Chapter 2
Performance Analysis of Call-Handling Processes in Buffered Cellular Wireless Networks

In this chapter effective numerical computational procedures to calculate QoS (Quality of Service) metrics of call-handling processes in mono-service Cellular Wireless Networks (CWN) with queues of either original (o-calls) or handover (h-calls) calls are proposed. Generalization of the results found here for integrated voice/data CWN is straightforward. Unlike classical models of mono-service CWN, here original and handover calls are assumed not to be identical in terms of time of radio channel occupancy. First we consider models of CWN with queues of h-calls in which for their prioritization a guard channels scheme is also used. We will then consider models of CWN with queues of o-calls and guard channels for h-calls. For both kinds of model the cases of limited and unlimited queues of patient and impatient calls are investigated. For the models with unlimited queues of heterogeneous calls the easily checkable ergodicity conditions are proposed. The high accuracy of the developed approximate formulae to calculate QoS metrics is shown.

2.1 Models with Queues for h-Calls

As mentioned in Chap. 1, the main method for prioritization of h-calls is the use of reserve channels (shared or isolated reservation). Another scheme for this purpose is the efficient arrangement of their queue in the base station. However, joint use of these schemes improves the QoS metrics of h-calls.

The required queue arrangement for h-calls can be realized in networks where microcells are covered by a certain macrocell, i.e. there exists a certain zone (handover zone – h-zone), within which mobile users can be handled in any of the neighboring cells. The time for the user to cross the h-zone is called the degradation interval. As the user enters the h-zone a check is made of the availability of free channels in a new cell. If a free channel exists, then the channel is immediately occupied and the h-procedure is considered to be successfully completed at the given stage; otherwise the given h-call continues to use the channel of the old (previous) cell while concurrently queuing for availability of a certain channel of a new cell. If the free channel does not appear in the new cell before completion of the degradation interval, then a forced call interruption of the h-call occurs.

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Herein we consider models of four types: (i) limited queuing of h-call and infinite degradation interval; (ii) limited queuing of h-call and finite degradation interval; (iii) unlimited queuing of h-call and infinite degradation interval; (iv) unlimited queuing of h-call and finite degradation interval.

In all mentioned types of model it is assumed that a cell contains $N > 1$ radio channels and o-calls (h-calls) enter the given cell by the Poisson law with intensity λ_o (λ_h), the time of channel occupancy by o-calls (h-calls) being an exponentially distributed random quantity with a mean of μ_o^{-1} (μ_h^{-1}). Note, that the time of channel occupancy considers both components of occupancy time: the call service time and mobility. If during the service time of any type of call the h-procedure occurs, then due to the lack of memory of exponential distribution the remaining time of the given call service in a new cell (now the h-call) also has an exponential distribution with the same mean.

The different types of call are handled by the scheme of guard channels (shared reservation), i.e. an entering o-call is received only when there exists not less than $g + 1$ free channels; otherwise the o-call is lost (blocked). A handover call is received with at least one free channel available; should all N channels be busy, the h-call joins the queue (limited or unlimited). In all model types it is assumed that at the moment a channel becomes free in a new cell the queue of h-calls (if it exists) is served by the FIFO (First-In-First-Out) process; with no queue the free channel stands idle.

2.1.1 Models with Finite Queues

First we consider the models of type (i), i.e. models with a limited queue of patient h-calls. In this model if all N channels are busy, then the entering h-call joins the queue with maximal size $B > 1$, if at least one vacant place is available; otherwise (i.e. when all places in the buffer are occupied) the h-call is lost. Since the degradation interval is infinite, the handover call cannot be lost should it be placed in a queue. In other words, the waiting h-calls are assumed to be patient. Note that this model is adequate for networks with slow velocity mobile users.

For a more detailed description of a cell's operation use is made of 2-D MC (Markov chain), i.e. the cell state at the arbitrary time instant is given by the vector $\mathbf{k} = (k_1, k_2)$, where k_i is the number of o-calls (h-calls) in the system, $i = 1,2$. Since o-calls are handled in a blocking mode and the system is conservative (i.e. with the queue available channel outages are not admitted), in the state $\mathbf{k}$ the number of h-calls in channels (k_2^{s}) and in the queue (k_2^{q}) are determined as follows:

$$k_2^{\mathrm{s}} = \min\{N - k_1, k_2\}, k_2^{q} = (k_1 + k_2 - N)^{+}, \tag{2.1}$$

where $x^{+} = \max(0, x)$. Therefore, the set of all possible states of the system is determined in the following way:

$$S := \left\{\mathbf{k}: k_1 = 0, 1, \ldots, N-g;\ k_2 = 0, 1, \ldots, N+B,\ k_1 + k_2^{\mathrm{s}} \leq N,\ k_2^{\mathrm{q}} \leq B\right\}. \tag{2.2}$$

Note 2.1. In the known works in view of the calls being identical in terms of channel occupation time, the state of a cell is described by a scalar magnitude which points out a general number of busy channels in a base station, i.e. as the mathematical model one applies 1-D MC. Since in the models studied the channel occupation time by different types of call is different, the description of a cell by a scalar magnitude is impossible in principle.

