy built by basic elements an *elementary* bond graph. Note that, due to the conventions for the reference directions of energy flows, power ports cannot always be connected directly by a bond. For instance, passive elements cannot be joint directly (Figure 2.38).

2.6 Pseudo Bond Graphs

In the previous section, basic elements have been introduced that enable a unified physical modelling approach for all energy domains on the basis of one of the two analogies discussed in Section 2.3. Both analogies, the classical force - voltage analogy as well as the dual mobility analogy, are in accordance with the general observation that in all energy domains, the amount of power transferred between two power ports may be expressed as the product of two power conjugated variables.

Table 2.8 Effort and flow variables in pseudo bond graphs

	Hydraulics	Thermodynamics	
Effort	Pressure	Temperature	Temperature
Flow	Mass flow	Heat flow	Enthalpy flow

This is essential in bond graph modelling. Nevertheless, in considering thermal or hydraulic systems, occasionally, it is convenient to choose effort and flow variables not as indicated in Table 2.4 and to accept that their product is *not* the power transferred between ports. Although the basic principle does no longer applies, again, basic elements describing physical effects in an idealised manner can be introduced. Moreover, the systematic construction of a bond graph from a schematic and, furthermore, the systematic derivation of equations from the bond graph still remain applicable. However, since such bond graphs do not represent energy flows, they are called *pseudo bond graphs* in the literature. In general, such pseudo bond graphs cannot be connected to true power bond graphs via transformers or gyrators. Their advantage is that modelling of thermodynamic systems may be become easier (cf. Chapter 10). On the other hand, it may be considered a disadvantage that pseudo bond graph modelling of physical effects is sometimes not quite convincing. If, for instance, in the magnetic domain, the magnetic flux is chosen as a flow instead of the flux rate, then the magnetic capacitor with the capacitance parameter C becomes a resistor with the resistance $R_{mag} = 1/C$. Table 2.8 shows some choices of effort and flow variables in the hydraulic and in the thermal domain for which the product is not the power transferred between ports. The choice of mass flow and an enthalpy flow takes into account that in open systems, an energy flow between subsystems is accompanied by a flow of mass. It is common to consider a control volume and to set up balances for energy, matter and momentum. Whereas a correct true bond graph representation is not always easy, a pseudo bond graph approach is similar to common engineering practice, especially in modelling process engineering systems. Often, pseudo bond graph modelling is considered to be more intuitive. True bond graph modelling of open thermal systems is dealt with in Chapter 10.

Example: Heat Transfer Through a Slab of Material

In the following, pseudo bond graph modelling shall be illustrated by considering heat transfer through an insulating slab of material. Since in this case, there is no flow of matter, according to Table 2.8, the absolute temperature T may be chosen as a effort variable and the heat flow $\dot{Q}$ as a flow variable (The latter variable already has the physical dimension of power).

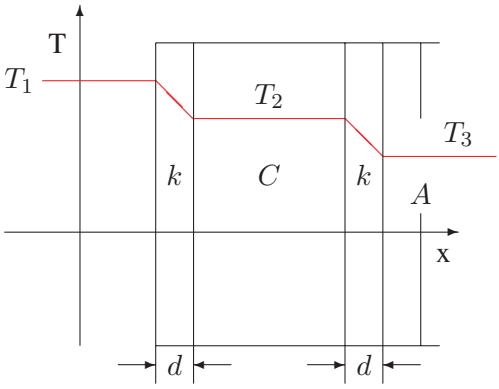


Fig. 2.39 Isolation between two temperatures

Consider two insulating parallel layers of thickness d and area A as depicted in Figure 2.39. The schematic may represent a piece of a wall separating a room of temperature T_1 from the colder surrounding at temperature T_3.

Inside the wall, between the two layers, a uniform temperature distribution is assumed. If k denotes the thermal conductance coefficient of the two layers, then Fourier's law reads

$$\dot{Q}_1 = \frac{kA}{d}\,(T_1 - T_2)\,. \tag{2.80}$$

According to the choice of effort and flow, Equation 2.80 may be represented by a combination of a resistor with the resistance parameter $R = d/(kA)$ and a 1-junction accounting for the temperature difference.

Due to the heat flows entering and leaving the lump of material between the two insulating layers, the spatially uniformly distributed temperature T_2 changes. The amount of heat stored in the wall is

$$Q_1(t) - Q_3(t) = C\,(T_2(t) - T_{20})\,, \tag{2.81}$$

where $T_{20} := T_2(t = 0)$. Since Equation 2.81 relates temperature T_2 to the displacements Q_1, Q_2, it may be considered to be the constitutive equation of a 1-port capacitor with the capacitance parameter C. This capacitor model assumes that the change of the volume between the two layers can be neglected. Let c denote the specific heat at constant volume and m the mass of the material lump between the two layers, then $C = cm$. The difference of heat flows rates can be represented by a 0-junction connected to the 1-port capacitor. If heat production inside the room (temperature T_1) is represented by a temperature source and the surrounding, considered to be a reservoir of infinite capacity at temperature T_3, is represented by a thermal effort sink, then the combination of the two resistors and the capacitor results in a coarse model of the wall displayed in Figure 2.40.

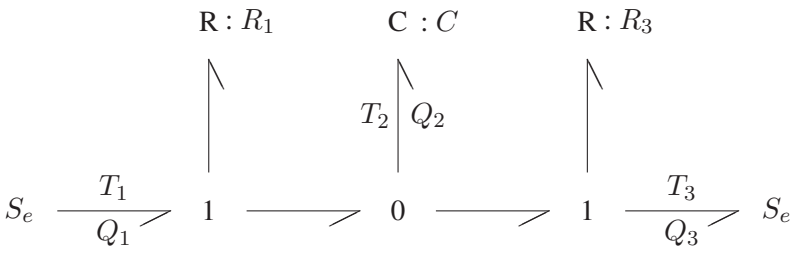


Fig. 2.40 Coarse pseudo bond graph model of a wall

2.7 Systematic Construction of Bond Graphs

Having introduced basic bond graph elements for representing fundamental physical processes and discussed reference directions for energy flows, the question now is how these elements can be combined in a systematic manner in order to come up with a bond graph model of a real physical process at the bottom of a model hierarchy. In other words, how can a non-hierarchical bond graph model be constructed in a systematic manner from a given system schematic. Procedures for each energy domain have been given by Karnopp, Margolis and Rosenberg in their textbook [23]. Breedveld gives a uniform formulation for all non-mechanical energy domains (electrical, magnetic, hydraulic, thermal domain). Since this method does not represent reference nodes in the bond graph and accounts for simplifications of structures in constructing the bond graph, the result does not exhibit the close topological affinity with the system schematic that an initial bond graphs shows if the procedure of Karnopp, Margolis and Rosenberg is applied. It may be a matter of personal preference, whether one or the other of the two procedures or a modification of them is used. At least for students, it might be useful if the construction of an initial bond graph is guided by the topology of the system schematic and if the initial result clearly resembles the system schematic. In subsequent steps, the initial bond graph can be simplified and completed following the given rules. In this book, we follow the procedure of Karnopp, Margolis and Rosenberg. In the following, the construction of bond graphs for planar motion of mechanical systems will be discussed first and then illustrated by an example.

2.7.1 Construction of Bond graphs for Mechanical Subsystems (Translation and Fixed-axis Rotation)

The starting point for a systematic construction of bond graphs for mechanical subsystems are distinct velocities and angular velocities.

