

# Contents

## Part I Population Growth Models in Random and Varying Environments

<b>1</b>	<b>A refinement of limit theorems for the critical branching processes in random environment</b> .....	3
	Vladimir Vatutin	
1.1	Introduction and main results .....	3
1.2	Branching in conditioned environment .....	8
1.3	Proof of Theorems 1.1 and 1.2 .....	17
	References .....	18
<b>2</b>	<b>Branching processes in stationary random environment: The extinction problem revisited</b> .....	21
	Gerold Alsmeyer	
2.1	Introduction .....	21
2.2	Classical results revisited .....	24
2.3	Main result and a counterexample .....	25
2.4	Some useful facts from Palm-duality theory .....	28
2.5	Proofs .....	29
	References .....	36
<b>3</b>	<b>Environmental versus demographic stochasticity in population growth</b> .....	37
	Carlos A. Braumann	
3.1	Introduction .....	37
3.2	Density-independent models and their local behavior .....	39
3.3	Density-independent models and extinction .....	44
3.4	Density-dependent models for environmental stochasticity .....	45
3.5	Conclusions .....	49
	References .....	52

<b>4</b>	<b>Stationary distributions of the alternating branching processes</b> . . . . .	<b>53</b>
	Penka Mayster	
4.1	Introduction . . . . .	53
4.2	Alternating branching process . . . . .	55
4.3	Alternating branching process with explicit immigration . . . . .	56
4.4	Reproduction by $n$ cycles . . . . .	58
4.5	Criticality . . . . .	59
4.6	Stationary distribution in random environment . . . . .	62
4.7	Unconditional probability generating functions . . . . .	63
4.8	Feed-back control . . . . .	64
	References . . . . .	66

## Part II Special Branching Processes

<b>5</b>	<b>Approximations in population-dependent branching processes</b> . . . . .	<b>71</b>
	Fima C. Klebaner	
5.1	Introduction and a motivating example . . . . .	71
5.2	A Representation of the process and its re-scaled version . . . . .	73
	5.2.1 Re-scaled process: Dynamics plus small noise . . . . .	74
	5.2.2 Dynamics without noise in binary splitting . . . . .	74
5.3	Time to extinction . . . . .	75
5.4	The size of the population after a long time provided it has survived . . . . .	75
5.5	Case of small initial population . . . . .	76
	5.5.1 Probability of becoming large and time for it to happen . .	77
5.6	Behaviour before extinction . . . . .	77
	References . . . . .	78
<b>6</b>	<b>Extension of the problem of extinction on Galton–Watson family trees</b> . . . . .	<b>79</b>
	George P. Yanev	
6.1	Introduction . . . . .	79
6.2	Critical phenomenon . . . . .	80
6.3	Distribution of the number of complete and disjoint subtrees, rooted at the ancestor . . . . .	83
6.4	Ratio of expected values of $Z_n$ s provided infinite subtrees exist . .	84
6.5	Geometric offspring distribution . . . . .	86
6.6	Poisson offspring distribution . . . . .	89
6.7	One-or-many offspring distribution . . . . .	91
6.8	Concluding remarks . . . . .	93
	References . . . . .	93
<b>7</b>	<b>Limit theorems for critical randomly indexed branching processes</b> . .	<b>95</b>
	Kosto V. Mitov, Georgi K. Mitov and Nikolay M. Yanev	
7.1	Introduction . . . . .	95
7.2	A conditional limit theorem for random time change . . . . .	97

7.3	Renewal processes	100
7.4	BGW branching processes starting with random number of particles	103
7.5	Limit theorems for the process $Y(t)$	105
7.6	Concluding remarks	107
	References	108
<b>8</b>	<b>Renewal measure density for distributions with regularly varying tails of order <math>\alpha \in (0, 1/2]</math></b>	<b>109</b>
	Valentin Topchii	
8.1	Introduction	109
8.2	Effects of attraction to a stable law	111
8.3	Asymptotics of renewal function density	114
	References	118
<b>Part III Limit Theorems and Statistics</b>		
<b>9</b>	<b>Approximation of a sum of martingale differences generated by a bootstrap branching process</b>	<b>121</b>
	Ibrahim Rahimov	
9.1	Introduction	121
9.2	Main theorems	123
9.3	Array of processes	127
	References	133
<b>10</b>	<b>Critical branching processes with immigration</b>	<b>135</b>
	Márton Ispány and Gyula Pap	
10.1	Introduction	135
10.2	Branching and autoregressive processes	136
10.3	Functional limit theorems	138
10.4	Nearly critical branching processes with immigration	140
10.5	Conditional least squares estimators	142
	References	145
<b>11</b>	<b>Weighted conditional least squares estimation in controlled multitype branching processes</b>	<b>147</b>
	Miguel González and Inés M. del Puerto	
11.1	Introduction	147
11.2	Probability model	148
11.3	Weighted conditional least squares estimator of the offspring mean matrix	150
	References	155