The elements of the generating matrix corresponding to 2-D MC, $q(\mathbf{k}, \mathbf{k}')$, $\mathbf{k}, \mathbf{k}' \in S$, are determined as follows (see Fig. 2.1):

$$q(\mathbf{k}, \mathbf{k}') = \begin{cases} \lambda_{\mathrm{o}} & \text{if } k_1 + k_2 \leq N - g - 1, \mathbf{k}' = \mathbf{k} + \mathbf{e}_1, \\ \lambda_{\mathrm{h}} & \text{if } \mathbf{k}' = \mathbf{k} + \mathbf{e_2}, \\ k_1 \mu_{\mathrm{o}} & \text{if } \mathbf{k}' = \mathbf{k} - \mathbf{e_1}, \\ k_2^s \mu_{\mathrm{h}} & \text{if } \mathbf{k}' = \mathbf{k} - \mathbf{e_2}, \\ 0 & \text{in other cases.} \end{cases} \tag{2.3}$$

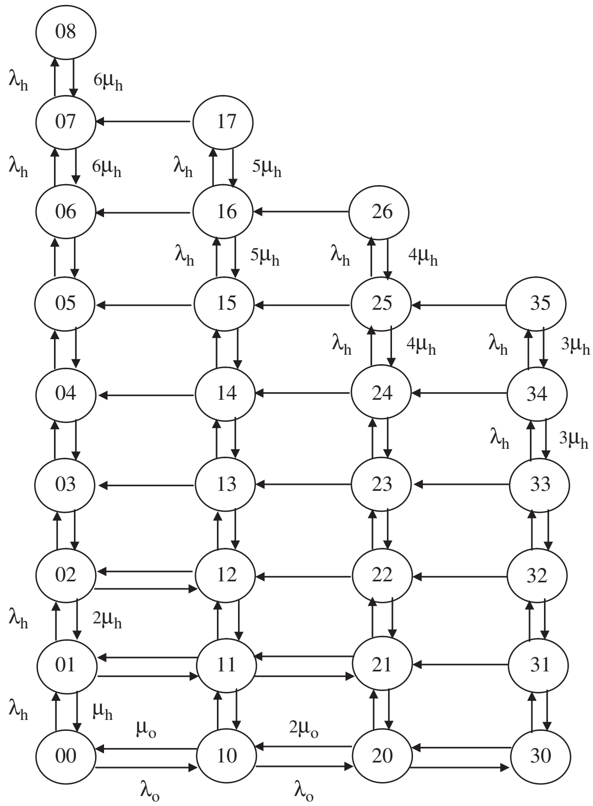

Fig. 2.1 State transition diagram for the model with a limited queue of patient h-calls, $N = 6$, $g = 3$, $B = 2$

Hence, the mathematical model of the given system is presented by 2-D MC with a state space (2.2) for which the elements of the generating matrix are determined by means of relations (2.3).

The stationary probability of state $\mathbf{k}$ is denoted by $p(\mathbf{k})$. Then in view of the model being Markovian, according to the PASTA theorem we find that the dropping probability of h-calls (P_{h}) and probability of blocking of o-calls (P_{o}) are determined in the following way:

$$P_{\mathrm{h}} := \sum_{k \in S} p(\mathbf{k}) \delta\left(k_2^{\mathrm{q}}, B\right), \tag{2.4}$$

$$P_{\mathrm{o}} := \sum_{k \in S} p(\mathbf{k}) I(k_1 + k_2^{\mathrm{s}} \geq N - g). \tag{2.5}$$

The average number of busy channels of the cell ($\tilde{N}$) and the average length of the queue of h-calls (L_{h}) are also determined via stationary distribution of the model:

$$\tilde{N} := \sum_{j=1}^{N} j \varsigma\left(j\right), \tag{2.6}$$

$$L_{\mathrm{h}} := \sum_{l=1}^{B} l \tau\left(l\right), \tag{2.7}$$

where

$$\varsigma\left(j\right) := \sum_{k \in S} p\left(\mathbf{k}\right) \delta\left(k_1 + k_2^{\mathrm{s}}, j\right) \text{ and } \tau\left(l\right) := \sum_{k \in S} p\left(\mathbf{k}\right) \delta\left(k_2^{\mathrm{q}}, l\right)$$

are the marginal distribution of a model.

Hence, to find QoS metrics (2.4), (2.5), (2.6), and (2.7) it is necessary to determine the stationary distribution of the model $p(\mathbf{k}), \mathbf{k} \in S$, from the corresponding system of global balance equations (SGBE). This is of the following form:

$$\begin{aligned}
& p\left(\mathbf{k}\right)\left(\lambda_{\mathrm{o}} I\left(k_1 + k_2^{\mathrm{s}} \leq g - 1\right) + \lambda_{\mathrm{h}}\left(1 - \delta\left(k_2^{\mathrm{q}}, B\right) + \left(k_1 + k_2^{\mathrm{s}}\right)\mu\right)\right) \\
& = \lambda_{\mathrm{o}} p\left(\mathbf{k} - \mathbf{e}_1\right)\left(1 - \delta\left(k_1, 0\right)\right) I\left(k_1 + k_2^{\mathrm{s}} \leq g - 2\right) + \lambda_{\mathrm{h}} p\left(\mathbf{k} - \mathbf{e_2}\right)\left(1 - \delta\left(k_2, 0\right)\right) \\
& \quad + \left(k_1 + 1\right)\mu p\left(\mathbf{k} + \mathbf{e_1}\right) I\left(k_1 < N - g - 1\right) + (k_2^{\mathrm{s}} + 1)\mu p\left(\mathbf{k} + \mathbf{e_2}\right) I\left(k_2^{\mathrm{s}} < N\right), \\
& \quad \mathbf{k} \in S,
\end{aligned} \tag{2.8}$$

$$\sum_{k \in S} p\left(\mathbf{k}\right) = 1 \tag{2.9}$$

However, to solve the last problem one requires laborious computation efforts for large values of N and B since the corresponding SGBE (2.8) and (2.9) have no explicit solution. As was mentioned in Sect. 1.2, very often the solution of such problems is evident if the corresponding MC has a reversibility property and hence there exists a stationary distribution for it of the multiplicative type. However, by applying Kolmogorov criterion for 2-D MC one can easily demonstrate that the given MC is not reversible. Indeed, the necessary reversibility condition states that if the transition from state (i, j) into the state (i', j') exists, then the reverse transition also exists. However, for the MC considered this condition is not fulfilled. So by the relations (2.3) in the given MC the transition $(k_1, k_2) \to (k_1 - 1, k_2)$ with intensity $k_1\mu_o$ exists, where $k_1 + k_2 > N{-}g$, while the inverse transition does not exist.