1. Identify distinct inertial velocities and angular velocities and represent them by a 1-junction.

   They should be annotated by a name in order to express which 1-junction represents which velocity. Ground (zero absolute velocity) is often represented by a 1-junction with a zero velocity (flow) source (Sf) attached. For the sake of a closer structural similarity between a given system schematic and an initial bond graph, the 1-junction representing zero absolute velocity may be repeatedly used in various places of the bond graph.

2. Connect 1-port C energy stores representing springs and 1-port resistors for dashpots to a 0-junction and insert them between proper pairs of 1-junctions.

   Insert 2-port transformers, or 2-port gyrators between proper pairs of 1-junctions.

   Springs and dampers react to a velocity *difference*. They provide a force or a moment acting equally on both velocity points. Therefore, their corresponding bond graph element is connected to a 0-junction that accounts for the velocity *difference*. The 0-junction is inserted between the two 1-junctions representing the velocities. In Figure 2.41, the symbol 'X' may be replaced either by a 'C' or an 'R' element. In order to ensure that there is a velocity difference at the port of the element, one bond must be oriented towards the 0-junction, the other one must point away from the 0-junction and the third one must be inward to the element port. Transformers couple velocities. For gyrators, two velocities are either inputs or both are outputs. Consequently, both types of elements are inserted between appropriate pairs of 1-junctions. The symbol 'TG' stands for 'TF' or 'GY' (Figure 2.42).
3. Add 1-port inertia elements to their respective 1-junction.

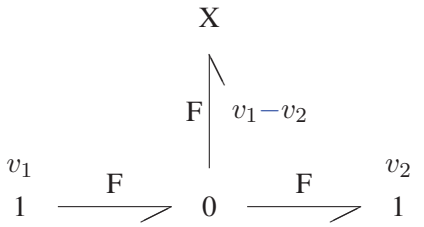


Fig. 2.41 Springs and dampers between two velocities

Fig. 2.42 2-port transformer or gyrator between two velocities

Since the motion of the centre of gravity of a rigid body, or the rotation with respect to a fixed axis is defined with reference to an inertial frame, stores of kinetic energy are attached directly to the 1-junction representing their velocity or angular velocity. The body's kinetic (co-)energy is a function of its velocity.

4. Add 1-port sources and 1-port sinks to appropriate 1-junctions.

   Sources and sinks represent boundary conditions. For instance, each body is subject to gravity force, or a motor provides an angular velocity. Since 1-junctions not only represent velocities or angular velocities, but also the sum of forces or moments acting on a body, sources or sinks are attached directly to the proper 1-junction.

5. Orientation of bonds

   Having inserted all elements in the bond graph, reference directions for energy flows are assigned to the bonds. For that purpose, it is useful to assume empty energy stores and an energy flow from the sources through the junction structure into energy stores, dissipators and sinks. Following this view, orientations of bonds must comply with rules already discussed for each bond graph element. In particular, orientations at 0-junctions connected to the power port of a C energy store or R element must ensure a velocity difference as depicted in Figure 2.41.

6. Simplification of the bond graph

   Finally, all 1-junctions representing a velocity or angular velocity equal to zero are eliminated along with all adjacent bonds. Resulting 2-port junctions with a through orientation of adjacent bonds are replaced by a single bond (Figure 2.43a). The symbol 'J' may be a 0- or 1-junction. If both adjacent bonds of a 2-port junction have an inward orientation, the junction cannot be condensed into a single bond. Such node changes the sign of one of the two power conjugated variables. Consider the 0-junction shown in Figure 2.44. Power conservation and the equality of efforts entails $f_1 + f_2 = 0$. 2-port 1-junctions with inward oriented adjacent bonds may be used in bond graphs of mechanical systems to explicitly represent internal forces in a body that appear in a free body diagram.

   Apparently, two junctions of the same type, either a 0- or a 1-junction, can be condensed into a single junction (Figure 2.43b).

A general recommendation finally added to the previously commented construction procedure is to add labels to elements so that they can be easily identified and distinguished from other elements of the same type. Typically, the label is a parameter if only one of the latter exists for a particular element. Otherwise, it can be either a function of the parameter, for example $\sin\alpha$ where α is an angle, a time depen-

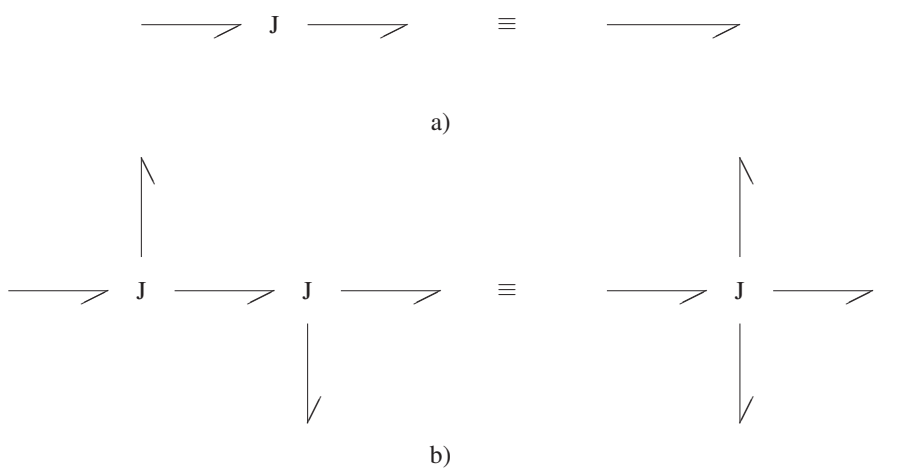


Fig. 2.43 Simplifications of junctions. **a** Replacing junction with two bonds by a single bond. **b** Condensing connected junctions of the same type into one junction

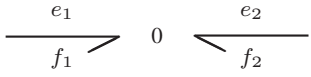


Fig. 2.44 Change in sign at a 2-port junction: $f_2 = -f_1$

dent function for a source, a name that may point to a set of parameters or just an identifier of the element.

Example: Rolling Cylinder on an Inclined Plane

For illustration of the systematic construction of a bond graph of a mechanical system in planar motion, consider the example depicted in Figure 2.45. In this example, a cylinder of radius r and mass m connected to a spring of stiffness k is rolling on a plane inclined at an angle α. It has a moment of inertia J with respect to an axis perpendicular to the paper plane through its centre of gravity. The centre of gravity of the rigid body moves at velocity $v(t)$ parallel to the inclined plane. At the same time, the cylinder is rotating with the angular velocity $\omega(t)$. At the contact point between the cylinder and the inclined plane, a viscous friction force is acting that is proportional to the relative velocity v_r. The contact point as part of the rolling cylinder has the velocity $v - v_t = v - r \times \omega$, whereas the velocity of the contact point as part of the plane is zero. Hence, the relative velocity v_r, effecting the viscous friction force, reads $v_r = v - r \times \omega$. The step by step construction of a bond graph model is shown in Figures 2.46 and 2.47.

For the sake of simplicity, linear characteristics have been assumed for the spring and the viscous friction. Furthermore, it is assumed that the inclined plane is station-

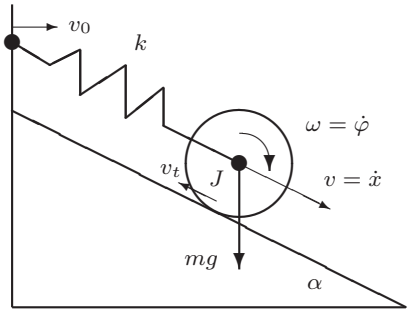


Fig. 2.45 Rolling cylinder on an inclined plane

ary ($v_0 \equiv 0$). The 2-port junctions are then condensed into bonds and the result is depicted in Figure 2.47.

