

Contents

1	Introduction	1
1.1	Notation, Definitions and Basic Properties	1
1.2	Examples	7
1.3	Feasibility and Reachability	9
1.3.1	Multi-machine Interactive Production Process: A Managerial Application	9
1.3.2	MMIPP: Synchronization and Optimization	10
1.3.3	Steady Regime and Its Reachability	11
1.4	About the Ground Set	12
1.5	Digraphs and Matrices	13
1.6	The Key Players	16
1.6.1	Maximum Cycle Mean	17
1.6.2	Transitive Closures	21
1.6.3	Dual Operators and Conjugation	29
1.6.4	The Assignment Problem and Its Variants	30
1.7	Exercises	36
2	Max-algebra: Two Special Features	41
2.1	Bounded Mixed-integer Solution to Dual Inequalities: A Mathematical Application	41
2.1.1	Problem Formulation	41
2.1.2	All Solutions to SDI and All Bounded Solutions	42
2.1.3	Solving BMISDI	43
2.1.4	Solving BMISDI for Integer Matrices	45
2.2	Max-algebra and Combinatorial Optimization	48
2.2.1	Shortest/Longest Distances: Two Connections	48
2.2.2	Maximum Cycle Mean	49
2.2.3	The Job Rotation Problem	49
2.2.4	Other Problems	51
2.3	Exercises	52

3	One-sided Max-linear Systems and Max-algebraic Subspaces	53
3.1	The Combinatorial Method	53
3.2	The Algebraic Method	57
3.3	Subspaces, Generators, Extremals and Bases	59
3.4	Column Spaces	64
3.5	Unsolvable Systems	67
3.6	Exercises	69
4	Eigenvalues and Eigenvectors	71
4.1	The Eigenproblem: Basic Properties	71
4.2	Maximum Cycle Mean is the Principal Eigenvalue	74
4.3	Principal Eigenspace	76
4.4	Finite Eigenvectors	82
4.5	Finding All Eigenvalues	86
4.6	Finding All Eigenvectors	95
4.7	Commuting Matrices Have a Common Eigenvector	97
4.8	Exercises	98
5	Maxpolynomials. The Characteristic Maxpolynomial	103
5.1	Maxpolynomials and Their Factorization	105
5.2	Maxpolynomial Equations	111
5.3	Characteristic Maxpolynomial	112
5.3.1	Definition and Basic Properties	112
5.3.2	The Greatest Corner Is the Principal Eigenvalue	114
5.3.3	Finding All Essential Terms of a Characteristic Maxpolynomial	116
5.3.4	Special Matrices	123
5.3.5	Cayley–Hamilton in Max-algebra	124
5.4	Exercises	126
6	Linear Independence and Rank. The Simple Image Set	127
6.1	Strong Linear Independence	127
6.2	Strong Regularity of Matrices	130
6.2.1	A Criterion of Strong Regularity	130
6.2.2	The Simple Image Set	135
6.2.3	Strong Regularity in Linearly Ordered Groups	137
6.2.4	Matrices Similar to Strictly Normal Matrices	138
6.3	Gondran–Minoux Independence and Regularity	138
6.4	An Application to Discrete-event Dynamic Systems	144
6.5	Conclusions	146
6.6	Exercises	146
7	Two-sided Max-linear Systems	149
7.1	Basic Properties	150
7.2	Easily Solvable Special Cases	151
7.2.1	A Classical One	151

7.2.2	Idempotent Matrices	152
7.2.3	Commuting Matrices	153
7.2.4	Essentially One-sided Systems	153
7.3	Systems with Separated Variables—The Alternating Method	156
7.4	General Two-sided Systems	162
7.5	The Square Case: An Application of Symmetrized Semirings	164
7.6	Solution Set is Finitely Generated	169
7.7	Exercises	176
8	Reachability of Eigenspaces	179
8.1	Visualization of Spectral Properties by Matrix Scaling	181
8.2	Principal Eigenspaces of Matrix Powers	186
8.3	Periodic Behavior of Matrices	188
8.3.1	Spectral Projector and the Cyclicity Theorem	188
8.3.2	Cyclic Classes and Ultimate Behavior of Matrix Powers	193
8.4	Solving Reachability	196
8.5	Describing Attraction Spaces	202
8.5.1	The Core Matrix	203
8.5.2	Circulant Properties	204
8.5.3	Max-linear Systems Describing Attraction Spaces	206
8.6	Robustness of Matrices	212
8.6.1	Introduction	212
8.6.2	Robust Irreducible Matrices	213
8.6.3	Robust Reducible Matrices	215
8.6.4	M -robustness	220
8.7	Exercises	223
9	Generalized Eigenproblem	227
9.1	Basic Properties of the Generalized Eigenproblem	228
9.2	Easily Solvable Special Cases	230
9.2.1	Essentially the Eigenproblem	230
9.2.2	When A and B Have a Common Eigenvector	230
9.2.3	When One of A, B Is a Right-multiple of the Other	231
9.3	Narrowing the Search for Generalized Eigenvalues	233
9.3.1	Regularization	233
9.3.2	A Necessary Condition for Generalized Eigenvalues	234
9.3.3	Finding $\max_{\lambda} C(\lambda) $	235
9.3.4	Narrowing the Search	236
9.3.5	Examples	238
9.4	Exercises	241
10	Max-linear Programs	243
10.1	Programs with One-sided Constraints	243
10.2	Programs with Two-sided Constraints	245
10.2.1	Problem Formulation and Basic Properties	245
10.2.2	Bounds and Attainment of Optimal Values	247

10.2.3 The Algorithms	251
10.2.4 The Integer Case	253
10.2.5 An Example	255
10.3 Exercises	257
11 Conclusions and Open Problems	259
References	261
Index	269



<http://www.springer.com/978-1-84996-298-8>

Max-linear Systems: Theory and Algorithms

Butkovič, P.

2010, XVIII, 274 p., Hardcover

ISBN: 978-1-84996-298-8