To overcome these difficulties one suggests employing the approximate method of calculating the stationary distribution of the 2-D MC. It is acceptable for models of micro- and picocells for which the intensity of h-calls entering greatly exceeds that of o-calls and the talk time generated by an h-call is short. In other words, here it is assumed that $\lambda_h >> \lambda_o$, $\mu_h >> \mu_o$ (for respective comments see Sect. 1.2).

Consider the following splitting of the state space (2.2):

$$S = \bigcup_{j=0}^{N-g} S_j, \quad S_j \bigcap S_{j'} = \varnothing, \; j \neq j', \tag{2.10}$$

where

$$S_j := \{\mathbf{k} \in S : k_1 = j\}, j = \overline{0, N-g}.$$

The sets S_j are combined in merged state $<j>$ and the merging function with the domain (2.2) is introduced:

$$U(\mathbf{k}) = <j> \;\text{ if } \mathbf{k} \in S_j, \; j = \overline{0, N-g}. \tag{2.11}$$

The merging function (2.11) specifies the merged model, which is 1-D MC with state space $\tilde{S} := \left\{<j>: j = \overline{0, N-g}\right\}$.

The above assumption about relations of loading parameters of different call types provides the fulfillment of conditions necessary for correct application of phase-merging algorithms: intensities of transitions between states inside each class $S_j, j = 0, 1, \ldots, N{-}g$, essentially exceed intensities of transitions between states from different classes.

To find the stationary distribution of the initial model one needs a preliminary determination of the stationary distribution of split and merged models. The stationary distribution of the jth split model with space of states S_j is denoted by $\rho^j(i), j = 0, 1, \ldots, N-g, \; i = 0, 1, \ldots, N+B-j$, i.e. $\rho^j(i)$ is the stationary probability of state $(j, i) \in S_j$ in jth split model. It is determined as the stationary distribution of classical queuing system $M/M/N{-}j/B$ with the load ν_h Erl, i.e.

$$\rho^j(i) = \begin{cases} \dfrac{\nu_h^i}{i!}\rho^j(0) \text{ if } i = \overline{1, N-j}, \\ \dfrac{\nu_h^i}{(N-j)!(N-j)^{i+j-N}}\rho^j(0) \text{ if } i = \overline{N-j+1, N-j+B}, \end{cases} \tag{2.12}$$

where

$$\rho^j(0) = \left(\sum_{i=0}^{N-j} \frac{v_{\mathrm{h}}^i}{i!} + \frac{1}{(N-j)!} \sum_{i=N-j+1}^{N-j+B} \frac{v_{\mathrm{h}}^i}{(N-j)^{i+j-N}} \right)^{-1}.$$

To find the stationary distribution of the merged model one should preliminarily determine the elements of the generating matrix corresponding to 1-D MC denoted by $q(< j', < j'' >), < j' >, < j'' > \in \tilde{S}$. The following relations determine the mentioned parameters:

$$q(< j' >, < j'' >) = \begin{cases} \lambda_{\mathrm{o}} \cdot \Lambda(j'+1) & \text{if } j'' = j'+1, \\ j'\mu & \text{if } j'' = j'-1, \\ 0 & \text{in other cases,} \end{cases} \tag{2.13}$$

where

$$\Lambda(i+1) = \rho^i(0) \sum_{j=0}^{N-g-i-1} \frac{v_{\mathrm{h}}^j}{j!},\ i = \overline{0, N-g-1}.$$

The last relations imply the stationary distribution of the merged model $\pi(< j >), < j > \in \tilde{S}$, to be determined as the corresponding distribution of the appropriate birth-and-death process. In other words

$$\pi(< j >) = \frac{v_{\mathrm{o}}^j}{j!} \prod_{i=1}^{j} \Lambda(i)\pi(< 0 >),\ j = \overline{1, N-g}, \tag{2.14}$$

where

$$\pi(< 0 >) = \left(1 + \sum_{i=1}^{N-g} \frac{v_{\mathrm{o}}^i}{i!} \prod_{j=1}^{i} \Lambda(j) \right)^{-1}.$$

Summing up everything stated above one can offer the following approximate formulae for calculating the QoS metrics (2.4), (2.5), (2.6), and (2.7) of the given model:

$$P_{\mathrm{h}} \approx \sum_{j=0}^{N-g} \rho^j(N+B-j)\pi(< j >); \tag{2.15}$$

$$P_{\mathrm{o}} \approx \sum_{j=0}^{N-g} \sum_{i=N-g-j}^{N+B-j} \rho^j(i)\pi(< j >); \tag{2.16}$$

$$\tilde{N} \approx \sum_{j=1}^{N-g} j \sum_{i=0}^{j} \rho^i (j-i) \pi(< i >)$$
$$+ \sum_{j=N-g+1}^{N-1} j \sum_{i=0}^{N-g} \rho^i (k-i) \pi(< i >) + N \sum_{i=0}^{N-g} \sum_{j=N-i}^{N+B-i} \rho^i (j) \pi(< i >); \tag{2.17}$$

$$L_{\mathrm{h}} \approx \sum_{j=1}^{N+B-g} j \sum_{i=0}^{g} \rho^i (j) \pi(< i >) + \sum_{j=N+B-g+1}^{N+B} j \sum_{i=0}^{N+B-j} \rho^i (j) \pi(< i >). \tag{2.18}$$

Now we can generalize the obtained results to the model with a limited queue of h-calls with a finite degradation interval [i.e. to the model of type (ii)]. Unlike the previous model in this model it is assumed that the degradation interval for h-calls are independent, equally distributed random quantities having exponential distribution with the finite mean γ^{-1}.