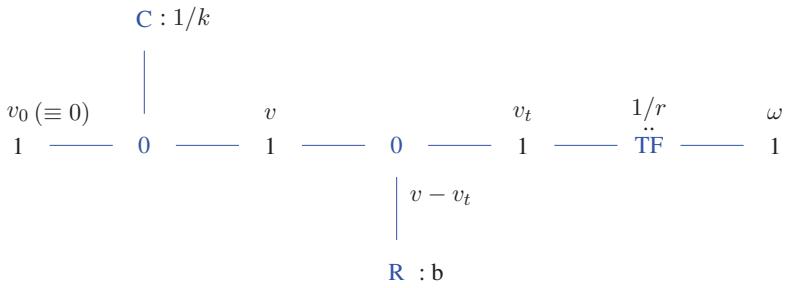
We will come back to this example in the next chapter after having introduced the concept of *computational causality* (Section 3.2) and a procedure for systematically deriving equations from a bond graph.

For modelling more complex planar mechanical motion, and in particular for the description of three-dimensional motion of rigid multibody systems, it is customary to use several moving body fixed reference frames. Since quantities are related to local reference frames, transformations between reference frames are needed. In bond graphs, such transformations can be represented by MTF elements. Their modulus depends on displacements. Three-dimensional motion of more complex mechanical systems is considered in Chapter 8. In the following, we will continue by considering the systematic construction of bond graph for non-mechanical subsystems. The notion of non-mechanical subsystems shall express that the modelling will focus on electrical, magnetic, hydraulic, acoustic, or thermal properties. Mechanical properties are assumed to be negligible. For instance, if a body is heated, it is assumed that the mechanical work due to its expansion can be neglected.

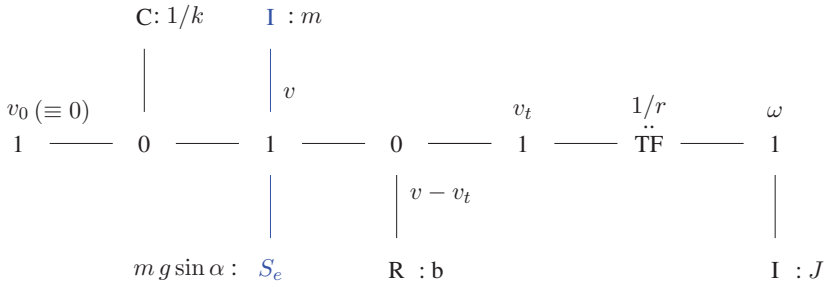
2.7.2 Construction of Bond Graphs for Non-mechanical Subsystems

The starting point for the construction of a bond graph for non-mechanical systems are distinct *efforts*. Apart from the interchanged role of efforts and flows due to the classical force voltage analogy, the procedure is quite similar to the one for mechanical systems.

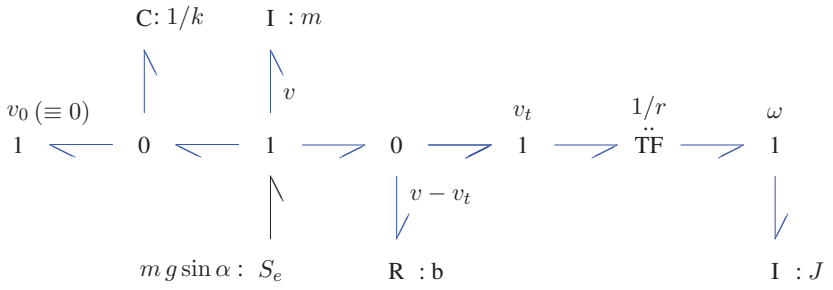
Step 1: Identification of distinct velocities and angular velocities



Step 2: Inserting C stores, dampers and TF elements

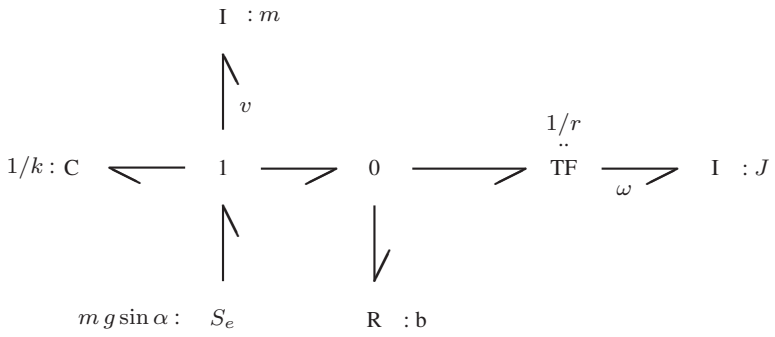


Step 3 u. 4: Adding inertias, sources and sinks



Step 5: Orientation of bonds

Fig. 2.46 Step by step construction of a bond graph model



Step 6: Simplification of the bond graph

Fig. 2.47 Figure 2.46 (continued)

1. Identify distinct efforts and represent them by 0-junctions.

   This means that according to Table 2.4, electrical, or magnetic potentials, absolute pressures in hydraulic subsystems, or absolute temperatures in the thermal domain are represented by 0-junctions. Like 1-junctions in bond graphs of mechanical subsystems, they should be labelled by a name in order to distinguish them.

2. The non-mechanical power port of an energy store, a resistor, a 2-port transformer, a 2-port gyrator, or a source is connected to a 1-junction to be inserted between a proper pair of 0-junctions.

   In case of an electrical transformer, the 1-junction at both ports of the TF element represent the currents through the coils of the transformer. In bond graphs of hydraulic systems, C elements are inserted via a 1-junction between the 0-junction of an absolute pressure and the 0-junction of the atmospheric pressure. In bond graphs of thermal systems, the thermal port of a C element is attached directly to the 0-junction of an absolute temperature.

   In electrical circuits, for instance, there is a voltage drop across an element with two terminals. In order to ensure a difference of potentials, one bond must be oriented toward the 1-junction, a second one away from the junction, and the third one towards the power port of the element (either an energy store or a resistor). The reference direction through the 1-junction corresponds to the reference direction of the current through the element with two terminals. There must also be

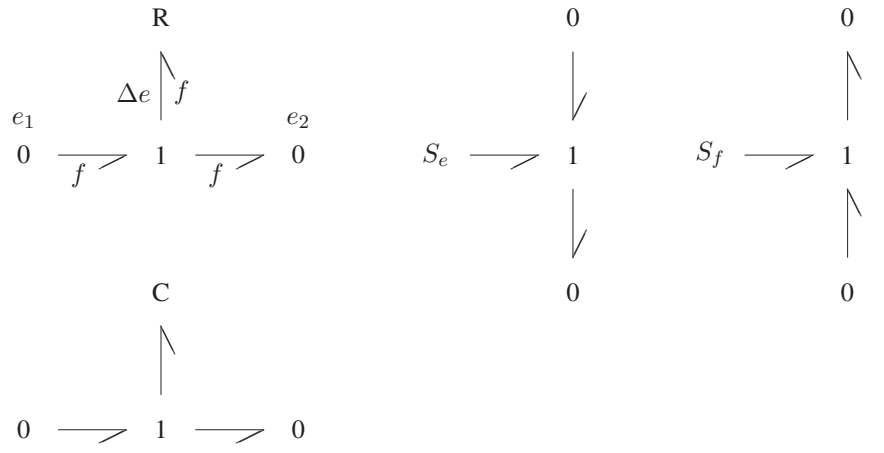


Fig. 2.48 Reference directions of energy flows for non-mechanical 1-port elements

