Game Systems in Team Sports

Jaime Gil-Lafuente

Abstract Spectator team sports are now being studied more frequently by those who have, or wish to have, responsibility of the smooth functioning of club sports. Technical directors, managers, trainers, and individuals in other similar roles look for "formulas" that can help them to obtain good qualifying results and present in-person spectators and the television and general audience with an interesting and visually stimulating show. Recently, the "game system" concept has become popular, providing a small set of formulas of attack and containment capable of giving unique quality or a distinct identity to a team. Because terms like "game system" are often repeated and circulated by word of mouth without precise knowledge of what they really mean, we attempt to establish a definition for this term. This definition, in turn, will provide the basis for a methodology for neutralizing the problems that impede or hinder the achievement of the objectives sought through the development of the game.

1 Concept and Content of Game Systems

We deem the "game system" a homogenous set of circulatory diagrams of the ball (or equivalent object) that can be represented by networks or arborescence (as well as anti-arborescence) whose objective is to ensure the best possible performance of the available athletes through victories over their opponents. The proposed definition includes three fundamental elements:

1. A homogenous set of circulatory diagrams. The notion of the "system" in sports has to do with characteristics that differentiate one team's method from other systems or simply from other ways of playing the game. Thus, it seems logical to think about the existence of a basic "method" to make the ball move (for example), from which there can exist variants that are adopted or not according to events on a particular game or during a specific competition.

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2. Representability through networks or arborescence. When one is attempting an in-depth study of a team's plays, circulatory diagrams as a form of visualization are almost essential. Thus, as we frequently see in sports television programs, a trainer will show on a blackboard or screen how the ball should circulate and/or how the players should move. Reticular diagrams and arborescence (and also anti-arborescence) are among the tools currently used for this purpose.
3. Success in sports. The quest for overall success through victory in each contest, game or match is integral to the spectacle of sports. Normally, victory is achieved when a team attains the greater number of points (e.g., through goals or baskets), while blocking or limiting those of its opponent. Circulatory diagrams should provide "lines" of movement by which offensive players can achieve goals and those by which they can intercept the opponent.

These reflections on the definition of the "game system" have permitted the introduction of other concepts that will be the object of subsequent investigation. We refer to networks, sub-networks, schemes, and cuts, all of which we attempt to introduce into a unitary model.

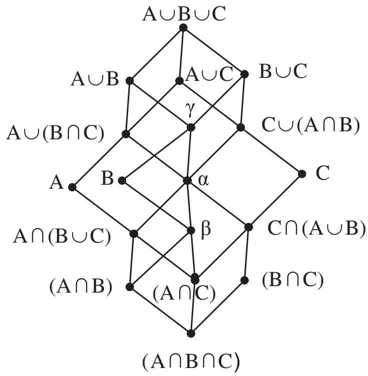
2 Game Diagrams

Having clarified this fundamental terminology, we will move on to a much more practical phase, situating ourselves in the position of the trainer or an analogous person—who, before the athletes "executed" their instructions, plotted on the board a positional diagram of some (or all) of the players and indicated how the ball should circulate.

First, with the object of beginning the study in the simplest manner possible, we will suppose that the sports official, manager or trainer (we will use this term for simplicity) establish that the ball should only be "touched" once by each athlete and that a maximum of just three athletes should be involved in a play. This is not an unreasonable scenario to begin with, and it allows the introduction of more complex diagrams.

As is well known, the possible diagrams form a free distributive lattice with three generators. The number of elements in this lattice is 18. Hence, that is also the number of possible diagrams. Before representing the lattice and the 18 diagrams from which it is derived, we must remember that the number of elements of the distributive n-lattice with generators increases rapidly as n increases. This soon makes it practically impossible to represent all possible diagrams. Thus, for example, when $n = 2$, the number of possible diagrams is four; if $n = 3$, then the number of possible diagrams is 18, as we have noted; when $n = 4$, now the number of diagrams is 166; when $n = 5$, the number of diagrams rises to 7,578; and finally, when $n = 6$, the number of possible diagrams becomes extremely high at 7,828,352. Obviously, this study will not consider every possibility, but will instead choose from among them according to their utility, highlighting

Fig. 1



$$\alpha = (A \cap B) \cup (B \cap C) \cup (C \cap A)$$
$$\beta = B \cap (A \cup C)$$
$$\gamma = B \cup (A \cap C)$$

those that are likely to be adopted in reality and by their homogeneity in leading to a "game system". Granted these observations, we will continue with our simple example, presenting the network for three athletes (or three generators), see Fig. 1.

Established clarifications in relation with the possibility to examine all or part of the diagrams to be developed by my team, we will go through the representation of each one of the more useful and pertinent ones. This might seem to call for establishing the theoretical basis on which the concept that we propose is based, but for educational purposes, we will first continue presenting our case, leaving the theoretical justifications for later.

The 18 vertices of the grid show the 18 possible diagrams with three players who only have the ball once in the game. We will assign to these three athletes the capital letters A, B, and C. When the operator ∪ appears in the grid, the players are acting in "parallel" (one player or the other plays) and when ∩ appears, the players are acting in a "series" (one player plays "and" then the other does). We can now represent the 18 diagrams using the reticular/grid formula. With the objective of maintaining consistency with the grid, we are going to arrange the diagrams in the same position as the vertices of the network.

At each end of the network, one sees the beginning of a play (in soccer, it could be the ball being played out by the goalkeeper) and the end of the play (for example, the ball reaching the other goal). These parts of the play are designated by I and F, respectively (see Fig. 2).

Based on these networks, it is possible to formulate certain pertinent questions.

1. Not all 18 plays merit the attention of the sports manager. Some will perhaps consider them unviable, while others, although they think them possible, will find that they are not consistent with what is perceived of as "modern play".