## Part IV Applications in Cell Kinetics and Genetics

<b>12</b>	<b>Branching processes in cell proliferation kinetics</b>	159
	Nikolay M. Yanev	
12.1	Introduction	159
12.2	Distributions of discrete marks over a proliferating cell populations	161
12.3	Distributions of continuous labels in branching populations of cells	162
12.4	Age and residual lifetime distributions for branching processes	164
12.5	Branching processes with immigration as models of leukemia cell kinetics	168
12.6	Age-dependent branching populations with randomly chosen paths of evolution	171
12.7	Multitype branching populations with a large number of ancestors	173
12.8	Concluding remarks	177
	References	177
<b>13</b>	<b>Griffiths–Pakes branching process as a model for evolution of Alu elements</b>	179
	Marek Kimmel and Matthias Mathaes	
13.1	Introduction	179
13.2	Alu repeat sequences	180
13.2.1	Background on Alus	180
13.2.2	Alu sequence data used in this study	180
13.3	Discrete branching process of Griffiths and Pakes with infinite allele mutations	181
13.3.1	Linear fractional offspring distribution	184
13.4	Fitting results	186
13.5	Discussion	187
	References	189
<b>14</b>	<b>Parametric inference for Y-linked gene branching models: Expectation-maximization method</b>	191
	Miguel González, Cristina Gutiérrez and Rodrigo Martínez	
14.1	Introduction	191
14.2	The probability model	192
14.3	The estimation problem: The expectation-maximization method	195
14.3.1	Determining the distribution of $\mathcal{F}Rr_N   (\mathcal{F}M_N, \alpha, \theta_R, \theta_r)$	197
14.3.2	The expectation-maximization method	199
14.4	Simulation study	200
	References	204

## Part V Applications in Epidemiology

<b>15 Applications of branching processes to the final size of SIR epidemics</b>	207
Frank Ball and Peter Neal	
15.1 Introduction	208
15.2 Early stages of epidemic	211
15.3 Final outcome of Reed–Frost epidemic	213
15.3.1 Preliminaries	213
15.3.2 Many initial infectives	216
15.3.3 Few initial infectives	219
15.3.4 Central limit theorem	221
References	222
<b>16 A branching process approach for the propagation of the Bovine Spongiform Encephalopathy in Great-Britain</b>	225
Christine Jacob, Laurence Maillard-Teyssier, Jean-Baptiste Denis and Caroline Bidot	
16.1 Introduction	225
16.2 Initial branching model	226
16.3 Limit process as $N_0 \rightarrow \infty$	229
16.4 Behavior of the BGW limit process	233
16.4.1 Extinction probability	234
16.4.2 Extinction time distribution	235
16.4.3 Size of the epidemic	235
16.5 Estimation	235
16.5.1 Observations	236
16.5.2 Model and parameters	236
16.5.3 Prior distributions	237
16.5.4 Algorithm and software	237
16.5.5 Main results	238
16.6 Conclusion	239
References	240
<b>17 Time to extinction of infectious diseases through age-dependent branching models</b>	241
Miguel González, Rodrigo Martínez and Maroussia Slavtchova-Bojkova	
17.1 Introduction	241
17.2 Model of epidemic spread	243
17.3 The epidemic's time to extinction	244
17.4 Determining vaccination policies	246
17.4.1 Vaccination based on the mean value of the time to extinction	247
17.4.2 Analyzing the control measures for avian influenza in Vietnam	248
17.5 Concluding remarks	251

17.6	Proofs .....	251
	References .....	256
<b>18</b>	<b>Time to extinction in a two-host interaction model for the macroparasite <i>Echinococcus granulosus</i> .....</b>	<b>257</b>
	Dominik Heinzmann	
18.1	Introduction .....	257
18.2	Prevalence-based interaction model .....	258
18.3	Approximating branching processes .....	259
18.4	Coupling .....	260
18.5	Time to extinction .....	262
18.6	Numerical illustration .....	263
	References .....	265
 <b>Part VI Two-Sex Branching Models</b>		
<b>19</b>	<b>Bisexual branching processes with immigration depending on the number of females and males .....</b>	<b>269</b>
	Shixia Ma and Yongsheng Xing	
19.1	Introduction .....	269
19.2	The bisexual process with immigration .....	270
19.3	The asymptotic growth rate .....	271
19.4	Limit behavior for the supercritical case .....	273
	References .....	276
<b>20</b>	<b>Two-sex branching process literature .....</b>	<b>279</b>
	Manuel Molina	
20.1	Introduction .....	279
20.2	The Daley's two-sex branching process .....	280
20.3	Discrete time two-sex branching processes .....	284
20.3.1	Processes with immigration .....	284
20.3.2	Processes in varying or in random environments .....	285
20.3.3	Processes depending on the number of couples in the population .....	285
20.3.4	Processes with control on the number of progenitor couples .....	287
20.3.5	Others classes of two-sex processes .....	287
20.4	Continuous time two-sex branching processes .....	287
20.5	Applications .....	288
20.5.1	Application in the field of the Epidemiology .....	289
20.5.2	Applications in the field of the Genetics .....	289
20.5.3	Applications in population dynamics .....	290
20.6	Some suggestions for research .....	290
	References .....	291
<b>Index .....</b>		<b>295</b>

Workshop on Branching Processes and Their  
Applications

González, M.V.; Puerto, I.M.; Martínez, R.; Molina, M.;  
Mota, M.; Ramos, A. (Eds.)

2010, XX, 296 p. 15 illus., Softcover

ISBN: 978-3-642-11154-9