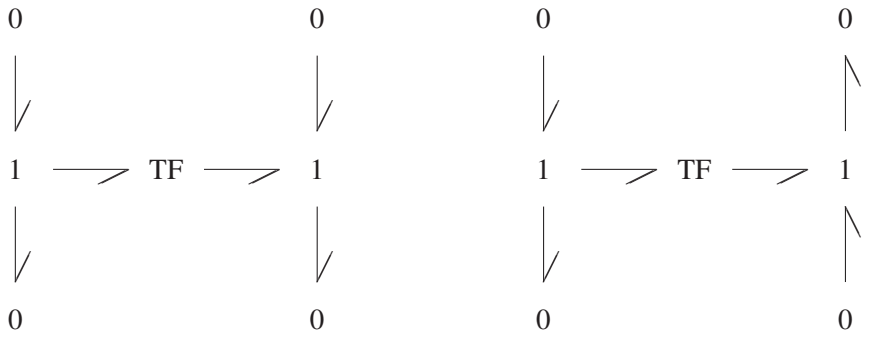


Fig. 2.49 Inserting a 2-port transformer

a through reference direction at a 1-junction if a source is connected to it (Figure 2.48). For each 1-junction connected to a port of a transformer or a gyrator, the reference direction of the energy flow must be through the junction according to Figure 2.49.

3. Once all elements have been inserted, reference directions for energy flows are assigned assuming empty energy stores and energy flows from the sources through the junction structure into energy stores, dissipators and sinks. Following this view, orientations of bonds must comply with rules already discussed for each bond graph element.

4. In bond graphs of electrical systems, choose a potential as reference and eliminate its corresponding 0-junction along with all adjacent bonds. If two sub-circuits of an electrical circuit are connected via an isolating transformer, a reference potential must be chosen in each sub-circuit.

   In hydraulic subsystems, it is common to choose the atmospheric pressure of the return reservoir. After elimination of its associated 0-junction along with all adjacent bonds, 0-junctions represent gage pressures. This results in a simplification of the construction of bond graphs for hydraulic systems. If 0-junctions represent gage pressures, then C elements are attached directly to a proper 0-junction. As TF elements in bond graphs of hydraulic systems relate a pressure to its associated mechanical force and, at the same time, the volume flow of an incompressible fluid flow to its associated translational velocity, the hydraulic port of the TF element is connected to a 0-junction of a gage pressure while its mechanical port is connected to the 1-junction of a velocity.

5. After elimination of reference nodes along with adjacent bonds, the bond graph is simplified, as has been discussed for bond graphs of mechanical subsystems.

Finally, it is good practice to add labels to elements so that they can be easily identified and distinguished from other elements of the same type. This has already been emphasised in the introduction of bond graph elements and as a general comment to the previous construction procedure for mechanical subsystems.

Example: Electrical Network with an Isolating Transformer

Consider the electrical network depicted in Figure 2.50. The conversion of the circuit diagram into a non-simplified bond graph is straightforward. Distinct nodes in the circuit are represented by 0-junctions. Bond graph elements corresponding to elements with two pins are inserted by means of a 1-junction. The result of the first three steps is shown in Figure 2.51. One can clearly see the topological affinity of the bond graph with the circuit diagram. The circled 0-junctions indicate reference potentials. After their elimination and subsequent simplification of the graph, the bond graph depicted in Figure 2.52 is obtained.

Before we follow the above procedure for the construction of a bond graph of a hydraulic subsystem, some preceding remarks have to made.

Hydraulic Capacitance of an Oil Filled Volume

As indicated in Table 2.6, the compliance of a fluid can be represented by a C energy store. If E_{fluid} denotes its bulk modulus and Q_c the volume flow into a pressurised volume V filled with a fluid of density ϱ assumed to be spatially uniform, then the increase of fluid in the volume due to its compression in the sealed volume results in a pressure increase (Equation 2.82)

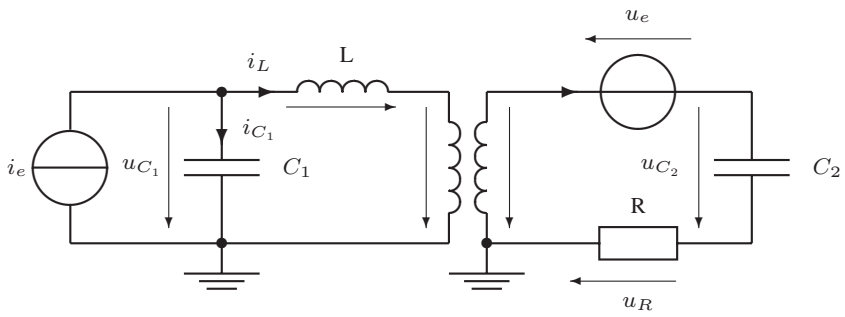


Fig. 2.50 Electrical circuit with an isolating transformer

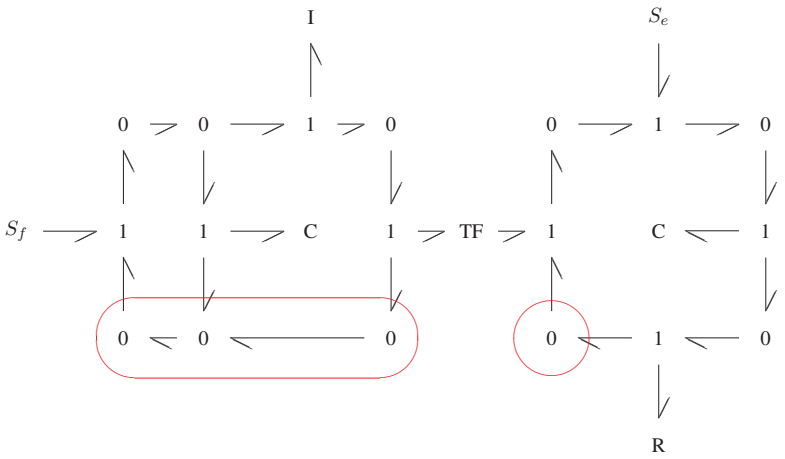


Fig. 2.51 Bond graph corresponding to the electrical circuit in Figure 2.50 after step 1) to 3)

$$p(t) - p(t=0) \;=\; E_{fluid} \times \frac{Q_c dt}{V} \;. \tag{2.82}$$

That is, the integral of the volume flow Q_c with respect to time is related to the *absolute* pressure, not to a difference, $p_1(t) - p_2(t)$, of two pressures at points 1 and 2 in space represented by two 0-junctions in a bond graph. Consequently, hydraulic C energy storage elements are always inserted between the 0-junction of a pressure assumed to be uniformly distributed in a volume and the 0-junction representing the atmospheric pressure p_0. Since the latter one is usually taken as reference, the corresponding 0-junction is removed from the bond graph with all adjacent bonds. Thus, a hydraulic C energy storage element is to be connected *directly* to the 0-junction representing the pressure, p, in the pressurised volume V. From Equa-

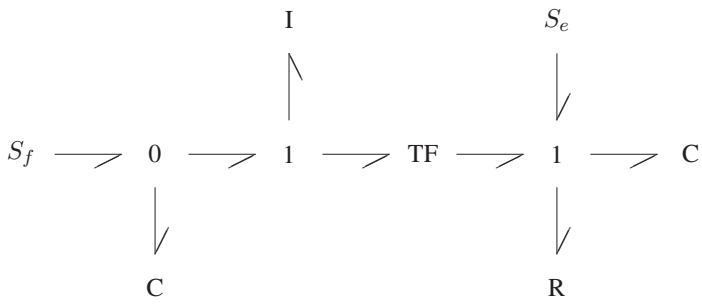


Fig. 2.52 Simplified bond graph corresponding to the circuit example in Figure 2.50

tion 2.82, we obtain for the capacitance parameter of a hydraulic C energy store $C_{hy} = V/E_{fluid}$.