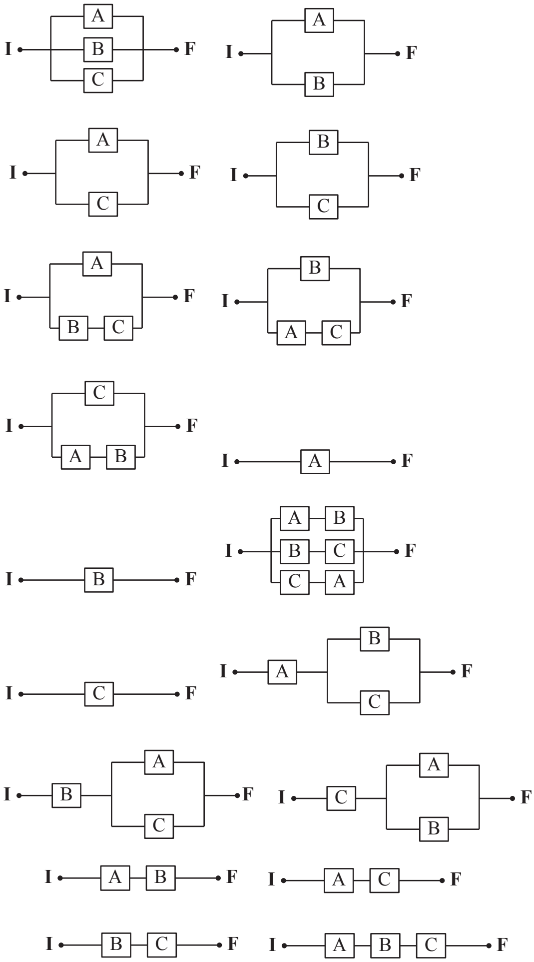


Fig. 2

Fortunately, because of this, there are distinct "game systems". Thus, out of all possibilities, the sports official will chose the systems that best fit his way of thinking.

2. The placement of two players "in order" (in a horizontal line)—for example, with *A* first and *B* afterwards—does not predetermine their order. The same diagram is valid with *B* first and *A* afterwards in our example. This principle of interchangeability is valid for any players who act in series, but not when they act in parallel, acting one "or" the other.
3. The placement of two players in the same line does not require them to run or move in a straight line; rather, the ball passes from one to another. We have drawn the networks using rectangular figures to remain consistent

with the reticular analysis done in other fields, but this does not mean that the athlete has to follow a pattern of precisely angulated lines. It would be perfectly acceptable, and in many cases desirable, for example, to follow a straight line towards F. What is important is maintenance of the sequence of players.

4. It will be observed that there does not appear to be any sign that indicates the direction of the game in any of these networks. Thus, the ball can circulate either from I to F or the reverse (forwards and backwards). We will tell that in theory, oriented arches/arcs do not exist under these circumstances. However, it is always possible to introduce this element. Then, the associated lines will become arrows, and the players will only be able to circulate in the direction indicated by the arrows (also called arcs/arches). In a network, there, for example, may be one oriented arc/arch among others without any specific orientation (that allow circulation in one direction and the other).

To move from the descriptive to the operational realm, we introduce a more complex diagram (reticular representation) than those shown so far, including five athletes: A, B, C, D, and E. We initially maintained the hypothesis that the ball only passes once to each of them. The manager, after adequate advice, presents the network shown in Fig. 3.

With this approach, the objective of the sports official is to determine the ball's shortest "path" from I to F or to avoid its going from F to I. For the former, this means examining all of the possibilities that the proposed diagram offers, the "minimum ties" that permit the advancement of the ball from the origin of the play to the end.

In this simple example, it appears that there are at least three possible schemes: $\{C\}$, $\{D, E\}$, and $\{A, B, E\}$. Graphically, we can represent these as in Fig. 4.

On the other hand, if it were the rival team's trainer who were using this diagram, the sports official for our team would be working to block those planned movements. In other words, the objective then would be to "cut" the planned movements. The task is now to find the "minimum lengths" that achieve this.

In this simple scenario, one can find the minimum lengths (which are as follows) by sight:

$$\{C, D, A\}, \{C, D, B\}, \text{ and } \{C, E\}.$$

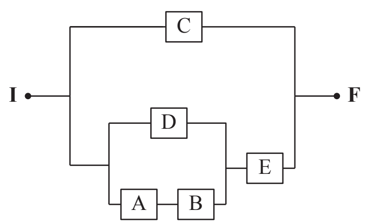
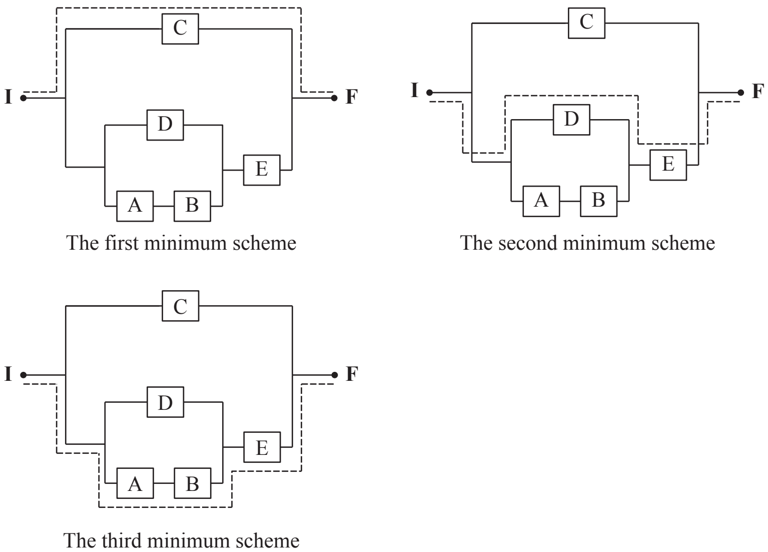


