

# Preface

Over the past 10-15 years, we have seen a revival of general Lévy processes theory as well as a burst of new applications. In the past, Brownian motion or the Poisson process have been considered as appropriate models for most applications. Nowadays, the need for more realistic modelling of irregular behaviour of phenomena in nature and society like jumps, bursts, and extremes has led to a renaissance of the theory of general Lévy processes. Theoretical and applied researchers in fields as diverse as quantum theory, statistical physics, meteorology, seismology, statistics, insurance, finance, and telecommunication have realised the enormous flexibility of Lévy models in modelling jumps, tails, dependence and sample path behaviour. Lévy processes or Lévy driven processes feature slow or rapid structural breaks, extremal behaviour, clustering, and clumping of points.

Tools and techniques from related but distinct mathematical fields, such as point processes, stochastic integration, probability theory in abstract spaces, and differential geometry, have contributed to a better understanding of Lévy jump processes.

As in many other fields, the enormous power of modern computers has also changed the view of Lévy processes. Simulation methods for paths of Lévy processes and realisations of their functionals have been developed. Monte Carlo simulation makes it possible to determine the distribution of functionals of sample paths of Lévy processes to a high level of accuracy.

This development of Lévy processes was accompanied and triggered by a series of Conferences on Lévy Processes: Theory and Applications. The First and Second Conferences were held in Aarhus (1999, 2002), the Third in Paris (2003), the Fourth in Manchester (2005), and the Fifth in Copenhagen (2007).

To show the broad spectrum of these conferences, the following topics are taken from the announcement of the Copenhagen conference:

- Structural results for Lévy processes: distribution and path properties
- Lévy trees, superprocesses and branching theory
- Fractal processes and fractal phenomena
- Stable and infinitely divisible processes and distributions
- Applications in finance, physics, biosciences and telecommunications
- Lévy processes on abstract structures
- Statistical, numerical and simulation aspects of Lévy processes
- Lévy and stable random fields.

At the Conference on Lévy Processes: Theory and Applications in Copenhagen the idea was born to start a series of Lecture Notes on Lévy processes to bear witness of the exciting recent advances in the area of Lévy processes and their applications. Its goal is the dissemination of important developments in theory and applications. Each volume will describe state of the art results of this rapidly evolving subject with special emphasis on the non-Brownian world. Leading experts will present new exciting fields, or surveys of recent developments, or focus on some of the most promising applications. Despite its special character, each article is written in an expository style, normally with an extensive bibliography at the end. In this way each article makes an invaluable comprehensive reference text. The intended audience are PhD and postdoctoral students, or researchers, who want to learn about recent advances in the theory of Lévy processes and to get an overview of new applications in different fields.

Now, with the field in full flourish and with future interest definitely increasing it seemed reasonable to start a series of Lecture Notes in this area. The present volume is the first in the series, and future volumes will appear over time under the common name “Lévy Matters”, in tune with the developments in the field. “Lévy Matters” will appear as a subseries of the Springer Lecture Notes in Mathematics, thus ensuring wide dissemination of the scientific material. The expository articles in this first volume have been chosen to reflect the broadness of the area of Lévy processes.

The first article by Ken-iti Sato characterises extensions of the class of selfdecomposable distributions on  $\mathbb{R}^d$ . They are given as two families each with two continuous parameters of classes of distributions of improper stochastic integrals  $\lim_{t \rightarrow \infty} \int_0^t f(s) dX_s$  for appropriate non-random functions  $f$  and Lévy processes  $X$ . Many known classes appear as limiting cases in some parameters: the Thorin class, the Goldie-Steutel-Bondesson class, and the class of completely selfdecomposable distributions. Moreover, the theory of fractional integrals of measures is built.

The second article by Thomas Duquesne discusses Hausdorff and packing measures of stable trees. Stable trees are a special class of Lévy trees, which form a class of random compact metric spaces, and were introduced by Le Gall and Le Jan (1998) as the genealogy of continuous state branching processes. It is shown that level sets of stable trees are the sets of points situated at a given distance from the root. In contrast to Brownian trees, for non-Brownian stable trees there is no exact packing measure for level sets, i.e. the sets of points situated at a given distance from the root.

The third (and last) article by Oleg Reichmann and Christoph Schwab presents numerical solutions to Kolmogorov equations, which arise for instance in financial engineering, when Lévy or additive processes model the dynamics of the risky assets. Solution algorithms based on wavelet representations for the Dirichlet and free boundary problems connected to barrier and American style contracts are presented. Lévy copulas are used for a systematic construction of parametric multivariate Feller-Lévy processes. Numerical aspects of the implementation and Monte Carlo path simulation techniques are addressed.

We take the possibility to acknowledge the very positive collaboration with the relevant Springer staff and the Editors of the LN Series, and the (anonymous) referees of the three articles.

We hope that the readers of this and subsequent volumes enjoy learning about the high potential of Lévy processes in theory and applications. Researchers with ideas for contributions to further volumes in the Lévy Matters series are invited to contact any of the Editors with proposals or suggestions.

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Lévy Matters I

Recent Progress in Theory and Applications:

Foundations, Trees and Numerical Issues in Finance

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