The state space of the given model is defined by (2.2) also. But for the given model the elements of the generating matrix corresponding to 2-D MC, $q(\mathbf{k}, \mathbf{k}')$, $\mathbf{k}, \mathbf{k}' \in S$ are determined as follows (see Fig. 2.2):

$$q(\mathbf{k}, \mathbf{k}') = \begin{cases} \lambda_{\mathrm{o}} & \text{if } k_1 + k_2 < N - g - 1,\ \mathbf{k}' = \mathbf{k} + \mathbf{e_1}, \\ \lambda_{\mathrm{h}} & \text{if } \mathbf{k}' = \mathbf{k} + \mathbf{e_2}, \\ k_1 \mu & \text{if } \mathbf{k}' = \mathbf{k} - \mathbf{e_1}, \\ k_2^s \mu \delta\left(k_2{}^q, 0\right) + \left(k_2{}^s \mu + k_2{}^q \gamma\right)\left(1 - \delta\left(k_2{}^q, 0\right)\right) & \text{if } \mathbf{k}' = \mathbf{k} - \mathbf{e_2}, \\ 0 & \text{in other cases.} \end{cases} \tag{2.19}$$

The stationary probability of blocking of o-calls (P_{o}) in this model is determined also by the formula (2.5). However, in this model losses of h-calls occur in the following cases:

(a) If at the moment of its entering the queue there are already B calls of the given type;
(b) If the interval of its degradation is completed before it gains access to a free channel.

Hence the given QoS metric is determined as follows:

$$P_{\mathrm{h}} := \sum_{k \in S} p(\mathbf{k}) \delta\left(k_2^{\mathrm{q}}, B\right) + \frac{1}{\lambda_{\mathrm{h}}} \sum_{i=1}^{B} i\gamma \sum_{j=0}^{N-g} p\,(j, N + i - j). \tag{2.20}$$

In the last formulae the first term of the sum defines the probability of the event corresponding to case (a) above, while the second term of the sum defines the probability of even corresponding to case (b) above.

For the given model the average number of busy channels of the cell ($\tilde{N}$) and the average length of the queue of h-calls (L_{h}) are also determined analogously to

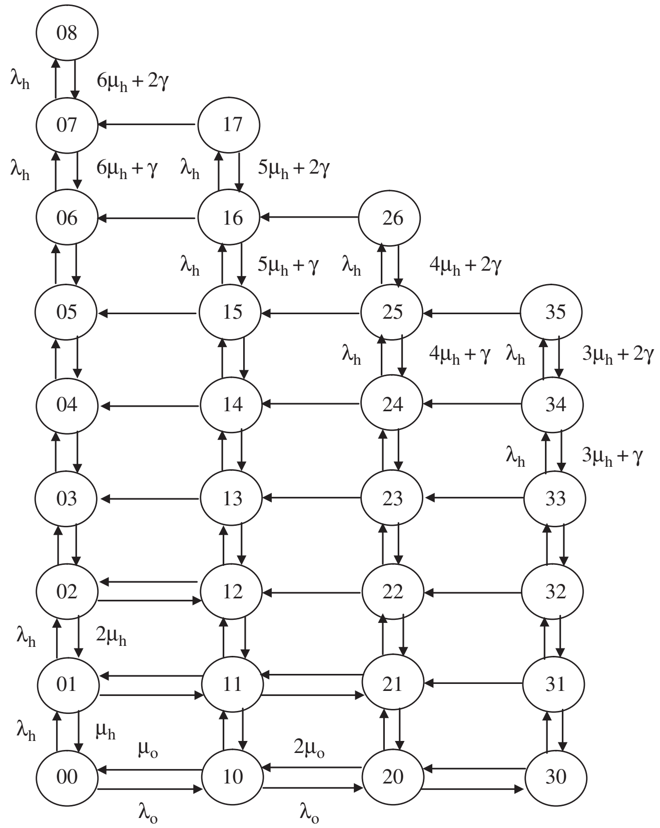

Fig. 2.2 State transition diagram for the model with a limited queue of impatient h-calls, $N = 6$, $g = 3$, B=2

(2.6) and (2.7). And SGBE for the given model is also derived in a similar way to (2.8) and (2.9). However, as in the model with patient h-calls, to solve the indicated SGBE requires laborious computation efforts for large values of N and B since the corresponding SGBE has no explicit solution. But to overcome these difficulties use is made of the approximate method proposed above for the model with patient h-calls.

In this case the splitting (2.10) of state space (2.2) is considered also and by function (2.11) an appropriate merged model with state space $\tilde{S} := \left\{ < j >: j = \overline{0, N-g} \right\}$ is constructed. But in this case the stationary distribution in the jth split model is determined as follows:

$$\rho^j(i) = \begin{cases} \dfrac{v_{\mathrm{h}}^i}{i!} \rho^j(0) \quad \text{if } i = \overline{1, N-j}, \\ \dfrac{v_{\mathrm{h}}^{N-j}}{(N-j)!} \displaystyle\prod_{l=1}^{i+j-N} \dfrac{\lambda_{\mathrm{h}}^l}{(N-j)\,\mu + l\gamma} \rho^j(0) \text{ if } i = \overline{N-j+1, N-j+B}, \end{cases} \tag{2.21}$$

where

$$\rho^j(0) = \left(\sum_{i=0}^{N-j} \frac{v_h^i}{i!} + \frac{v_h^{N-j}}{(N-j)!} \sum_{i=N-j+1}^{N-j+B} \prod_{l=1}^{i+j-N} \frac{\lambda_h^l}{(N-j)\,\mu + l\gamma} \right)^{-1}, \; j = \overline{0, N-g}.$$

Furthermore, the stationary distribution of the merged model $\pi(<j>), <j> \in \tilde{S}$ is determined in a similar way to (2.14). Note that in this case the terms in (2.13) and (2.14) are calculated by taking into account formulae (2.21).

