Chapter 16
Fringe Distortion Effects

From the LDA principle described in Chap. 3, the necessary condition for accurate LDA measurements is the uniformity of the fringe spacing in the measurement volume. The uniform fringe spacing can be achieved if the measurement volume that is formed by two laser beams coincides with two beam waists. Each deviation from this requirement will lead to fringe distortion in the measurement volume and hence to measurement errors. Because of the non-uniformity of the fringe spacing along the measurement volume a uniform constant laminar flow for instance will then be measured as a flow with velocity fluctuations. Measurements of both the mean velocity and the turbulence quantities thus suffer from systematic errors.

Fringe distortions in LDA measurement volumes have been historically considered as the consequence of improper optical layout. Two most well-known forms of the optical layout causing the fringe distortion have been shown in Fig. 16.1. The visible non-uniformity of the fringe spacing in these two cases is either along or across the measurement volume. Corresponding detailed investigations to characterize the non-uniformity of two such different fringe patterns have been performed for instance by Hanson (1973, 1975), Durst and Stevenson (1975) and Miles and Witze (1994, 1996). According to Hanson (1973, 1975) linear distributions of the fringe spacing exist in both longitudinally (Fig. 16.1a) and laterally (Fig. 16.1b) distorted measurement volumes. The influence of the fringe distortion on the measurement accuracy has been investigated by Zhang and Eisele (1997, 1998c) with respect to the longitudinal fringe distortion. As it has already been shown in Fig. 15.14, the first type of the fringe distortion in the measurement volume is confirmed as exactly taking place, when the tangential velocity of the flow in a circular pipe is measured without matching the refractive index of the fluid. It is obviously the most representative fringe distortion encountered in the practical applications. The second type of fringe distortion across the measurement volume (Fig. 16.1b), however, is still considered as merely a matter of the improper optical layout.

Another type or the third type of possible fringe distortions in the LDA measurement volume, as shown in Fig. 14.14b for a special case, is related to the astigmatism due to the laser beam refractions. Because of the irregular distribution of the beam waists around the LDA measurement volume and hence the complexity of the form of respective wave front of two laser beams, this type of fringe distortion may not yet

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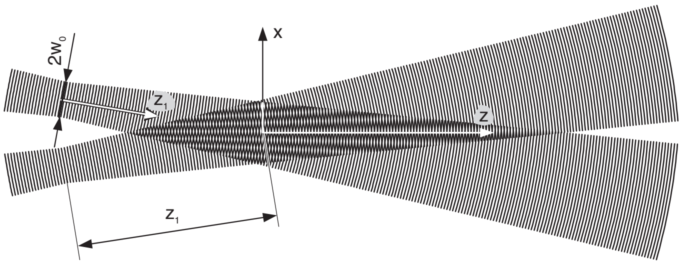
(a)



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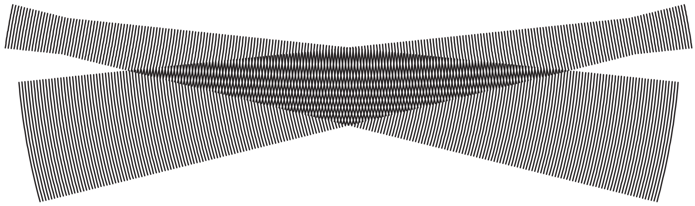

Fig. 16.1 Fringe distortions (first and second types) in the measurement volume; The third type of the possible fringe distortion is referred to Fig. 14.14b

be well characterized. A further type of fringe distortions in the measurement volume is known as the local fringe distortion which is caused by laser light diffraction through particles in the transmission path of the laser beams (Ruck 1991).

The outcome of the fringe distortion in the LDA measurement volume is the systematic measurement error in both the mean velocity and the turbulence quantities. In all cases of measuring the flow turbulence, the fringe distortion results in the broadening of the Doppler frequency and hence the overestimation of the turbulence intensity. This overestimation obviously depends on both the form and the scale of the complex fringe distortion. As an LDA user one is indeed interested in knowing the extent of respective measurement errors and the possibility of correcting them. For this purpose, the influence of the most representative fringe distortion in form of Fig. 16.1a on the flow measurement is considered here, in order to give a reference as well as to make a criterion for error estimations. The analysis assumes the linear distribution of the fringe spacing along the measurement volume length.

16.1 Linear Longitudinal Distribution of the Fringe Spacing

According to Fig. 16.1a the crossing of two Gaussian beams takes place after their respective waists located at equal distance from the beam intersection point. The same fringe distortion with the same consequence in LDA flow measurements will be given when the beam crossing is found prior to both beam waists. The assumption of linear distribution of the fringe spacing in the LDA measurement volume is based on earlier investigations of this type of fringe distortion and contributes to the simplification of calculations.