Lossless Hydraulic Tee Junction

Moreover, we recall that the total pressure accounts for the hydrostatic pressure, the dynamic pressure and a gravitational term proportional to a difference of vertical heights. If the energy transported by the fluid is considered, then the kinetic energy and the internal energy of the fluid must be taken into account in addition to the hydraulic energy. It appears that bond graph modelling of thermofluid systems may be complicated. It becomes rather simple for so-called *hydrostatic* systems with low velocity flows and hydrostatic pressures of high values. Consequently, the total amount of power transferred between two ports may be approximated by the product of hydrostatic pressure and volume flow. Under this assumption, a lossless tee junction in a hydraulic circuit can be represented by a 0-junction like nodes in electrical networks. However, if fluid velocities cannot be neglected, dynamic pressures have to be taken into account. Consequently, the hydraulic tee junction can no longer be represented by a potential junction. Consider the hydraulic tee junction shown in Figure 2.53a). Assuming an incompressible fluid, the mass balance becomes

$$Q_1 + Q_2 + Q_3 \;=\; 0\;. \tag{2.83}$$

Let v_i $(i = 1, 2, 3)$ denote the corresponding fluid velocities. With the assumption that the pipes have the same cross sectional area A and that the fluid is not accelerated inside the tee junction, the momentum balance reads

$$A(p_1 - p_2) \;=\; -(\varrho\, Q_1 v_1 - \varrho\, Q_2 v_2)\;. \tag{2.84}$$

Finally, the energy balance is

$$(p_1 + \frac{1}{2}v_1^2)Q_1 + (p_2 + \frac{1}{2}v_2^2)Q_2 + (p_3 + \frac{1}{2}v_3^2)Q_3 \;=\; 0\;. \tag{2.85}$$

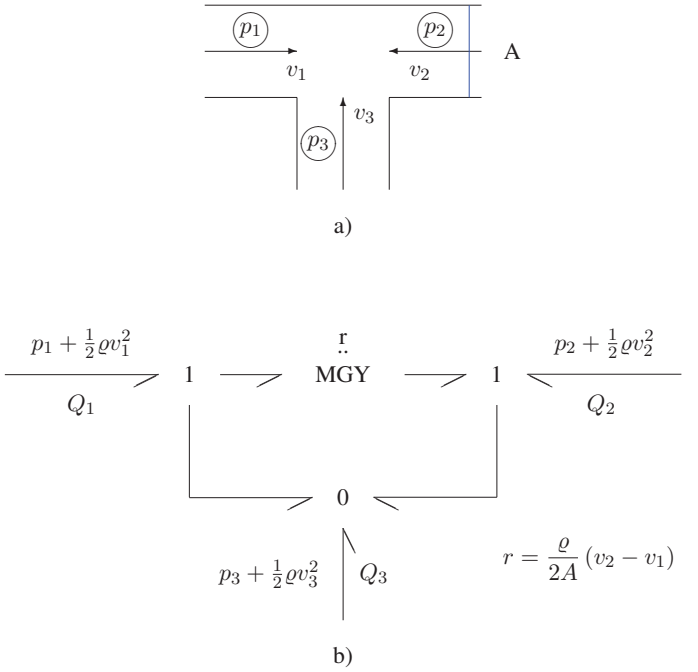


Fig. 2.53 Hydraulic tee junction. **a** Schematic. **b** Bond graph of a lossless hydraulic tee junction accounting for dynamic pressures (Breedveld, 1984)

All three balances can be represented by the bond graph depicted in Figure 2.53.

Apparently, the modulated gyrator disappears and the bond graph reduces to a 0-junction if $v_1 = v_2$.

Hydraulic Ram

At the piston of a hydraulic ram, the hydrostatic pressure, p, in the chamber is converted into a mechanical force, F, acting on the piston of cross sectional area A. At the same time, the piston's displacement at (slow) velocity, v, entails a volume flow $\dot{V}$.

$$F = A \times p \tag{2.86a}$$

$$\dot{V} = A \times v \tag{2.86b}$$

As these equations relate efforts and simultaneously relate associated flows, they are the constitutive equations of a transformer, of which the hydraulic port is to be connected to the 0-junction of the gage pressure, p, and its mechanical port to the 1-junction representing the translational velocity v.

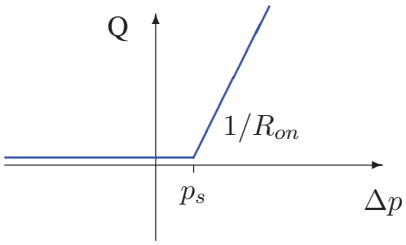


Fig. 2.54 Piecewise linear approximation of the static characteristic of a the pressure relief valve

Pressure Relief Valve

If the fast dynamics of a pressure relief valve are neglected, then it may be modelled as a switched resistor. If the pressure on the pump surmounts an allowed limit p_s, the valve is considered to open immediately and some volume flow returns to the tank passing an opening. In this view, the pressure relief valve may be compared to an electrical diode. Its characteristic may be approximated by the piecewise linear curve depicted on Figure 2.54, where Δp denotes the pressure drop across the valve and Q the volume flow through the valve.

Example: Hydrostatic Drive

With these remarks, a bond graph model of a hydraulic drive can be developed systematically by following the procedure for non-mechanical subsystems.

In the simple example of a hydrostatic drive (Figure 2.55), a constant volume flow pump, protected by a pressure relief valve, delivers a hydraulic volume flow into a double acting unsymmetrical cylinder with cross sectional areas A_1 and A_2. Its piston moves a mechanical load (not depicted) against a spring of stiffness k. The initial word bond graph in Figure 2.56 directly corresponds with the circuit schematic. In the word bond graph, an effort source accounts for possible disturbances on the mechanical load.

Since the aim is to demonstrate the step by step construction of a bond graph and not to develop a model accounting for given requirements, it is sufficient to build a rather simple model. The constant flow pump may be represented by a flow source and the pressure relief valve (PRV) by a nonlinear resistor neglecting the valve's fast dynamics.