Making use of the above results and omitting the known intermediate mathematical calculations the following formulae to calculate the QoS metrics of a network with a limited queue and finite interval of degradation of h-calls are obtained:

$$P_h \approx \sum_{i=0}^{N-g} \rho^i(N+B-i)\pi(i) + \frac{1}{\lambda_h} \sum_{i=1}^{B} i\gamma \sum_{j=0}^{N-g} \rho^j\,(N+i-j)\,\pi\,(j); \tag{2.22}$$

$$P_o \approx \sum_{j=0}^{N-g} \sum_{i=N-g-j}^{N+B-j} \rho^j(i)\pi(<j>); \tag{2.23}$$

$$\begin{aligned} \tilde{N} \approx & \sum_{j=1}^{N-g} j \sum_{i=0}^{j} \rho^i(j-i)\pi(<i>) \\ & + \sum_{j=N-g+1}^{N-1} j \sum_{i=0}^{N-g} \rho^i(j-i)\pi(<i>) + N \sum_{j=0}^{N-g} \pi(<j>) \sum_{i=N-j}^{N+B-j} \rho^j(i); \end{aligned} \tag{2.24}$$

$$L_h \approx \sum_{j=1}^{N+B-g} j \sum_{i=0}^{g} \rho^i(j)\pi(<i>) + \sum_{j=N+B-g+1}^{N+B} j \sum_{i=0}^{N+B-j} \rho^i(j)\pi(<i>). \tag{2.25}$$

2.1.2 Models with Infinite Queues

We now consider the model of type (iii), i.e. a model of a cell with an unlimited queue of h-calls and an infinite degradation interval. The set of all possible states of the given model is determined in the following way:

$$S := \left\{ \mathbf{k}{:}k_1 = 0, 1, \ldots, N-g;\; k_2 = 0, 1, \ldots;\; k_1 + k_2^s \le N \right\}. \tag{2.26}$$

The number of h-calls in the queue and in channels and the elements of the generating matrix are calculated in a similar way to (2.1) and (2.3), respectively (see Fig. 2.3).

The required QoS metrics in this case is also calculated via a stationary distribution of the model that is determined from the corresponding SGBE of infinite

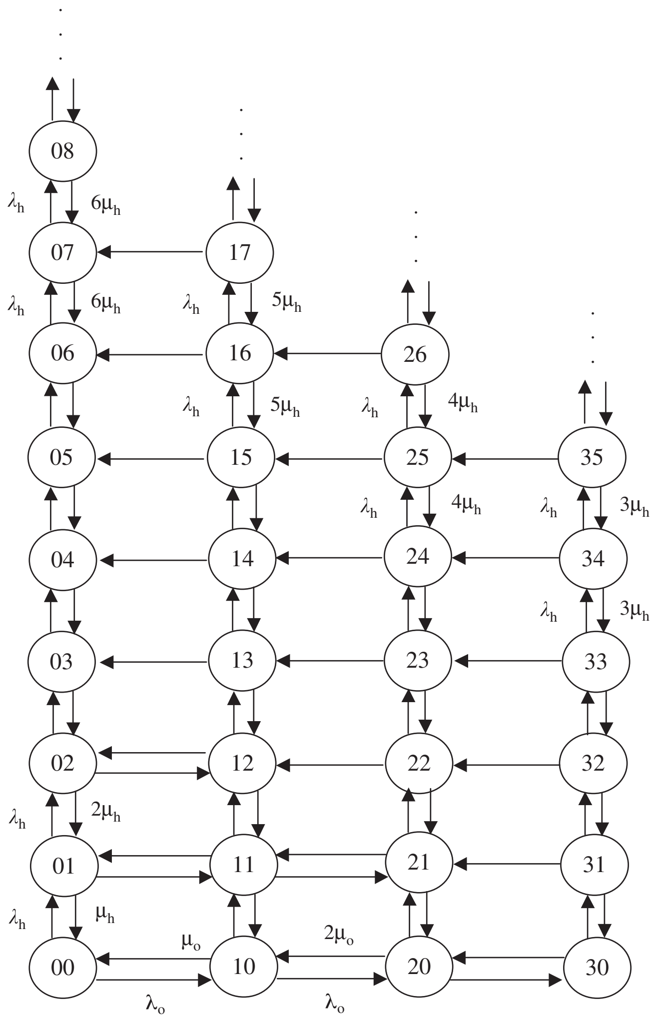

Fig. 2.3 State transition diagram for the model with an unlimited queue of patient h-calls, $N = 6$, $g = 3$

dimension. However, the employment of the method of two-dimensional generating functions for finding the stationary distribution of the given model from the mentioned SGBE is related to well-known computational and methodological difficulties. In relation to this we shall apply the above-described approach to calculating the stationary distribution of the model.

Without repeating the above procedures we just note that here we also made use of the split scheme of the state space (2.26) analogous to (2.10). Since the selection of the split scheme completely specifies the structures of the split and merged models, below only minor comments are made on the formulae suggested.