Fig. 3



The first minimum scheme

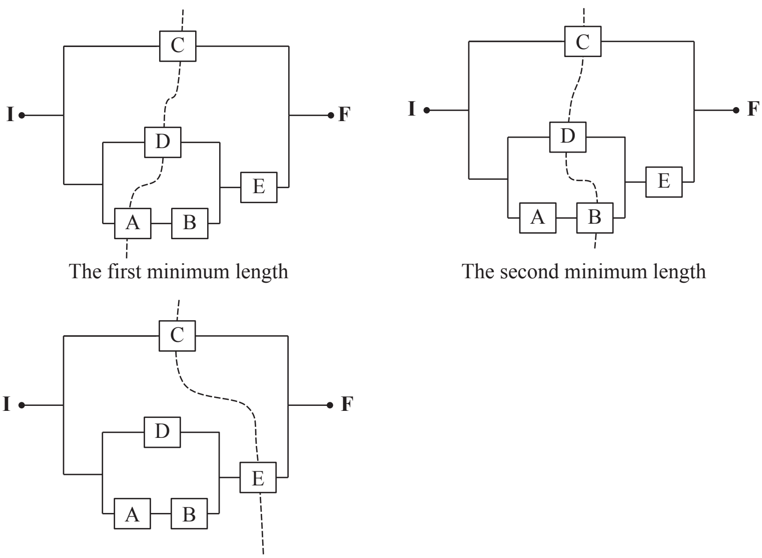
The second minimum scheme

The third minimum scheme

Fig. 4

Graphically, we can represent these as in Fig. 5.

In this first example, intended as an introduction to a possible theory of game systems, two fundamental concepts have appeared: those of “minimum ties” and “minimum lengths”.

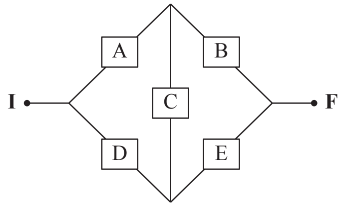


The first minimum length

The second minimum length

Fig. 5

Fig. 6



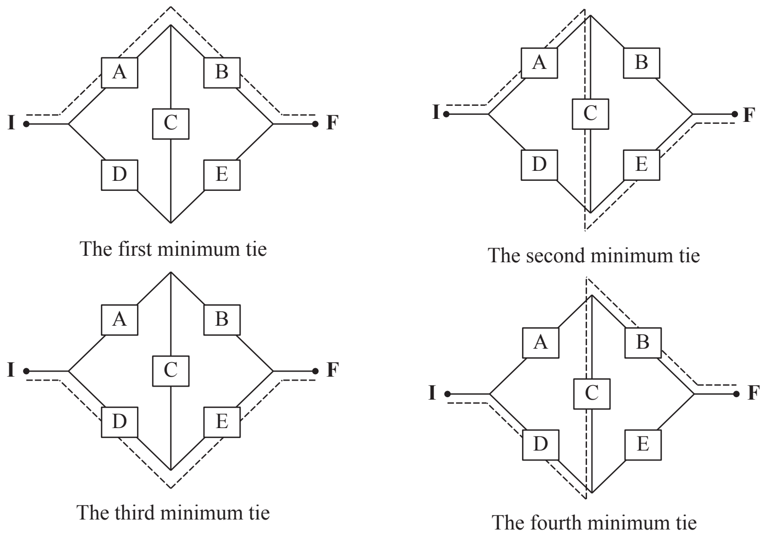


Fig. 7

Considering the same basic construction, we present another example. Here, the same players assume different positions, such that the circulation of the ball will also be different. The suggested diagram is as in Fig. 6.

The first objective is to develop an offensive play. There are now four "minimum ties":

$$\{A, B\}, \{A, C, E\}, \{D, E\}, \text{ and } \{B, C, D\}.$$

We can graphically represent these as in Fig. 7.

For the second objective, defensive action, there are also four "minimum lengths":

$$\{A, D\}, \{B, E\}, \{A, C, E\}, \{B, C, D\}.$$

See Fig. 8 for an illustration. It is thus observed that if the team playing defense can successfully block the marked players through the discontinuous line in each network, the opposing team's play is destined to fail. We can now ask ourselves

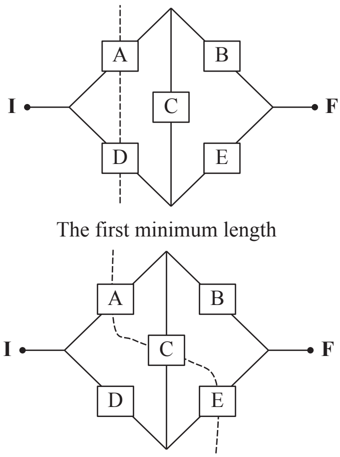


Fig. 8

what happens when, because of a player's limitations or habits, the movement of the ball is limited to one direction. To consider this possibility, we can use the same diagram but assume that player C can only move the ball in the direction marked by the arrow noted in Fig. 9.

This condition significantly changes the situation. In effect, in terms of offense, the four available minimum schemes have been reduced to the following three:

$$\{A, B\}, \{A, C, E\}, \text{ and } \{D, E\}.$$

In our illustration, there would no longer be a fourth minimum tie. Something similar happens with defense. The four minimum lengths have been reduced to the following three:

$$\{A, D\}, \{A, C, E\}, \text{ and } \{B, E\}.$$

In our illustration, the fourth minimum length would disappear.

With these examples, we believe to have demonstrated the practical meaning of two fundamental concepts: those of "minimum ties" and "minimum lengths". However, we should also note an underlying hypothesis: "If a player can make a

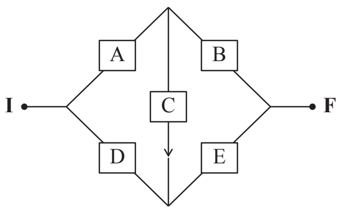


Fig. 9

play himself, he will not utilize other players". This hypothesis is based on the idea that in the game, taking place with the same number of players on each team, if the player who has the ball is free (of marking), his teammates will not be free; therefore, it assumes an unnecessary risk by involving another marked player.

When a player is marked and unable to escape, then the intervention of another player appears to be necessary. Despite this, it is also possible to dispense with this assumption (which we have now made explicit), although we are going to maintain it for our purposes, so as not to break the thread of the exposition.

3 Generalization of the Concepts of Our Diagrams

Until now, we have focused on diagrams in which each player intervenes only once in each play. Now we will consider a model that allows players, if applicable, to occupy another position on the court or field once the ball is released such that they can receive it again. This represents a type of generalization of previously presented player involvement. Again, we will begin with an example shown in Fig. 10.

