

Contents

1	Parametrized Plane Curves	1
1.1	Definitions	1
1.2	Geometric Equivalence	2
1.2.1	Open Curves	2
1.2.2	Closed Curves	2
1.3	Unit Tangent and Normal	3
1.4	Arc Length and Curvature	3
1.5	Expression in Coordinates	5
1.5.1	Cartesian Coordinates	5
1.5.2	Polar Coordinates	6
1.6	Euclidean Invariance	6
1.7	Enclosed Area and Green (Stokes) Formula	7
1.8	Rotation Index and Winding Number	9
1.9	More on Curvature	10
1.10	Discrete Curves and Curvature	11
1.10.1	Least-Squares Approximation	11
1.10.2	Curvature and Distance Maps	12
1.11	Invariance	14
1.11.1	Euclidean Invariance	15
1.11.2	Scale Invariance	17
1.11.3	Special Affine Transformations	18
1.11.4	Generalized Curvature	19
1.12	Characterization of a Bounded Convex Set	22
1.13	Non-Local Representations	25
1.13.1	Semi-Local Invariants	25
1.13.2	The Shape Context	27
1.13.3	Conformal Welding	28
1.14	Implicit Representation	31
1.14.1	Introduction	31
1.14.2	Implicit Polynomials	32
1.15	Invariance for Affine and Projective Transformations	34
1.15.1	Affine Group	35
1.15.2	Projective Group	37
1.15.3	Affine Curvature	39

1.15.4	Projective Curvature	39
2	Medial Axis	43
2.1	Introduction	43
2.2	Structure of the Medial Axis	44
2.3	The Skeleton of a Polygon	44
2.4	Voronoi Diagrams	46
2.4.1	Voronoi Diagrams of Line Segments	46
2.4.2	Voronoi Diagrams of the Vertices of Polygonal Curves .	47
2.5	Thinning	48
2.6	Sensitivity to Noise	48
2.7	Recovering the Initial Curve	49
2.8	Generating Curves from Medial and Skeletal Structures	51
2.8.1	Skeleton with Linear Branches	51
2.8.2	Skeletal Structures	56
3	Moment-Based Representation	59
3.1	Introduction	59
3.2	Moments of Inertia and Registration	59
3.3	Algebraic Moments	60
3.4	Rotation Invariance	62
4	Local Properties of Surfaces	65
4.1	Curves in Three Dimensions	65
4.2	Regular Surfaces	67
4.2.1	Examples	67
4.2.2	Changing Coordinates	69
4.2.3	Implicit Surfaces	70
4.3	Tangent Plane and Differentials	70
4.3.1	Tangent Plane	70
4.3.2	Differentials	71
4.4	Orientation and Normals	73
4.5	Integration on an Orientable Surface	73
4.6	The First Fundamental Form	76
4.6.1	Definition and Properties	76
4.6.2	The Divergence Theorem on Surfaces	78
4.7	Curvature and Second Fundamental Form	81
4.8	Curvature in Local Coordinates	83
4.9	Implicit Surfaces	84
4.9.1	The Delta-Function Trick	84
4.10	Gauss–Bonnet Theorem	87
4.11	Triangulated Surfaces	88
4.11.1	Definition and Notation	88
4.11.2	Estimating the Curvatures	88