The uniform velocity distribution within the measurement volume length is assumed. From its measurement by $u = \Delta x \cdot \nu_D$ with non-uniform fringe spacing, one obtains the related non-uniform Doppler frequency as

$$\frac{1}{\nu_D}\frac{d(\nu_D)}{dz} = -\frac{1}{\Delta x}\frac{d(\Delta x)}{dz} \tag{16.1}$$

According to Hanson (1973, 1975) from accounting for the relative shift in Doppler frequency, the fringe spacing gradient is expressed as

$$\frac{1}{\Delta x}\frac{d(\Delta x)}{dz} = \frac{1}{R} \tag{16.2}$$

Herein R is the radius of curvature of two Gaussian beam wave fronts at the beam crossing point. It is calculated, according to Eq. (3.63), with the spot size (radius w_0) at the beam waist and the distance (z_1) of the beam waist from the beam crossing point as follows

$$R = z_1\left[1 + \left(\frac{\pi w_0^2}{\lambda z_1}\right)^2\right] = z_1\left[1 + \left(\frac{z_R}{z_1}\right)^2\right] \tag{16.3}$$

In this equation, z_R represents the Rayleigh length, as given in Eq. (3.66).

From Eq. (16.2) it can be shown that, by assuming $|z/R| << 1$ within the region of the measurement volume, the longitudinal fringe spacing varies linearly over the length of the measurement volume. That is with $k = \Delta x_0/R$

$$\Delta x = kz + \Delta x_0 \tag{16.4}$$

In this equation, Δx_0 is the fringe spacing at the centre of the measurement volume ($z = 0$). According to Hanson (1973), this value of fringe spacing is equal to that in the undistorted measurement volume ($\lambda/2\sin\alpha$).

16.2 Fringe Distortion Number and the Apparent Mean Velocity

It should be mentioned that the fringe distortion in the measurement volume also influences the measurement of mean velocities. Indeed, the error in the mean velocity will not disappear, even if linear fringe distortion according to Eq. (16.4) takes place. This can be easily demonstrated by assuming the uniform flow of velocity u_0 through the measurement volume. From the measured Doppler frequency and the specified constant fringe spacing in the software, the flow velocity is calculated as follows:

$$u = \Delta x_0 \nu_\mathrm{D} = \Delta x_0 \frac{u_0}{\Delta x} \tag{16.5}$$

It is inversely related to the fringe spacing which linearly changes along the measurement volume. For this reason the ensemble average of velocities u from measurements is in no cases equal to the actual flow velocity u_0 and is therefore denoted as the apparent mean. For the general case a turbulent flow with random velocity fluctuations is considered to have a mean velocity equal to $\overline{u}$. The apparent mean velocity is calculated by the arithmetic average as

$$\overline{u}_\mathrm{app} = \Delta x_0 \frac{1}{N} \sum_{\mathrm{i}=1}^{\mathrm{N}} \frac{u_\mathrm{i}}{\Delta x_\mathrm{i}} \tag{16.6}$$

On the side of the measurement volume, the measurement volume will longitudinally be divided into m partial volumes of equal distance. In each partial volume, the fringe spacing can be considered to be constant. On the side of the flow, the same and constant statistical flow properties are assumed to exist among all partial volumes. This also includes the assumption that particles have equal probability in passing through every partial volume. With respect to $N = m \cdot n$ and the mean velocity equal to $\overline{u}$ Eq. (16.6) is then written as

$$\overline{u}_\mathrm{app} = \Delta x_0 \frac{1}{N} \sum_{\mathrm{j}=1}^{\mathrm{m}} \left(\frac{1}{\Delta x_\mathrm{j}} \sum_{\mathrm{i}=1}^{\mathrm{n}} u_\mathrm{i} \right) = \Delta x_0 \overline{u} \frac{1}{m} \sum_{\mathrm{j}=1}^{\mathrm{m}} \frac{1}{\Delta x_\mathrm{j}} \tag{16.7}$$

By extending m to infinity and with substitution of Δx_j by Eq. (16.4) the summation in the above equation can be presented by the corresponding integral calculation, so that with $\Delta z/2$ as the half length of the measurement volume

$$\overline{u}_\mathrm{app} = \overline{u} \frac{\Delta x_0}{\Delta z} \int_{-\Delta z/2}^{\Delta z/2} \frac{\mathrm{d}z}{kz + \Delta x_0} \tag{16.8}$$

To simplify the calculation results, the fringe distortion number is introduced as defined by

$$\gamma = \frac{k\,\Delta z/2}{\Delta x_0} = \frac{\Delta z/2}{R} \tag{16.9}$$

It represents the relative change of the fringe spacing at the end of the measurement volume. Usually it is a small value because of $\Delta z << R$.