After the first three steps of the procedure for the systematic construction of bond graphs of non-mechanical subsystems, the bond graph depicted in Figure 2.57 is obtained. Like the initial bond graph of the electrical network, it shows a close topological affinity to the hydraulic circuit schematic. The fluid compliance in the trapped oil volumes on both sides of the piston has been accounted for by two C energy stores. If the return tank pressure is chosen as a reference, elimination of the corre-

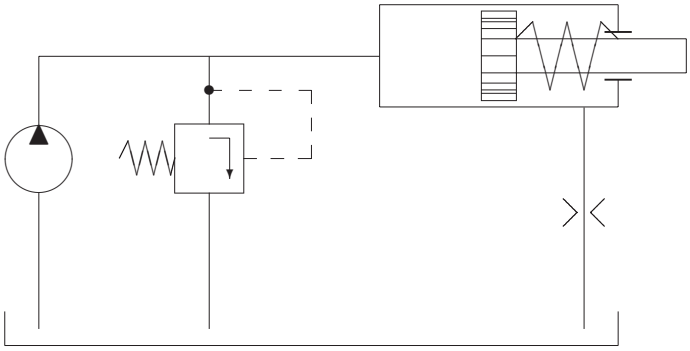


Fig. 2.55 Hydraulic drive

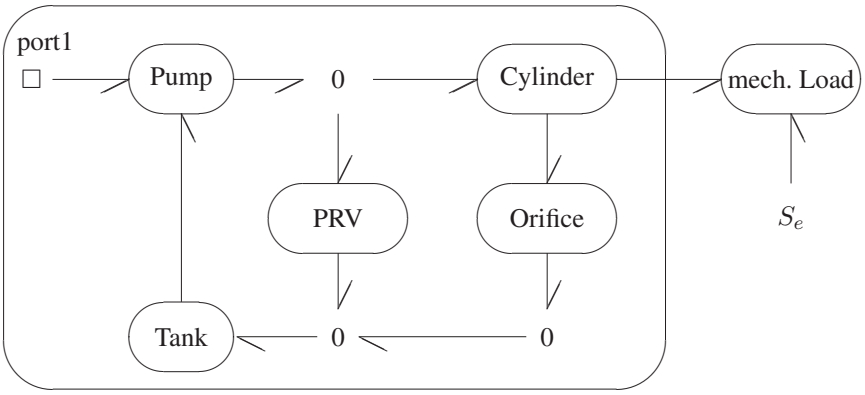


Fig. 2.56 Word bond graph of the hydraulic drive

sponding 0-junctions and subsequent simplification yields the bond graph shown in Figure 2.58.

Example: Electromechanical System

For systems with subsystems in different energy domains, it is obvious to construct a bond graph model for each subsystem following the steps of the corresponding procedure and to connect the submodels by models of the energy transducers. For illustration, consider the mechanical slider crank mechanism driven by an electrical DC motor with constant excitation as shown in Figure 2.59. The armature winding has a self-inductance L and a resistance R. The rod connecting the disk of inertia J to the slider of mass m is assumed to be massless. The piston is moving against an external disturbance force. Figure 2.60 shows the corresponding bond graph of the electromechanical system.

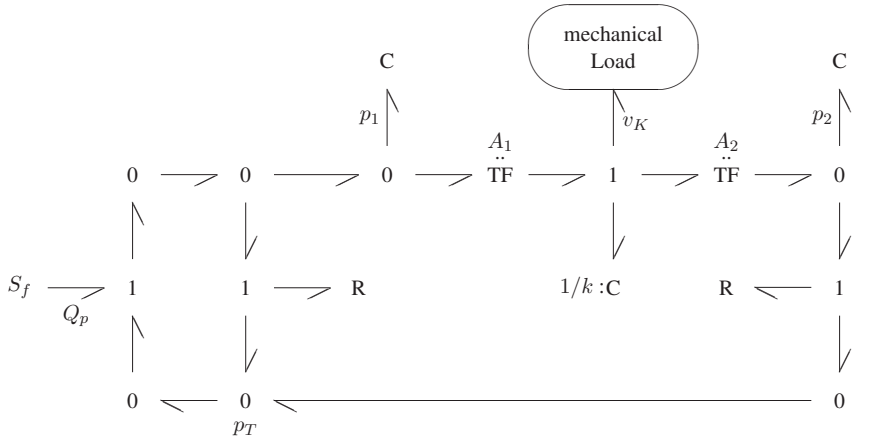


Fig. 2.57 Initial bond graph of the hydraulic drive

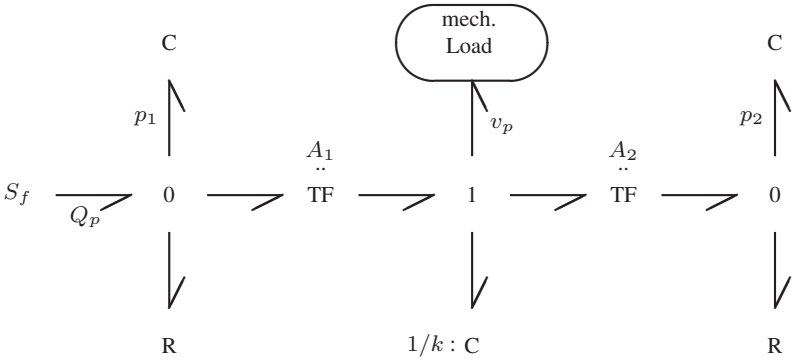


Fig. 2.58 Simplified bond graph of the hydraulic drive

The bond graph of the electrical subsystem shows a clear similarity to the topology of the circuit schematic. In the bond graph of the mechanical subsystem, the 1-junction of the translational velocity, v, of the piston is connected to the 1-junction of the angular velocity, ω, by a displacement modulated transformer. Its modulus, $T(\phi)$, is controlled by the angle, ϕ, of the crank given by the geometry. The motor converting electrical energy into mechanical energy has been simply modelled by the gyrator. It transforms the electrical current, i_a, through the armature winding into a mechanical moment and simultaneously converts the angular velocity, ω, into a voltage. Its ratio is the torque constant k_T. That is, the inertia of the rotor and bearing friction are neglected. Moreover, the shaft between the motor and the pump is considered an ideal power conservative connection represented by a bond.

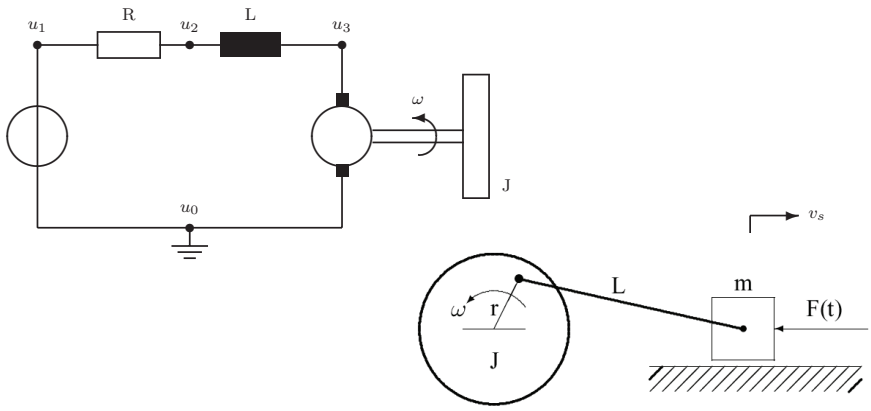


Fig. 2.59 Simple example of an electromechanical system

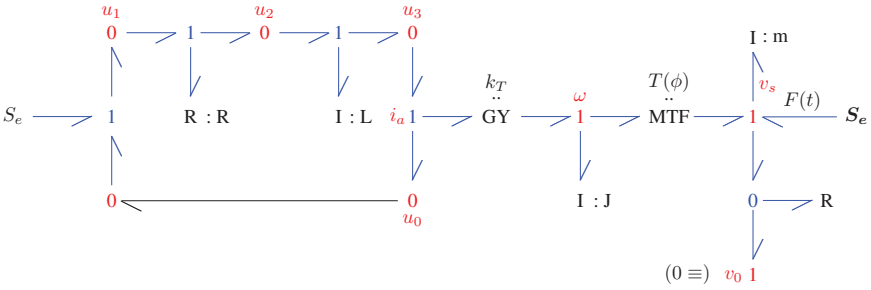


Fig. 2.60 Bond graph of the electromechanical system in Figure 2.59

Torsion and inertia effects are neglected. The friction between the piston and the plane is modelled by a resistor connected to a 0-junction that is inserted between the 1-junctions of the piston and the plane. An external disturbance force $F(t)$, acting on the piston, has been accounted for by an effort source. Mechanical inertias are connected directly to the two 1-junctions representing the angular velocity ω of the disk and the translational velocity v of the piston.