5 Isocontours and Isosurfaces	105
5.1 Computing Isocontours	105
5.2 Computing Isosurfaces	107
6 Evolving Curves and Surfaces	115
6.1 Curve Evolution	115
6.1.1 Grassfires	119
6.1.2 Curvature Motion	121
6.1.3 Implicit Representation of the Curvature Motion	123
6.1.4 More on the Implementation	124
6.2 Surface Evolution	126
6.2.1 Mean Curvature Surface Flow	132
6.3 Gradient Flows	132
6.4 Active Contours	136
6.4.1 Introduction	136
6.4.2 First Variation and Gradient Descent	136
6.4.3 Numerical Implementation	137
6.4.4 Initialization and Other Driving Techniques	138
6.4.5 Evolution of the Parametrization	139
6.4.6 Geometric Methods	142
6.4.7 Controlled Curve Evolution	143
6.4.8 Geometric Active Surfaces	144
6.4.9 Designing the V Function	144
6.4.10 Inside/Outside Optimization	145
6.4.11 Sobolev Active Contours	147
7 Deformable templates	149
7.1 Linear Representations	149
7.1.1 Energetic Representation	150
7.2 Probabilistic Decompositions	152
7.2.1 Deformation Axes	152
7.3 Stochastic Deformations Models	154
7.3.1 Generalities	154
7.3.2 Representation and Deformations of Planar Polygonal Shapes	155
7.4 Segmentation with Deformable Templates	157
8 Ordinary Differential Equations and Groups of Diffeomorphisms	161
8.1 Introduction	161
8.2 Flows and Groups of Diffeomorphisms	165
8.2.1 Definitions	165
8.2.2 Admissible Banach Spaces	171
8.2.3 Induced Group of Diffeomorphisms	172
8.2.4 A Distance on G_V	172
8.2.5 Properties of the Distance	174

9 Building Admissible Spaces	177
9.1 Reproducing Kernel Hilbert Spaces	177
9.1.1 The Scalar Case	177
9.1.2 The Vector Case	181
9.2 Building V from Operators	185
9.3 Invariance of the Inner Product	188
9.3.1 Invariance: the Operator Side	189
9.3.2 Invariance: the Kernel Side	192
9.3.3 Examples of Radial Kernels	195
9.4 Mercer's Theorem	197
9.5 Thin-Plate Interpolation	199
9.6 Asymptotically Affine Kernels	200
10 Deformable Objects and Matching Functionals	203
10.1 General Principles	203
10.2 Variation with Respect to Diffeomorphisms	204
10.3 Labeled Point Matching	207
10.4 Image Matching	208
10.5 Measure Matching	213
10.5.1 Matching Densities	215
10.5.2 Dual RKHS Norms on Measures	217
10.6 Matching Curves and Surfaces	219
10.6.1 Curve Matching with Measures	220
10.6.2 Curve Matching with Vector Measures	224
10.6.3 Surface Matching	225
10.6.4 Induced Actions and Currents	229
10.7 Matching Vector Fields	233
10.8 Matching Fields of Frames	237
10.9 Tensor Matching	238
10.10 Pros and Cons of Greedy Algorithms	242
10.11 Summary of the Results of Chapter 10	243
10.11.1 Labeled Points	243
10.11.2 Images	243
10.11.3 Measures	244
10.11.4 Plane Curves	244
10.11.5 Surfaces	245
10.11.6 Vector Fields	246
10.11.7 Frames	246
10.11.8 Tensor Matching	247
11 Diffeomorphic Matching	249
11.1 Linearized Deformations	249
11.2 The Monge–Kantorovich Problem	251

11.3 Optimizing Over Flows	254
11.4 Euler–Lagrange Equations and Gradient	255
11.5 Conservation of Momentum	259
11.5.1 Properties of the Momentum Conservation Equation .	260
11.5.2 Existence of Solutions	262
11.5.3 Time Variation of the Eulerian Momentum	263
11.5.4 Hamiltonian Form of EPDiff	265
11.5.5 Case of Measure Momenta	266
11.6 Optimization Strategies for Flow-Based Matching	267
11.6.1 Gradient Descent in \mathcal{X}_V^2	268
11.6.2 Gradient in the Hamiltonian Form	271
11.6.3 Gradient in the Initial Momentum	277
11.6.4 Shooting	281
11.6.5 Gradient in the Deformable Object	284
11.6.6 Image Matching	287
11.6.7 Pros and Cons of the Optimization Strategies	292
11.7 Numerical Aspects	293
11.7.1 Discretization	293
11.7.2 Kernel-Related Numerics	298
12 Distances and Group Actions	303
12.1 General Principles	303
12.1.1 Distance Induced by a Group Action	303
12.1.2 Distance Altered by a Group Action	305
12.1.3 Transitive Action	305
12.1.4 Infinitesimal Approach	308
12.2 Invariant Distances Between Point Sets	308
12.2.1 Introduction	308
12.2.2 Space of Planar N -Shapes	309
12.2.3 Extension to Plane Curves	313
12.3 Parametrization-Invariant Distances Between Plane Curves .	314
12.4 Invariant Metrics on Diffeomorphisms	317
12.4.1 Geodesic Equation	318
12.4.2 A Simple Example	320
12.4.3 Gradient Descent	321
12.4.4 Diffeomorphic Active Contours	321
12.5 Horizontality	323
12.5.1 Riemannian Projection	323
12.5.2 The Momentum Representation	325
13 Metamorphosis	331
13.1 Definitions	331
13.2 A New Metric on M	332
13.3 Euler–Lagrange Equations	333
13.4 Applications	334