In this network, it can be observed that players A, B and C now move from their original "lines" to other positions; they have the potential to touch the ball on more than one occasion and with or without the intervention of another player. In this diagram, the offensive position allows the minimum schemes $\{A\}$, $\{B\}$, and $\{C, D\}$, where we can visualize these via the graphics as in Fig. 11. For the defensive position, two minimum lengths appear, $\{A, B, C\}$, $\{A, B, D\}$, whose representations in the network are as in Fig. 12. Observe that, in the first diagram, we have shown the "cut line" in the more advanced positions of $\{A, B, C\}$. We would also have been able to do this in the rearmost positions without weakening the effectiveness of the exposition. As such, this can be considered: three cuts instead of the two that we have noted. Based on this new assumption, the cut $\{A, B, C\}$ would repeat in two configurations: one more advanced in the diagram than the other. In reality, there are three players who move forward and backwards in an area of the field or court.

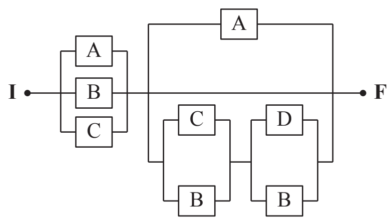
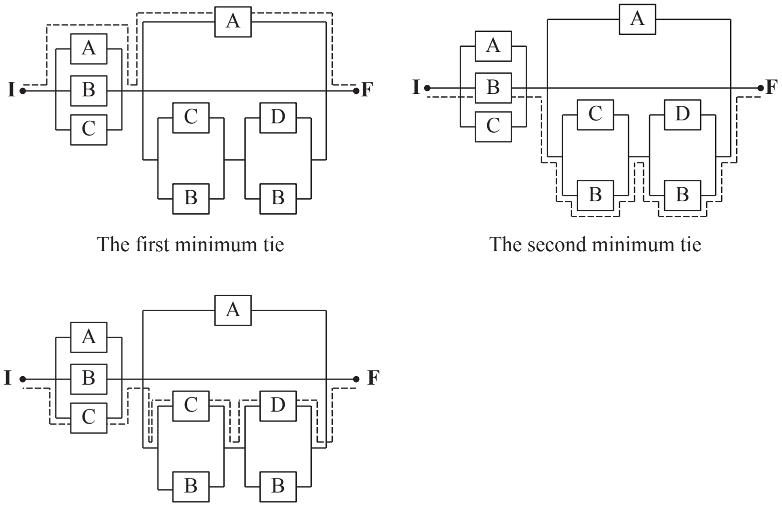


Fig. 10

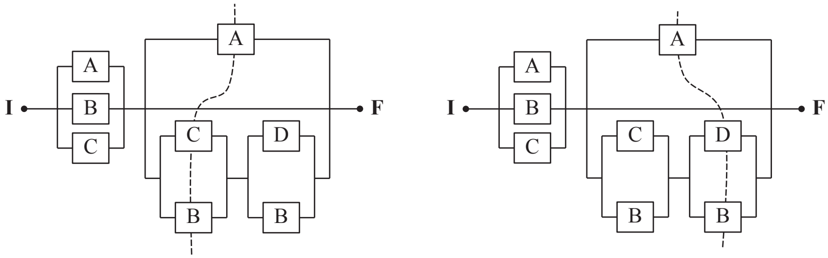


The first minimum tie

The second minimum tie

The third minimum tie

Fig. 11



First minimum length

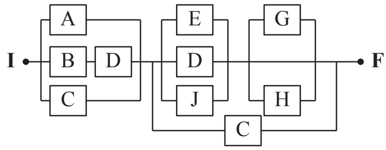
Second minimum length

Fig. 12

4 Paths and Cuts in Complex Diagrams

In the previous sections, we have shown, given certain assumptions, some possible diagrams of plays, all of them including 3, 4 or (at most) 5 players. However, there are few team sports in which the teams are this small. At the beginning of this paper, we highlighted that increasing the number of players increases the number of possible different diagrams astronomically. Thus, in assumptions that limit this

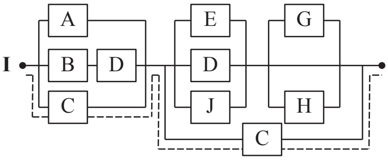
Fig. 13



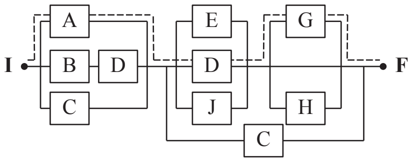
number (for example the non-repetition of a player in more than one position), the problem of how to operationalize a system with an increased number of players arises. For example, in soccer or football, the team will consist of a goalkeeper and ten more players. We have found a solution for use in the context of this and other sports with a similar number of active participants: American football, basketball and rugby, among others. The key is to decompose and compose global diagrams represented by networks. In more technical terms, the goal is to decompose a network into sub-networks and then reconstitute it. Fortunately, we know the proper techniques for doing this, sometimes with only minimal study of the diagrams. Below, we will present several examples. We begin with a complex network as in Fig. 13. This network includes eight players, a number sufficient to study this process of decomposition. In this diagram, one can observe that there are nine minimum offensive ties: $\{C\}$, $\{A, D, G\}$, $\{A, D, H\}$, $\{A, E, G\}$, $\{A, E, H\}$, $\{A, J, G\}$, $\{A, J, H\}$, $\{B, D, G\}$, and $\{B, D, H\}$. We graphically present these in Fig. 14.

The description and graphic representation of these minimum schemes encourages certain reflections. First of all, the number and complexity of the minimum schemes have increased even though we have limited ourselves to 8 players and only 2 of them engage in "repeat play": players C and D. Secondly, it is difficult to visualize all of the minimum ties without omitting some or making mistakes. We approach the first issue by decomposing the network into subnetworks. Regarding the second, we can simply say that there should be no problem in analytically determining the minimum schemes. We will address this question in greater depth later on but for the moment move on to consider defensive action.