2.7.3 Simplification of Some Bond Graph Structures

Once a bond graph has been derived from a system schematic following the steps of the construction procedures discussed above, it might be appropriate to look for further simplifications. This is useful for visual analysis of the graph, but also for

its automatic processing by software packages. If rule based simplifications are not carried out automatically, then the storage of variables that are not needed and unnecessary operations can be avoided if simplifications of the graph are done manually prior to any further model processing. Figure 2.61 shows some sub-structures in bond graphs and their corresponding equivalent simplification. Equivalences may be proven by setting up the equations for all nodes and subsequent simplification. Figure 2.62 depicts how a transformer or a gyrator together with another element can be condensed into one element.

2.8 Some Remarks on the Choice of Orientations in Bond Graphs

In the previous sections, the basic bond graph elements and two procedures for systematically constructing a bond graph from a system schematic orientations of bonds have been introduced. The rules for reference directions of energy flows do not determine a *unique* orientation for all bonds in a bond graph. There may be several admissible pattern of orientations for the bonds of the junction structure. However, this does not mean that once an undirected bond graph has been constructed, subsequently orientations of bonds in the junction structure may be chosen almost arbitrarily [7]. The result may easily be an inconsistent choice of signs. As Perelson has shown in [29], a bond graph derived from an electrical circuit may no longer represent the network if there is no through orientation of the energy flow at 1-junctions representing the current through an element with two pins.

In 1993, Lamp, Asher and Woodall [24] considered the reverse question under which condition a given bond graph can be implemented by a network. In [24], they observe that the bond graph structure reproduced in Figure 2.63 can be implemented by a network with one transformer (Figure 2.64). A particular feature of this bond graph structure is that the bond loop includes an odd number of bonds having the same energy reference direction. As a generalisation of this observation, they express the conjecture that each bond graph can be implemented as a network if a certain number of transformers and gyrators is used. This, however, means that either the functionality of 1- and 0-junctions is extended or networks are considered realisations of bond graphs even if they contain elements with no correspondence in the bond graph (see Perelson, footnote in [29]). In [30], Perelson shows that if 1-junctions connected to a port of an element have an adjacent bond oriented towards the junction and another pointing away from it (cf. Figure 2.48), then the directed bond graph is equivalent to the network from which it has been constructed. This has been taken into account when the procedure for the construction of bond graphs for non-mechanical subsystems (cf. Figure 2.48) was introduced.

With respect to practical engineering problems, the transformation of a system schematic into a bond graph is considered to be of primary concern. This holds in particular for multidisciplinary systems for which graphical model representations are either not formalised or which use domain specific symbols. Consequently, a