13.4.1	Labeled Point Sets	334
13.4.2	Deformable Images	336
13.4.3	Discretization of Image Metamorphosis	337
13.4.4	Application: Comparison of Plane Curves	338
A	Elements from Hilbert Space Theory	347
A.1	Definition and Notation	347
A.2	Examples	349
A.2.1	Finite-Dimensional Euclidean spaces	349
A.2.2	ℓ^2 Space of Real Sequences	349
A.2.3	L^2 Space of Functions	349
A.3	Orthogonal Spaces and Projection	350
A.4	Orthonormal Sequences	352
A.5	The Riesz Representation Theorem	353
A.6	Embeddings and the Duality Paradox	354
A.6.1	Definition	354
A.6.2	Examples	355
A.6.3	The Duality Paradox	357
A.7	Weak Convergence in a Hilbert Space	358
A.8	The Fourier Transform	359
B	Elements from Differential Geometry	361
B.1	Introduction	361
B.2	Differential Manifolds	361
B.2.1	Definition	361
B.2.2	Vector Fields, Tangent Spaces	362
B.2.3	Maps Between Two Manifolds	365
B.3	Submanifolds	365
B.4	Lie Groups	366
B.4.1	Definitions	366
B.4.2	Lie Algebra of a Lie Group	367
B.4.3	Finite-Dimensional Transformation Groups	368
B.5	Group Action	373
B.5.1	Definitions	373
B.5.2	Homogeneous Spaces	374
B.5.3	Infinitesimal Action	375
B.6	Riemannian Manifolds	375
B.6.1	Introduction	375
B.6.2	Geodesic Distance	377
B.6.3	Lie Groups with a Right-Invariant Metric	378
B.6.4	Covariant Derivatives	378
B.6.5	Parallel Transport	381
B.6.6	A Hamiltonian Formulation	381

C Ordinary Differential Equations	383
C.1 Differentials in Banach Space	383
C.2 A Class of Ordinary Differential Equations	385
C.2.1 Existence and Uniqueness	386
C.2.2 Flow Associated to an ODE	387
C.3 Numerical Integration of ODEs	390
D Optimization Algorithms	395
D.1 Directions of Descent and Line Search	395
D.2 Gradient Descent	396
D.3 Newton and Quasi-Newton Directions	398
D.4 Conjugate Gradient	400
D.4.1 Linear Conjugate Gradient	400
D.4.2 Nonlinear Conjugate Gradient	403
E Principal Component Analysis	405
E.1 General Setting	405
E.2 Computation of the Principal Components	406
E.2.1 Small Dimension	406
E.2.2 Large Dimension	407
E.3 Statistical Interpretation and Probabilistic PCA	408
F Dynamic Programming	411
F.1 Minimization of a Function on a Tree	411
F.2 Shortest Path on a Graph: a First Solution	412
F.3 Dijkstra Algorithm	413
F.4 Shortest Path in Continuous Space	414
References	419
Index	431



<http://www.springer.com/978-3-642-12054-1>

Shapes and Diffeomorphisms

Younes, L.

2010, XVI, 438 p. 36 illus., Hardcover

ISBN: 978-3-642-12054-1