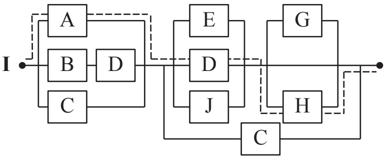
Here, there are four minimum lengths at play as follows: $\{A, B, C\}$, $\{A, C, D\}$, $\{C, D, E, J\}$, and $\{C, G, H\}$. They can be graphically represented as in Fig. 15. It can be observed that the first and second minimum lengths differ only in one player. In the first play, the "cancelation" must be at B, whereas it must be at D in the second play. One should note, however, that the two players are in line at the same place in the diagram; this seems to indicate that, for defensive purposes, there is a "redundancy" and therefore that one of them is superfluous. Nevertheless, leaving aside this issue for the moment, we see what can be done to simplify the work of analyzing the diagrams as they are represented here.



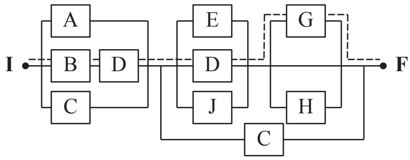
First minimum tie



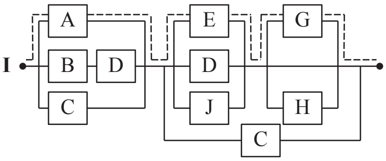
Second minimum tie



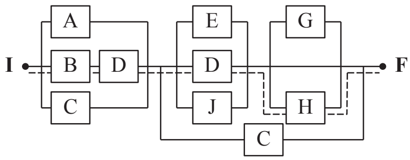
Third minimum tie



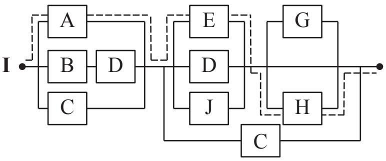
Fourth minimum tie



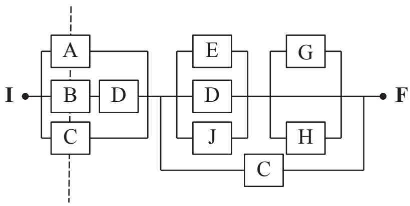
Fifth minimum tie



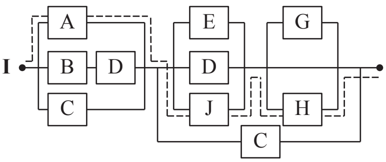
Sixth minimum tie



Seventh minimum tie



Eighth minimum tie



Ninth minimum tie

Fig. 14

5 Partial Analysis of a Global Diagram

Every complex diagram represented by a network/chain can be decomposed into various partial diagrams, each one of which can be conceptualized as a sub-network. In addition, representing a pair of theoretical rules, specified exactly with two operators, if they later want to return to form the initial network. Of course, there are various ways of decomposing the network; the same diagram of a play can yield various groups of sub-diagrams. In other words, according to necessities or desire,

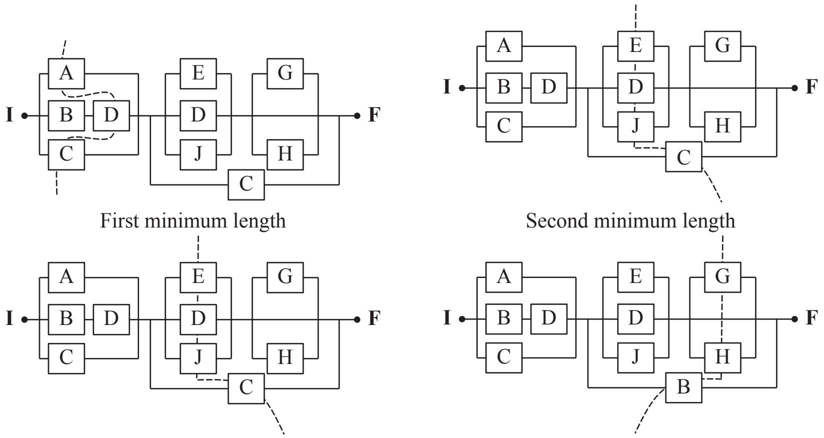


Fig. 15

the network can be cut into pieces in a distinct manner, and all of the possibilities are theoretically correct. In practice, the sports official will choose the method that seems more useful for the team's particular purposes.

Given this word of caution, we suppose that the trainer wishes to separately analyze the 4 sub-networks denoted by S_1, S_2, S_3, S_4 that appear in Fig. 16. As we have just noted, this can be considered a network of sub-networks. In effect, after theoretical merging of the components of a subgroup, the players of a sub-network act as just one player, as shown in Fig. 17. This is very useful—above all, for the recomposition of a network from sub-networks as we will see in the following. Based on this decomposition, it is possible to separately study each of the four sub-networks. We will begin with sub-network s_1, which is graphically represented as in Fig. 18. It is easy to establish the minimum schemes (offensive position):

$$\{A\}, \{B, D\}, \{C\}.$$

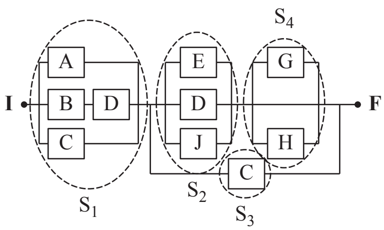


Fig. 16

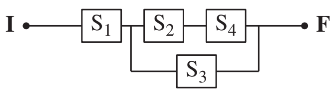
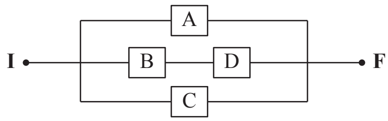


Fig. 17

Fig. 18



The same is true of the minimum lengths:

$$\{A, B, C\}, \{A, C, D\}.$$

Thus, it becomes very easy to make a partial study of this zone of play. It will not escape the reader that these segmented analyzes are helpful in devising positional changes or modifications to the structure of the sub-network. The study of the other sub-networks, which is presented below, is even easier.