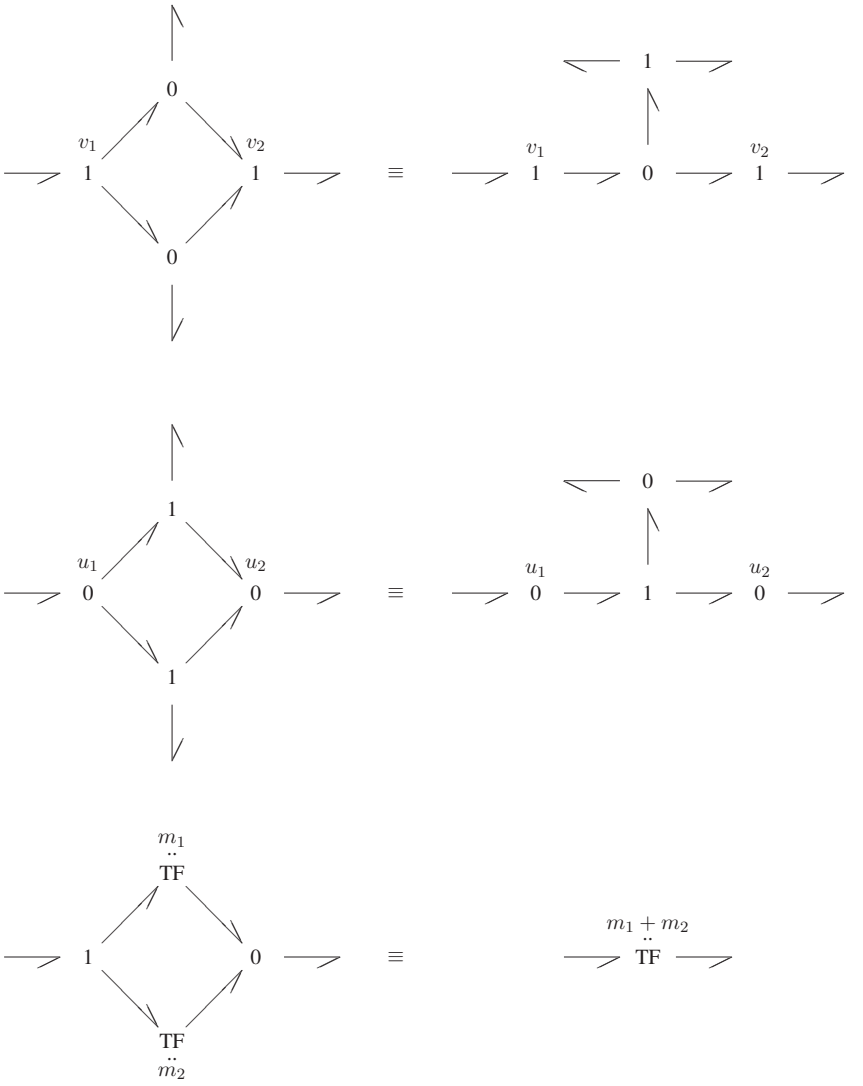


Fig. 2.61 Simplification of some sub-structures

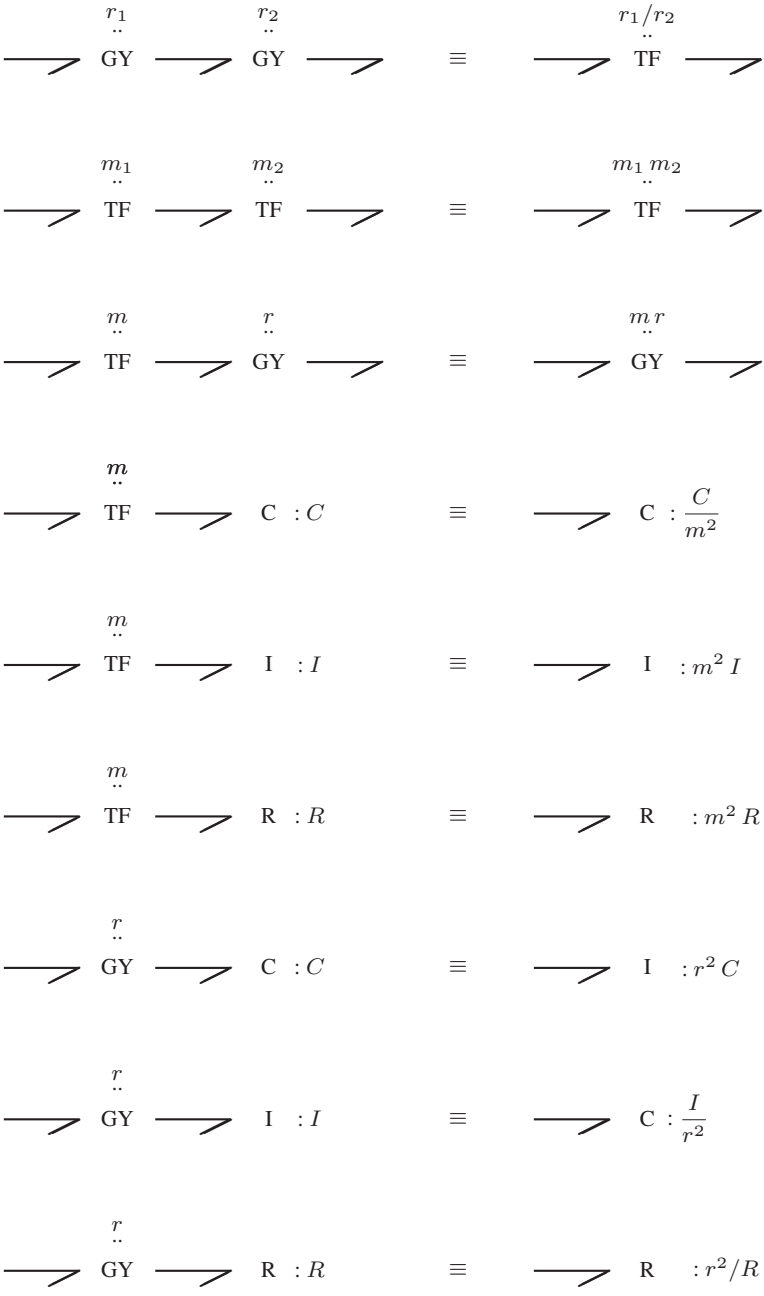


Fig. 2.62 Combination of a transformer or a gyrator with another element

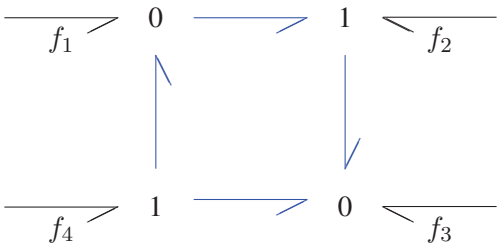


Fig. 2.63 Bond loop with an odd number of bonds having the same orientation

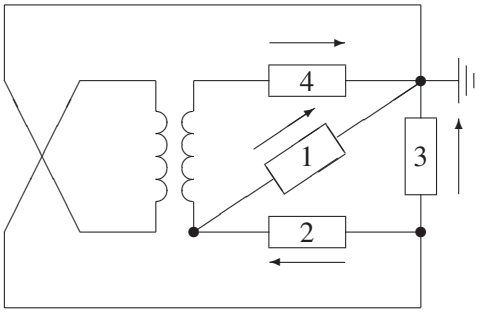


Fig. 2.64 Network realisation of the bond graph part in Figure 2.63 (Lamp, Asher and Woodall, 1993)

unified formalised graphical representation, e.g., a bond graph, is certainly a useful step towards a mathematical model. If the rules discussed above are taken into account, then the equivalence of the resulting bond graph with the initial representation is ensured. Moreover, since there are powerful software packages available for modelling and simulation that, besides other formalisms support bond graphs and are able to directly process them, there is little need to transform a bond graph of a multidisciplinary system into a network to have it processed by a program for network analysis. Therefore, the transformation of a given bond graph into a network is considered less important and will not be furthermore addressed in this book.

2.9 Conclusion

In this chapter, the fundamentals of bond graph based physical system modelling have been provided. The adjective *physical* emphasises that the intellectual decomposition of a system uses the view of an exchange of energy between subsystems and that there are elementary physical processes on the bottom of the model hierar-

chy for which fundamental balances, viz., conservation of energy, conservation of mass, or momentum hold.

Before starting the development of a bond graph model, it must be decided which physical effects are to be taken into account.[4] In bond graphs, these effects are represented either by a word free of choice for a subsystem, e.g., a transducer or a sensor, or a reserved symbol denoting a basic physical process, e.g., storage of potential energy in a spring. Like networks, bond graphs use the abstraction of spatial concentration of physical properties. It is a characteristic feature of bond graph based or network based physical systems modelling that an initial graphical model is developed by accounting for physical effects in a *qualitative* manner. Moreover, as long as the initial bond graph is not simplified, it shows a clear resemblance to the topological structure in the system schematic.

In contrast to block diagrams, in general, it is not necessary to consider functional relations in order to construct a bond graph. There is no need to precisely know how the power conjugated variables of an element are related. It is sufficient to know which type of an element the constitutive equations define. Consideration and manipulation of equations can be reserved to subtle cases in which it is unclear how coupled physical effects can be modelled under given assumptions. The aim of bond graph based physical modelling is not only to graphically express functional relations, but to come up with a model that complies with conservations laws of physics. Such a task is not easy for open thermodynamic systems in which mass enters and leaves a control volume conveying momentum and energy.

Since the conceptual starting point of bond graph modelling is the energy exchange between subsystems, this modelling approach is particularly suited for multidisciplinary dynamic systems in which several energy domains are involved. Moreover, the introduction of two general power conjugated variables and their assignment to physical quantities in each energy domain enable a uniform description of basic physical process, viz., the distribution or the storage of energy, and energy conversion. The uniform representation is reflected by a few elements that are the same in all energy domains apart from some details, e.g., that there is no inertia in thermodynamics. Like in networks, there are sources, energy stores and resistors. In contrast to networks, the range of symbols is limited to those representing fundamental effects. For instance, a diode can be considered to be a resistor with a special characteristic.

The choice of effort and flow variables is based on an analogy between mechanical and electrical quantities. There are two such analogies, the classical force voltage analogy and the mobility analogy. Both analogies lead to equivalent mathematical models. Following a widely used convention in the bond graph literature, the classical analogy is adopted throughout this monograph.

Especially for thermodynamic and process engineering systems, it is also common to use physical quantities as effort and flow, although their product does not have the physical dimension of power. With such choices, it is convenient to develop

[4] The proper choice of physical effects that need be included in a model clearly has an impact on the complexity of a model. This subject is briefly addressed in Section 6.9. A recent review of proper modelling techniques is given in the 2008 article [15] by Ersal and his co-authors.

pseudo bond graphs. They do have an intuitive meaning, but cannot be coupled to true bond graphs via transformers or gyrators in general.

True bond graphs as well as pseudo bond graphs can be systematically constructed from a system schematic. For that purpose, there are two procedures, one for mechanical and one for non-mechanical subsystems. Regarding the orientation of bonds, some rules are crucial in order to ensure that a bond graph complies with requirements from physics and is equivalent to the initial system representation.

The next chapter discusses how bond graphs can be augmented so that they are equivalent to a block diagram and how a mathematical model can be derived from a so-called *causally* augmented bond graph in a systematic manner. Often, causal information is added to a bond graph following a basic procedure introduced by Karnopp and Rosenberg. Once this procedure is available, some interesting aspects and approaches can be discussed.

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