Equation (16.8) is then calculated as

$$\frac{\overline{u}_\mathrm{app}}{\overline{u}} = \frac{1}{2\gamma} \ln \frac{1+\gamma}{1-\gamma} \approx 1 + \frac{1}{3}\gamma^2 \tag{16.10}$$

Herein the approximation has been made because of $\gamma << 1$.

This calculation result demonstrates that the measurement of the mean flow velocity will also be affected by the fringe distortion in the measurement volume, even though the linear fringe spacing distribution is assumed. The relative error, however, is mostly negligible because of $\gamma << 1$.

The fringe distortion number has been defined in Eq. (16.9) as a pure function of geometric parameters of the measurement volume. Since it specifies the relative change in the fringe spacing at the end of the measurement volume ($z = \Delta z/2$) against that at the measurement volume centre ($z = 0$), it also represents the frequency broadening $\Delta\nu_D/\nu_D$ at $z = \Delta z/2$.

According to Eq. (16.9) the fringe distortion number has also been shown to be a function of the radius of wave front curvature of two Gaussian beams at the beam crossing point. Substituting this curvature radius by Eq. (16.3) then yields

$$\gamma = \frac{\Delta z/2}{z_1\left[1 + (z_R/z_1)^2\right]} \tag{16.11}$$

For laser beams with given beam waist thickness $2w_0$ and hence given Raleigh length z_R the fringe distortion number has been shown as the function of the distance z_1 between the beam waist and the measurement volume centre. This functionality is illustrated in Fig. 16.2 for a given LDA optical set-up. The radius of the curvature of the Gaussian beam wave front has also been shown in function of the distance. At the Rayleigh length which is equal to $z_R = 30$ mm in this example, the fringe distortion number reaches its maximum.

Based on parameter quantifications in this example the fringe distortion number has been confirmed to be in the range of usually not exceeding 0.02. In applying this limit to Eq. (16.10), the error in the mean velocity is practically very small. By the way, the minimum radius of the curvature of the Gaussian beam wave front reads $R = 60$ mm. Compared with this value, the half length of the measurement volume ($\Delta z/2 = 1$ mm) is negligible. The assumption of $|z/R| << 1$ that leads to Eq. (16.4) is thus validated.

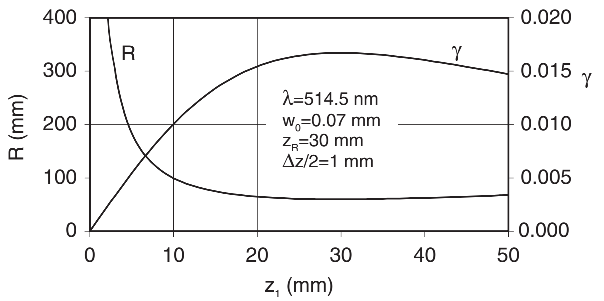

Fig. 16.2 Radius of the curvature of the Gaussian beam wave front and the fringe distortion number

16.3 Overestimation of the Flow Turbulence

As is well-known, the most significant outcome of the fringe distortion in LDA measurements is the Overestimation of the turbulence intensity. This is also called the broadening effect in turbulence measurements. In order to quantify this effect, a stationary turbulent flow is considered which is specified by a mean velocity $\bar{u}$ and a fluctuation velocity σ (standard deviation). From the viewpoint of statistics, the connection between the mean velocity and the standard deviation is given, according to Eq. (5.5), as

$$\sigma^2 = \overline{u^2} - \bar{u}^2 \tag{16.12}$$

with $\overline{u^2}$ as the mean square of velocity component u.

This relationship also applies in the case of velocity data from measurements undergoing the effect of fringe distortion in the measurement volume. Corresponding velocities are thus apparent, as given by

$$\sigma_{\text{app}}^2 = \overline{u_{\text{app}}^2} - \bar{u}_{\text{app}}^2 \tag{16.13}$$

The overestimation of the turbulence intensity and the related quantities as a result of the fringe distortion in the measurement volume is then determined by

$$\sigma_{\text{app}}^2 - \sigma^2 = \left(\overline{u_{\text{app}}^2} - \overline{u^2}\right) - \left(\bar{u}_{\text{app}}^2 - \bar{u}^2\right) \tag{16.14}$$