Sub-network S_2 is shown in Fig. 19. There are three minimum ties—$\{E\}$, $\{D\}$, and $\{J\}$—with only one minimum length: $\{E, D, J\}$. Sub-network S_3 is shown in Fig. 20. Here, there is evidently a minimum tie and a minimum length that coincide: $\{C\}$. Sub-network S_4 is shown in Fig. 21. There are two minimum ties, $\{G\}$ and $\{H\}$, and there is only one minimum length, $\{G, H\}$.

We have now analyzed the component parts of the global network. However, one cannot automatically fuse the partial minimum schemes and cuts into a global system of minimum schemes and cuts. In this phase of the study, we will determine them taking into account the entire network.

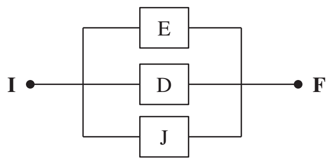


Fig. 19

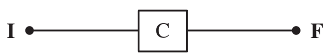


Fig. 20

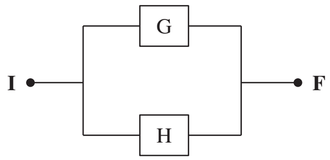


Fig. 21

6 Modification of the Diagrams for the Game

Suppose that the sports official wishes to modify the diagram through the sub-network S_1-displacing, for example, players B and D to move them to a new diagram, as is represented in Fig. 22. Once available, the new S_1 sub-network recombines with the others to help create the global network as previously stated. We have the diagram shown in Fig. 23. However, to modify the structure of one or more sub-networks is not the only useful option. Additionally, the sports official might decide to rearrange the network or sub-networks themselves. Thus, for example, the prior network of sub-networks shown in Fig. 24 could become the network shown in Fig. 25. Then, the general network looks as in Fig. 26.

For this network to be valid, one must reevaluate the minimum ties and cuts, which normally do not coincide with those in the existing network before the adjustments to the sub-networks. A question can be posed in relation to these changes. The change between S_1 and S_2 (with independence of the variation in the positioning of S_3) means, in a certain sense, moving players from mid-field or mid-court (those that form S_2) to defensive positions and, conversely, moving defensive players to positions at mid-field or mid-court, and this is not always possible in all team sports. The question to ask is what would happen if the players in the sub-networks interchange/exchange and the only thing that varied was the relative positions of the sub-networks? Evidently, not much would change. The same general network would

Fig. 22

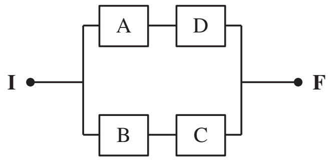


Fig. 23

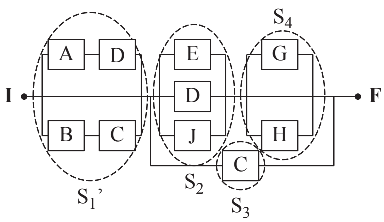


Fig. 24

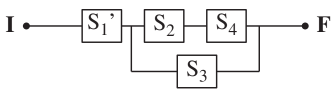


Fig. 25

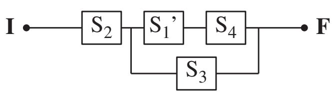
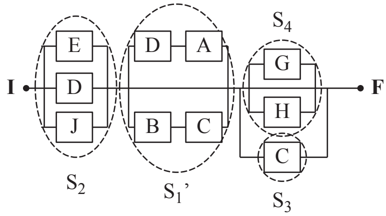


Fig. 26



exist, and the only modifications would be changes to which players were in what position. Easier still, one might preserve the original configuration of letters but just assign them to different players, as appropriate.

7 Study of the Game by Zones

To end our examination of this subject, we reverse our initial approach and assume that the trainer wishes to study the game based on the zones of the field or court, creating partial diagrams that can later be conveniently integrated into a general illustration. This process is similar to the last one we studied, but with small variations (the construction of the larger network occurs after the formation of each sub-network). We will consider this problem using an example. Let us say that the problem consists of two perfectly differentiable phases:

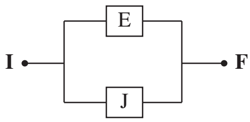
(1) Construction of the sub-networks
(2) Implementation of a network

Let us suppose that a trainer wishes to conduct a study of the circulation of the ball, dividing the field or court into four zones: Z_1 (rear or defensive), Z_2 (left of center), Z_3 (right of center), and Z_4 (front).

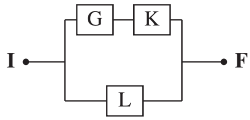
There are 11 players: 4 in zone Z_1, 2 in zone Z_2, 2 in zone Z_3, and 3 in zone Z_4. The sports official, after the corresponding analysis, decides to establish the diagrams shown in Fig. 27, represented by the respective sub-networks.

In constructing these sub-networks, it is not our intention to provide a real diagram that would routinely be used within a game system. We simply wish to present a totally arbitrary example. We now move to the second phase, constructing the network of sub-networks (see Fig. 28). In this, as in many other cases, the formation of the general network is immediate, see Fig. 29.

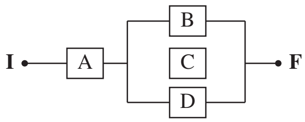
It is this network that one references when one needs to ascertain the minimum ties for the offensive positions and the minimum lengths for the defensive positions. This can be done independently of the partial analyzes that can be studied separately for each one of the 4 sub-networks.



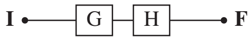
Sub-network S_2 of Zone Z_2



Sub-network S_4 of Zone Z_4



Sub-network S_1 of Zone Z_1



Sub-network S_3 of Zone Z_3

Fig. 27

Fig. 28

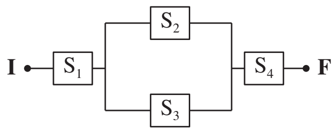
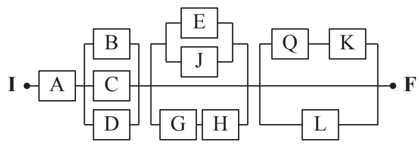


Fig. 29



8 The Function of the Structure of a Game Diagram

We have presented numerous examples of the construction and functioning of diagrams and sub-diagrams of plays, avoiding mathematical formulations until now. The decomposition and recomposition of a network into sub-networks and the various sub-networks in a general network have been our object of analysis based on simple logic. We have determined the minimum “ties”, “lengths”, or “cuts” in a simple visual manner, trusting in the intuition and high-quality work of sports officials. However, we believe that while this approach is interesting, a scientifically rigorous analysis of the subject requires us to create technical methods of automatically finding such solutions without the possibility of error or omission.

