

Preface

This book presents new results on applications of geometric algebra. The time when researchers and engineers were starting to realize the potential of quaternions for applications in electrical, mechanical, and control engineering passed a long time ago. Since the publication of *Space-Time Algebra* by David Hestenes (1966) and *Clifford Algebra to Geometric Calculus: A Unified Language for Mathematics and Physics* by David Hestenes and Garret Sobczyk (1984), consistent progress in the applications of geometric algebra has taken place. Particularly due to the great developments in computer technology and the Internet, researchers have proposed new ideas and algorithms to tackle a variety of problems in the areas of computer science and engineering using the powerful language of geometric algebra. In this process, pioneer groups started the conference series entitled “Applications of Geometric Algebra in Computer Science and Engineering” (AGACSE) in order to promote the research activity in the domain of the application of geometric algebra. The first conference, AGACSE’1999, organized by Eduardo Bayro-Corrochano and Garret Sobczyk, took place in Ixtapa-Zihuatanejo, Mexico, in July 1999. The contributions were published in *Geometric Algebra with Applications in Science and Engineering*, Birkhäuser, 2001. The second conference, AGACSE’2001, was held in the Engineering Department of the Cambridge University on 9–13 July 2001 and was organized by Leo Dorst, Chris Doran, and Joan Lasenby. The best conference contributions appeared as a book entitled *Applications of Geometric Algebra in Computer Science and Engineering*, Birkhäuser, 2002. The third conference, AGACSE’2008, took place in August 2008 in Grimma, Leipzig, Germany. The conference chairs, Eduardo Bayro-Corrochano and Gerik Sheuermann, edited this book using selected contributions that were peer-reviewed by at least two reviewers.

In the history of science, theories would have not been developed at all without essential mathematical concepts. In various periods of the history of mathematics and physics, there is clear evidence of stagnation, and it is only thanks to new mathematical developments that astonishing progress has taken place. Furthermore, researchers unavoidably cause fragmented knowledge in their various attempts to combine different mathematical systems. We realize that each mathematical system brings about some parts of geometry; however, together, they constitute a system that is highly redundant due to an unnecessary multiplicity of representations

for geometric concepts. In contrast, in the geometric algebra language, most of the standard matter taught in engineering and computer science can be advantageously reformulated without redundancies and in a highly condensed fashion.

This book presents a selection of articles about the theory and applications of the advanced mathematical language *geometric algebra* which greatly helps to express the ideas and concepts and to develop algorithms in the broad domains of computer science and engineering. The contributions are organized in seven parts.

The *first part* presents screw theory in geometric algebra, the parameterization of 3D conformal transformations in conformal geometric algebra, and an overview of applications of geometric algebra. The *second part* includes thorough studies on Clifford–Fourier transforms: the two-dimensional Clifford windowed Fourier transform; the cylindrical Fourier transform; applications of the 3D geometric algebra Fourier transform in graphics engineering; the 4D Clifford–Fourier transform for color image processing; and the use of the Hilbert transforms in Clifford analysis for signal processing. In the *third part*, self-organizing geometric neural networks are utilized for 2D contour and 3D surface reconstruction in medical image processing. The clustering and classification are handled using geometric neural networks and associative memories designed in the conformal geometric algebra. This part concludes with a retrospective of the quaternion wavelet transform, including an application for stereo vision. The *fourth part* for computer vision starts with a new cone-pixel camera using a convex hull and twists in conformal geometric algebra. The next work introduces a model-based approach for global self-localization using active stereo vision and Gaussian spheres. In the *fifth part*, the geometric characterization of M-conformal mappings is discussed, and a study of fluid flow problems is carried out in depth using quaternionic analysis. The *sixth part* shows the impressive space group visualizer for all 230 3D groups using the software packet for geometric algebra computations CLUCalc. The second author studies geometric algebra formalism as an alternative to distributed representation models; here convolutions are replaced by geometric products, and, as a result, a natural language for visualization of higher concepts is proposed. Another author studies computational complexity reductions using Clifford algebras and shows that graph problems of complexity class NP are polynomial in the number of Clifford operations required. The *seventh part* includes new developments in efficient geometric algebra computing: The first author presents an efficient blade factorization algorithm to produce faster implementations of the *Join*; with the software packet GALOOP, the second author symbolically reduces involved formulas of conformal geometric algebra, generating suitable code for computing using hardware accelerators. Another chapter shows applications of Grobner bases in robotics, formulated in the language of Clifford algebras, in engineering to the theory of curves, including Fermat and Bezier cubics, and in the interpolation of functions used in finite element theory.

We are very thankful to all book contributors, who are working persistently to advance the applications of geometric algebra. We do hope that the reader will find this collection of contributions in a broad scope of the areas of engineering and computer science very stimulating and encouraging. We hope that, as a result, we will see our community growing and benefitting from new and promising scientific

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