While the second term on the r.h.s. of this equation can be calculated by using Eq. (16.10) or simply set to zero, the first term requires similar calculations as those in Sect. 16.2. In response to each velocity event the Doppler frequency is again $u_{\text{i}}/\Delta x_{\text{i}}$. The apparent mean square of velocities in the above equation is basically calculated, in analogy to Eq. (16.6), by

$$\overline{u_{\text{app}}^2} = (\Delta x_0)^2 \frac{1}{N} \sum_{\text{i}=1}^{\text{N}} \left(\frac{u_{\text{i}}}{\Delta x_{\text{i}}}\right)^2 \tag{16.15}$$

By dividing the measurement volume into m partial volumes of equal distance and based on same assumptions that led to Eq. (16.7), the above equation is converted into

$$\overline{u_{\text{app}}^2} = (\Delta x_0)^2 \, \overline{u^2} \frac{1}{m} \sum_{\text{j}=1}^{\text{m}} \frac{1}{\left(\Delta x_{\text{j}}\right)^2} \tag{16.16}$$

The linear fringe spacing distribution according Eq. (16.4) is applied. By extending m to infinity, the summation in the above equation can be presented by the corresponding integral calculation, so that

$$\overline{u_{\mathrm{app}}^2} = (\Delta x_0)^2\, \overline{u^2} \frac{1}{\Delta z} \int\limits_{-\Delta z/2}^{\Delta z/2} \frac{dz}{(kz + \Delta x_0)^2} \tag{16.17}$$

In using the fringe distortion number as defined by Eq. (16.9), one obtains

$$\overline{u_{\mathrm{app}}^2} = \frac{1}{1-\gamma^2}\overline{u^2} \approx \left(1+\gamma^2\right)\overline{u^2} \tag{16.18}$$

Substitution of Eqs. (16.10) and (16.18) in Eq. (16.14) and with respect to $\overline{u^2} = \sigma^2 + \bar{u}^2$ yields

$$\frac{\sigma_{\mathrm{app}}^2 - \sigma^2}{\bar{u}^2} = \gamma^2 \left(\frac{\sigma^2}{\bar{u}^2} + \frac{1}{3}\right) \tag{16.19}$$

Because of $\sigma_{\mathrm{app}} + \sigma \approx 2\sigma$ (not for $\sigma = 0$) this equation can also be written as

$$\frac{\Delta\sigma}{\bar{u}} = \frac{1}{2}\gamma^2 \frac{\bar{u}}{\sigma} \left(\frac{\sigma^2}{\bar{u}^2} + \frac{1}{3}\right) \tag{16.20}$$

with $\Delta\sigma = \sigma_{\mathrm{app}} - \sigma$ as the overestimation of the standard deviation of the mean flow velocity.

The overestimation of the turbulence intensity obviously depends on both the real flow turbulence to be measured and the extent of the fringe distortion that is specified by the fringe distortion number. In regards Eq. (16.20), the overestimation of the flow turbulence is illustrated in Fig. 16.3 in the function of the real turbulence intensity for different fringe distortion numbers. For typical fringe distortions ($\gamma < 0.02$) the overestimation of the flow turbulence has been found to be not significant, especially in the measurement of flows with high turbulence intensity.

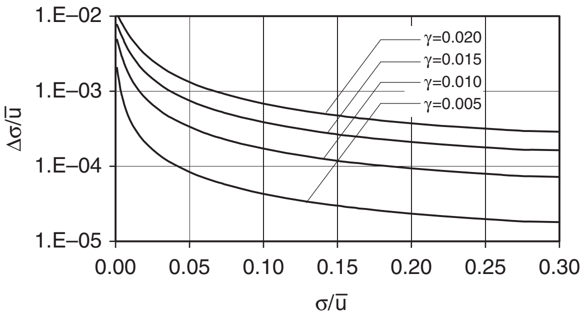

Fig. 16.3 Overestimations of the standard deviation in function of the real flow turbulence and the fringe distortion number γ

For a uniform laminar flow with $\sigma = 0$ the apparent turbulence intensity is calculated from Eq. (16.19) as

$$\frac{\sigma_{\mathrm{app}}}{\overline{u}} = \sqrt{1/3} \cdot \gamma \tag{16.21}$$

Calculations made in this chapter basically refer to the fringe distortion with linear longitudinal variation in the fringe spacing (Fig. 16.1a). The calculation results can be used as the reference, if other types of fringe distortions and the respective influences on measurements should be accounted for. In general, the influence of the second type of fringe distortions (Fig. 16.1b) on turbulence measurements is rather smaller than that treated above, as this influence was already considered by Hanson (1975) in the earlier time to be negligible. The same might also be true for the case accounting for the third type of fringe distortions that is caused by the optical aberration i.e. astigmatism according to Fig. 14.14.

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