Furthermore, some urgent questions arise. Among them is the reliability or unreliability of the game system or of the diagrams of plays. One can and should reflect on what happens if one of the players fails to execute a particular move during an encounter. In addition, one must of course attempt to optimize the functioning of the game system. Furthermore, other lesser questions deserve special attention. For example, when a player is expelled from the field or is hurt without a substitute being called in, the diagram of the play must be recomposed and the positions adjusted in the best way possible. It is important to know if the presence of the player on the field is superfluous because there are one or more players who are already doing the work that this player was assigned to do (how many times we have seen or heard that two players are "in the way"?). At times, the value of a player on the rival team makes particular attention necessary. In this case two players could mark the same player. These decisions, among others, have the potential to assure the success or failure of a team.

It is for these reasons that we will undertake this part of our study, considering the offensive position of the player, whom we denote as A. This individual may or may not perform adequately (they may have significant weaknesses). Therefore, we assign a binary variable to the functioning of A such that

If A yields, it will equal 1
If A does not yield, it will equal 0

We next consider the simplest case, formed by 2 players: A and B. Each one is assigned a binary variable, a for A and b for B. Only two diagrams can be formed that include only these two athletes.

1. If A yields "and" B yields, system S functions. The network, already known as the "network in series", is shown in Fig. 30.
   If one of the two does not yield the diagram no longer functions. Let us imagine all of the possible cases:
   A yields "and" B yields; S functions;
   A does not yield "and" B yields; S does not function;
   A yields "and" B does not yield; S does not function;
   A does not yield "and" B does not yield; S does not function.
2. If A yields "and/or" B yields, the system S functions. The network, also known as a "parallel network", is shown in Fig. 31.

Fig. 30

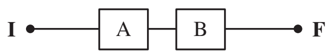
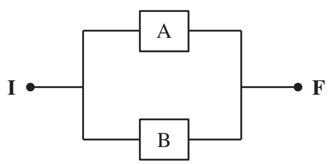


Fig. 31



It is sufficient then that one of the two players yields (although both may also do so) for the system to function. The possible cases are as follows:

A yields "and" *B* yields; *S* functions;
A does not yields "and" *B* yields; *S* functions;
A yields "and" *B* does not yield; *S* functions;
A does not yield "and" *B* does not yield; *S* does not function.

It can easily be proved that the only operators capable of faithfully reflecting the two diagrams are $(\cdot)$ and $(+)$. In effect, if a function is formed that represents these two situations and we call it $\varphi(a, b)$, then for the network in series

$$\varphi(a, b) = a \cdot b,$$

where the symbol $\cdot$ (product) semantically reflects the "and", and for the parallel network

$$\varphi(a, b) = a + b,$$

where the symbol $+$ (algebraic sum) semantically corresponds to "and/or".

Thus, it can be said that every diagram of play represented by a network possesses its own function φ that is called its "structural function". Thus, all of the previously presented networks have their own structural function.

In general, when various athletes $A, B, C, \ldots, K$ are involved in a play, the function of the structure will be depicted as follows:

$$\varphi(a, b, c, \ldots, k).$$

Below is a key example of what form the structural function of some of the networks previously studied takes. We begin with the diagram shown in Fig. 32. The function of the structure is as follows:

$$\varphi(a, b, c, d, e) = c + (d + a \cdot b) \cdot e$$
$$\varphi(a, b, c, d, e) = c + de + a \cdot b \cdot e$$

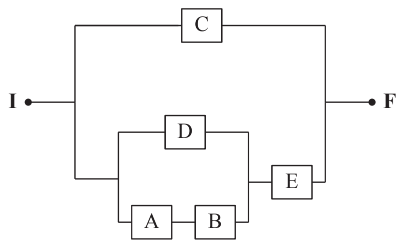
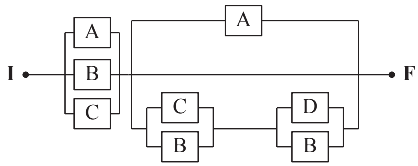


Fig. 32

Fig. 33



For the network shown in Fig. 33. we obtain the following function of the structure:

$$\begin{aligned}\varphi(a,b,c,d) &= (a+b+c)\cdot(a+(b+c)\cdot(b+d))\\ &= (a+b+c)(a+b+b\cdot d+c\cdot b+c\cdot d)\end{aligned}$$

By the property of absorption

$$b+b\cdot d+b\cdot c=b,$$

so

$$\varphi(a,b,c,d)=a+a\cdot b+a\cdot c\cdot d+a\cdot b+b+b\cdot c\cdot d+a\cdot c+b\cdot c+c\cdot d.$$

Finally, by absorption and using $a^2=a$ and $b^2=b$, we have

$$\varphi(a,b,c,d)=a+b+c\cdot d.$$

Lastly, we can consider one of the most complex networks studied in Fig. 34. It has a structural function as follows:

$$\begin{aligned}\varphi(a,b,c,d,e,j,g,h) &= (a+bd+c)(c+(e+d+j)(g+h))\\ &= (a+bd+c)(c+eg+eh+dg+dh+jg+jh)\\ &= ac+aeg+aeh+adg+adh+agj+ajh+bcd+bdeg\\ &\quad +bdeh+bdg+bdh+bdgj+bdjh+c+ceg+ceh+cdg\\ &\quad +cdh+cgj+cjh\\ &= c+adg+adh+aeg+aeh+agj+ajh+bdg+bdh\end{aligned}$$

Once we know how to obtain a structural function from a representative network of the diagram of a play, we are ready to consider how to obtain the "minimum ties or paths" and also the "minimum lengths".