The stationary distribution of the *j*th split model is determined as the stationary distribution of the classical queuing system *M/M/N–j/*∞ with the load ν_h Erl, i.e.

$$\rho^j(i) = \begin{cases} \dfrac{\nu_h{}^i}{i!}\rho^j(0), & i = \overline{1, N-j}, \\ \dfrac{\nu_h{}^i (N-j)^{N-j-i}}{(N-j)!}\rho^j(0), & i \geq N-j, \end{cases} \tag{2.27}$$

where

$$\rho^j(0) = \left(\sum_{i=0}^{N-j-1} \frac{\nu_h{}^i}{i!} + \frac{\nu_h{}^{N-j}}{(N-j)!} \cdot \frac{N-j}{N-j-\nu_h} \right)^{-1}, \ j = 0, 1, \ldots, N-g.$$

The ergodicity condition of the *j*th split model is $\nu_h < N\text{–}j$. Hence, for the stationary mode to exist in each split model the condition $\nu_h < g$ should be fulfilled. Note that the model ergodicity condition is independent of the o-calls load. This should have been expected since by the assumption $\lambda_h >> \lambda_o$, $\mu_h >> \mu_o$ o-calls are handled by the scheme with pure losses. In the particular case $g = 1$ we obtain the condition $\nu_h < 1$.

The stationary distribution of the merged model is $\pi(<j>), <j> \in \tilde{S}$ determined in a similar way to (2.14). However, in this case one should consider the fact that in the above formulae the parameters $\rho^j(0)$, $j = 0, 1, \ldots, N-g-1$, are calculated from (2.27).

After performing the necessary mathematical transformations one obtains the following approximate formulae for calculating the QoS metrics of the model with an unlimited queue of patient h-calls and guard channels available:

$$P_o \approx 1 - \sum_{j=0}^{N-g-1} \sum_{i=0}^{N-g-1-j} \rho^j(i)\, \pi(<j>); \tag{2.28}$$

$$\begin{aligned} \tilde{N} \approx & \sum_{j=1}^{N-g} j \sum_{i=0}^{j} \rho^i(j-i)\pi(<i>) + \sum_{j=N-g+1}^{N-1} j \sum_{i=0}^{N-g} \rho^i(k-i)\pi(<i>) \\ & + N \sum_{j=0}^{N-g} \pi(<j>) \left(1 - \sum_{i=0}^{N-j-1} \rho^j(i) \right); \end{aligned} \tag{2.29}$$

$$L_h \approx \sum_{i=0}^{N-g} \pi(<i>)\, \rho^i(0)\, \frac{\nu_h^{N+1-i}}{(N-i)!} \cdot \frac{N-i}{(N-i-\nu_h)^2}. \tag{2.30}$$

Now we consider the model of type (iv), i.e. a model with an unlimited queue of impatient h-calls with the guard channels available. In the given model the impatient h-call can be lost from the unlimited queue unless a single channel of a new cell is

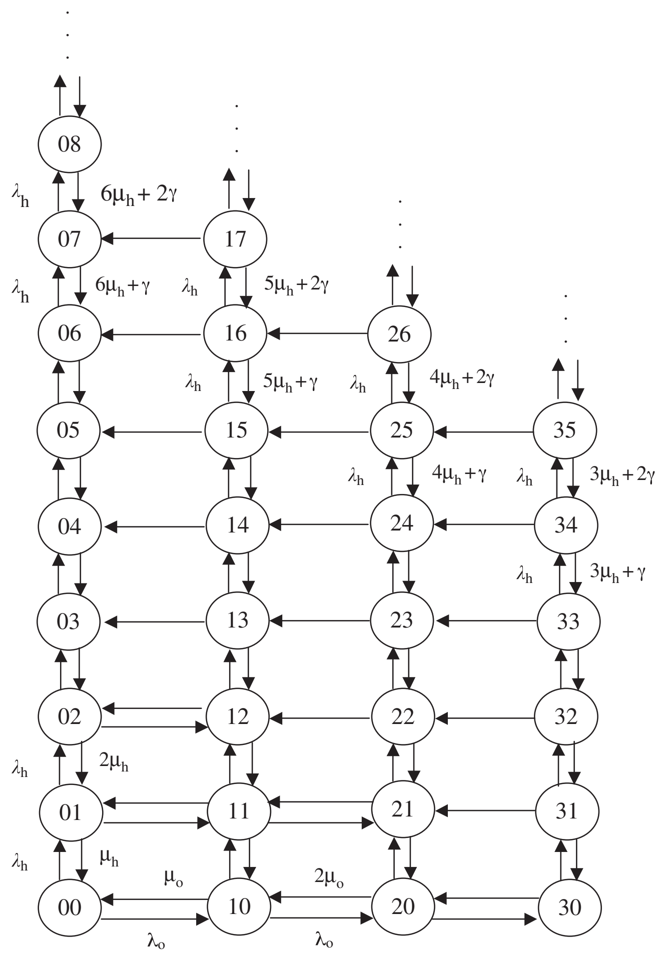

Fig. 2.4 State transition diagram for the model with an unlimited queue of impatient h-calls, $N = 6$, $g = 3$

freed before termination of its degradation interval. To obtain tractable results as in the model of type (ii) it is assumed that degradation intervals for all h-calls are independent, equally exponentially distributed random quantities with the finite mean γ^{-1}. The state space of this model is given by means of the set (2.26). However, thereby the elements of the generating matrix of the corresponding 2-D MC are determined in a similar way to (2.19) (see Fig. 2.4). Similarly to the previous model for the given model one can develop the appropriate SGBE for stationary probabilities of the system states. However, the above-mentioned difficulties of applying such SGBE in this case are even more complicated since to solve this one the approximate approach is used.

