# Preface

At the end of the nineteenth century Lyapunov and Poincaré developed the so called qualitative theory of differential equations and introduced geometric-topological considerations which have led to the concept of dynamical systems. In its present abstract form this concept goes back to G.D. Birkhoff.

This is also the starting point of Chapter 1 of this book in which uncontrolled and controlled time-continuous and time-discrete systems are investigated under the aspect of stability and controllability. Chapter 1 starts with time-continuous dynamical systems. After the description of elementary properties of such systems it focusses on stability in the sense of Lyapunov and gives applications to systems in the plane such as the mathematical pendulum, to general predator-prey models, and to evolution matrix games.

The time-discrete case is divided into the autonomous and the non-autonomous part where the latter is no more a dynamical system in the strong sense. It is the counter part of the time-continuous case where the right-hand side of the system of differential equations which describes the dynamics of the system depends explicitly on the time.

Controlled dynamical systems could be considered as dynamical systems in the strong sense, if the controls were incorporated into the state space. We, however, adopt the conventional treatment of controlled systems as in control theory. We are mainly interested in the question of controllability of dynamical systems into equilibrium states. In the non-autonomous time-discrete case we also consider the problem of stabilization.

Chapter 3 is concerned with chaotic behaviour of autonomous time discrete systems. We consider three different types of chaos: chaos in the sense of Devaney, disorder chaos and chaos in the sense of Li and Yorke. The chapter ends with two examples of strange (or chaotic) attractors.

The Appendix A is concerned with a dynamical method for the calculation of Nash equilibria in non-cooperative $n$-person games. The method is based on the fact that Nash equilibria are fixed points of certain continuous mappings of the Cartesian product of the strategy sets of the players into itself. This gives rise to an iteration method for the calculation of Nash equilibria the set of which can be considered as the $\Omega$-limit set of a time-discrete dynamical system.

In Appendix B we consider two optimal control problems in chemotherapeutic treatment of cancer. These two problems are somehow dual to each other and are shown to have solutions of the same type.

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