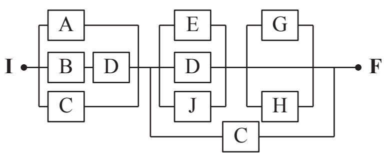


Fig. 34

9 Obtaining Minimum Schemes and Cuts from the Functional Structure

Let us begin with the definition of the "tie" or "path". A path or tie is formed by a subset of athletes such that if they adequately fulfill the desired offensive or defensive functions can make the game scheme work correctly. The adequate performance is evidently to overcome the obstacles that rival athletes introduce. When the diagram of the play is represented by a network, the scheme or path constitutes the path that the ball can take to circulate from point I to point F.

Based on the structural function φ, one can know with certainty if a subset of players forms a tie or path. As such, it is sufficient to give the value 1 to each Boolean variable that corresponds to the athletes considered. If the function takes the value of 1, what we have is a tie. Let us consider the last network as an example.

Recall that the structural function was as follows:

$$\varphi(a, b, c, d, e, j, g, h) = c + adg + adh + aeg + aeh + agj + ajh + bdg + bdh$$

It is proven that $\{B, D, J, H\}$ forms a scheme.

In effect, if we assign a value 1 to the Boolean variables b, d, j, and h, the value $\varphi(x)$ is also equal to 1, although the others are equal to 0:

$$\varphi(a, b, c, d, e, j, g, h) = 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 1 = 1.$$

The same happens with the set $\{C, D, G\}$ or the set $\{A, C\}$, for example.

In contrast, neither $\{A, B, D\}$ nor $\{E, D, G\}$ gives the value 1 for the function φ, as can be proven immediately.

We now arrive at the concept of the "minimum tie" or "minimum path". When a scheme or path does not contain players who form another scheme with reduced numbers, it is said that this scheme or path is minimal. In other words, if a play can be carried out from beginning to end with some athletes, others do not have to intervene.

All of the minimum schemes or paths can be determined without error or omission from the structural function. Therefore, it is sufficient to separate all of the combined Boolean sums. The components of each one form a minimum scheme or path.

The three previous structural functions can serve as an example and proof. Thus, the elements of the combination $\varphi(a, b, c, d, e) = c + de + abe$ are $\{C\}$, $\{D, E\}$, $\{A, B, E\}$, which coincide with those that were visually identified at the beginning of this paper.

The elements of the combination $\varphi(a, b, c, d) = a + b + cd$ are $\{A\}$, $\{B\}$, $\{C, D\}$; these results are also coincident with those that we arrived at visually.

Finally, the nine elements of the addends of the structural function

$$\varphi(a, b, c, d, e, j, g, h) = c + adg + adh + aeg + aeh + agj + ajh$$

correspond to the nine figures that were previously presented based on a visual inspection.

We now move on to the notion of the "cut". A cut is formed by a group of players who impede the development of the play from end to beginning (or theoretically from beginning to end). In reality, it is a line of contention that has the purpose of preventing realization of the play of the opposing team.

Using a structural function, one can determine if a subset of athletes does or does not form a cut. To do this, it is sufficient to substitute the Boolean variables in the function corresponding to the players in question with zeros. If the value of the function is null, we have a cut. We see this with one of the previous structural functions:

$$\varphi(a, b, c, d, e) = c + de + abe.$$

Here, the subset of players $\{A, C, E\}$ creates a cut, and we know this because in giving the Boolean variables a, c, and e the value of 0, we also create a null function:

$$\varphi(a, b, c, d, e) = 0 + d \cdot 0 + 0 \cdot b \cdot 0 = 0$$

The following is not, for example, a null function: $\{A, B, D, E\}$. This is because if the Boolean variable C is equal to one, then

$$\varphi(a, b, e, d, e) = 1 + 0 \cdot 0 + 0 \cdot 0 \cdot 0 = 1.$$

This connects to an important concept, that of "minimum length". By analogy, we will define "minimum length" as that subset of players that has impeded the movement of the ball if you cannot do without any of them. In other words, a minimum length should not include athletes who would not be necessary in a numerically reduced cut.

It is possible to find all of the minimum lengths in a network without error or omission. For this purpose, we suggest transforming the structural function of the network φ into its dual φ^* because the minimum "ties" or "paths" to φ^* are the minimum "cuts" of φ. This represents the players (elements) corresponding to the Boolean variables of the sums of φ^* of the minimum lengths of φ.

To find the dual function of φ^* of a structural function φ, it is sufficient to substitute $(\cdot)$ for $(+)$ and vice versa.

Let us look at some of the structural functions already presented. We can begin with the following:

$$\begin{aligned}
\varphi(a, b, c, d, e) &= c + de + abe \\
\varphi^*(a, b, c, d, e) &= c(d + e)(a + b + e) \\
&= c(ad + bd + de + ae + be + e) \\
&= acd + bcd + ce
\end{aligned}$$

Because the minimum schemes or paths of φ^* are

$$\{C, E\}, \{A, C, D\}, \{B, C, D\},$$

they will also be the minimum "cuts" of the structural function φ. It is observed that they coincide with the findings that we ascertained visually for the figures at the beginning of this paper.

Let us consider the following:

$$\begin{aligned} \varphi(a, b, c, d) &= a + b + cd \\ \varphi^*(a, b, e, d) &= ab \cdot (c + d) \\ &= abc + abd \end{aligned}$$

The minimum "ties" or "paths" φ^* that coincide with the minimum "cuts" of φ are $\{A, B, C\}, \{A, B, D\}$. Note that they coincide with those visually obtained.

We have described construction of the structural function of the representative network of the diagram of play and its dual function with the fundamental elements in the development of a theory of the game systems. Although these are not the only significant considerations, the concept of the minimum tie or path and that of the minimum length are important. We say this because the objective of the game system is perhaps not to arrive at the goal as quickly as possible and using the least possible number of players. The game system in soccer based on "touch", "touch" and more "touch" is enough to support this position. Nevertheless, this does not in any way invalidate the "general theory" that we intend to develop. In these assumptions, other concepts are substituted by minimum bonds/ties or paths and perhaps, although this is not fully established, that of the minimum length. Not to deviate too much from the methodological lines that we have traced here, we do not consider these alternatives here.

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