The stationary distribution of the jth split model in the given case is determined by

$$\rho^j(i) = \begin{cases} \dfrac{{v_\mathrm{h}}^i}{i!}\rho^j(0), & \text{if } i = \overline{1, N-j}, \\ \dfrac{{v_\mathrm{h}}^{N-j}}{(N-j)!} \displaystyle\prod_{k=N-j+1}^{i} \dfrac{\lambda_\mathrm{h}}{(N-j)\,\mu_\mathrm{h} + (k+j-N)\,\gamma}\rho^j(0), & \text{if } i \geq N-j+1, \end{cases} \tag{2.31}$$

where

$$\rho^j(0) = \left(\sum_{i=0}^{N-j} \frac{{v_\mathrm{h}}^i}{i!} + \frac{{v_\mathrm{h}}^{N-j}}{(N-j)!} \sum_{m=N-j+1}^{\infty} \prod_{k=N-j+1}^{m} \frac{\lambda_\mathrm{h}}{(N-j)\,\mu_\mathrm{h} + (k+j-N)\,\gamma}\right)^{-1}.$$

It is worth noting that in the given model at any permissible values of load parameters in the system there exists a stationary mode. It can be easily proved since the analysis of the ratio limit of two neighboring terms of a series shows that the numerical series

$$R := \sum_{m=N-j+1}^{\infty} \prod_{k=N-j+1}^{m} \frac{\lambda_\mathrm{h}}{(N-j)\,\mu_\mathrm{h} + (k+j-N)\,\gamma} \tag{2.32}$$

involved in determining $\rho^j(0)$ [see formulae (2.31)] converges at any positive values of load parameters of h-calls and degradation interval. However, unfortunately one does not manage to find the exact value of the sum of series (2.32) but we manage to find the following limits of this sum change:

$$\exp\left(\frac{\lambda_\mathrm{h}}{(N-j)\,\mu_\mathrm{h} + \gamma}\right) - 1 \leq R \leq \exp\left(\frac{\lambda_\mathrm{h}}{\gamma}\right) - 1. \tag{2.33}$$

Note 2.2. From the last relations one can see that in a particular case, when $(N-j)\,\mu_\mathrm{h} << \gamma$ or the quantity $(N-j)\,\mu_\mathrm{h}$ is sufficiently small, the approximate value of sum R can be used as the right-hand side of inequality (2.33).

The QoS metrics of the given model are determined like (2.28), (2.29), and (2.30) but now the described formulae involve the corresponding distributions for the model with impatient h-calls. In the given model losses of h-calls from the queue due to their impatience also occur. The probability of such an event is calculated as follows:

$$P_\mathrm{h} = \frac{1}{\lambda_\mathrm{h}} \sum_{n=1}^{\infty} n\gamma \sum_{i=0}^{N-g} p(i, N+n-i)\,. \tag{2.34}$$

2.1.3 Numerical Results

The algorithms suggested allow one to study the behavior of QoS metrics of the systems investigated in all admissible ranges of changes to their structural and load parameters. In order to be short, among the models of cells with a queue of h-calls, only the results for models with unlimited queues are given in detail.

Some results of numerical experiments for the model with patient h-calls at $N = 15$ and $\mu_o = 0.5$ are shown in Figs 2.5, 2.6, and 2.7. They completely confirmed all theoretical expectations. So, the probability of losing o-calls grows (see Fig. 2.5) and the average number of busy channels (see Fig. 2.6) and the average number of h-calls in the queue (see Fig. 2.7) falls as the number of guard channels increases. As changes to the average time of h-call delay coincide with the analogous dependence on their average number in a queue, the details of this function are not presented here.

Note that all the functions under study are increasing with respect to intensity of o-call traffic. However, unlike the function $\tilde{N}$ the rates of change of the functions P_o and L_h are sufficiently high. So, at $N = 15$, $\nu_o = 4$, and $\nu_h = 0.8$ the values of functions P_o and L_h at the points $g = 1$ and $g = 10$ equal $P_o(1) = 3.8\text{E-}04$, $P_o(10) = 2.8\text{E-}01$, $L_h(1) = 9.9\text{E-}05$, $L_h(10) = 1.2\text{E-}10$, and those of the function $\tilde{N}$ at these points equal 4.7985 and 3.6551.

The analysis of results of numerical experiments shows that regardless of the essential difference in loads of o-calls, as the number of guard channels grows the corresponding values of functions P_o and L_h become closer. For example, in two experiments at $N = 15$ the load parameters were selected in this way: (1) $\nu_o = 4$, $\nu_h = 0.8$; (2) $\nu_o = 2$, $\nu_h = 0.75$. For this data the following ratios hold: $P_o^1(1)/P_o^2(1) \approx 400$, $P_o^1(14)/P_o^2(14) \approx 1.01$; $L_h^1(1)/L_h^2(1) \approx 10^3$, $L_h^1(14)/L_h^2(14) \approx 3$,

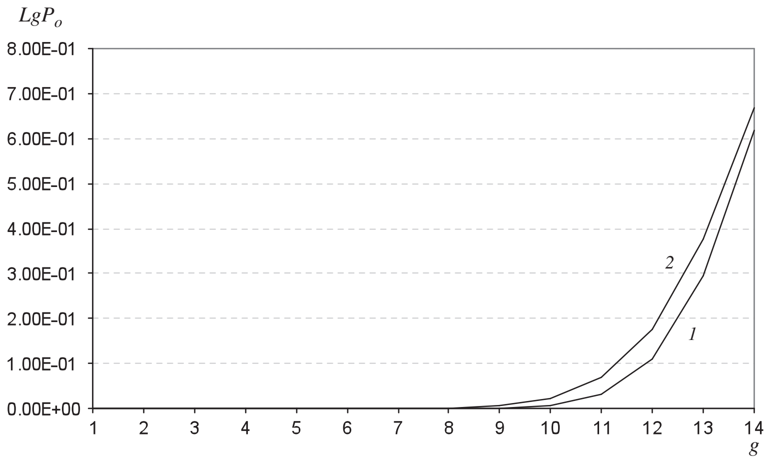

Fig. 2.5 P_o versus g for the model with an unlimited queue of h-calls in the case where $N = 15$; $\mu_o = 0.5$; $1 - \lambda_o = 2$, $\lambda_h = 4$ $\mu_h = 5$; $2 - \lambda_o = 1$, $\lambda_h = 3$ $\mu_h = 4$

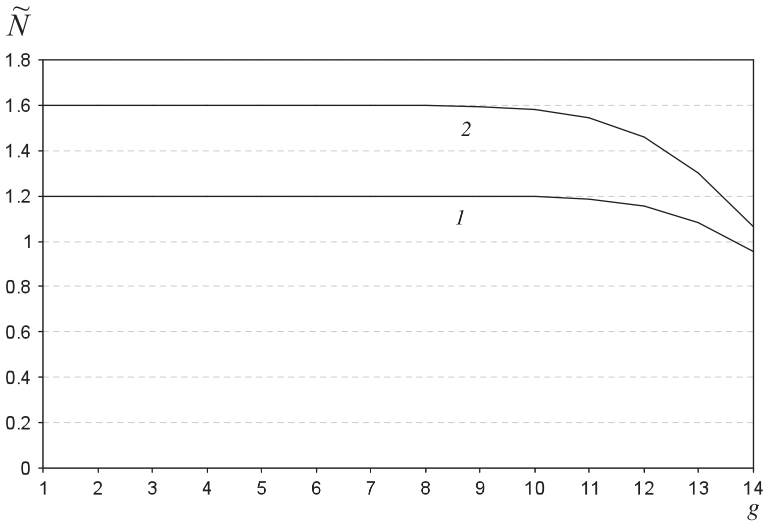

Fig. 2.6 $\tilde{N}$ versus g for the model with an unlimited queue of h-calls in the case where $N = 15$; $\mu_o = 0.5$; $1 - \lambda_o = 2$, $\lambda_h = 4$ $\mu_h = 5$; $2 - \lambda_o = 1$, $\lambda_h = 3$ $\mu_h = 4$

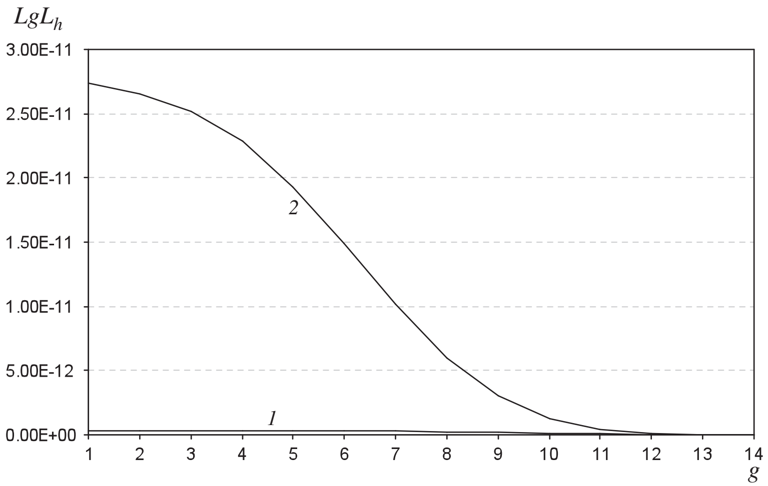

Fig. 2.7 L_h versus g for the model with an unlimited queue of h-calls in the case where $N = 15$; $\mu_o = 0.5$; $1 - \lambda_o = 2$, $\lambda_h = 4$ $\mu_h = 5$; $2 - \lambda_o = 1$, $\lambda_h = 3$ $\mu_h = 4$

where P_o^i (L_h^i) is the value of the function P_o (L_h) in the ith experiment, $i = 1,2$. The corresponding ratios for the function $\tilde{N}$are of the form $\tilde{N}^1(1)/\tilde{N}^2(1) \approx 1.8$,$\tilde{N}^1(14)/\tilde{N}^2(14) \approx 1.1$.

The results of numerical experiments for the model with impatient h-calls showed that the probability of losing o-calls also grew as the number of guard channels increased. In this model the probability of losing h-calls decreases with respect to the number of guard channels and the increase in the number of guard channels also decreases the average number of busy channels. Both functions increase with respect to the intensity of o-call traffic.

The analysis of QoS metrics of different models with equal initial data showed that in the model with patient h-calls the probability of losing o-calls was higher than in the model with impatient h-calls. This should have been expected since in the model with impatient h-calls there occur losses of h-calls from the queue thereby increasing the chances for o-calls to occupy a free channel. These comments also refer to other QoS metrics namely utilization of channels in the model with impatient h-calls is worse than in the model with patient h-calls. And the average length of the queue of h-calls in the model with patient h-calls is larger than that in the model with impatient h-calls.

Another goal of performing numerical experiments was the estimation of the proposed formula accuracy. The exact values (EV) of the QoS metrics for the model with patient h-calls at the identical time of channel occupancy by different types of calls are determined by the following formulae which are easily derived from classical one-dimensional birth-and-death processes:

$$P_{\mathrm{o}} = 1 - \sum_{k=0}^{N-g-1} \rho_k,$$

$$\tilde{N} = \sum_{k=1}^{N-1} k\rho_k + N\left(1 - \sum_{k=0}^{N-1} \rho_k\right),$$

$$L_{\mathrm{h}} = \frac{A\tilde{v}_{\mathrm{h}}^{N+1}}{(1-\tilde{v}_{\mathrm{h}})^2},$$

where

$$\rho_k = \begin{cases} \dfrac{v^k}{k!}\cdot\rho_0 & \text{if } k = \overline{1, N-g}, \\ \left(\frac{\lambda}{\lambda_{\mathrm{h}}}\right)^{N-g}\cdot\dfrac{v_h^k}{k!}\cdot\rho_0 & \text{if } k = \overline{N-g+1, N}, \end{cases}$$

$$\rho_0 = \left(\sum_{k=0}^{N-g}\frac{v^k}{k!} + \left(\frac{\lambda}{\lambda_{\mathrm{h}}}\right)^{N-g}\sum_{k=N-g+1}^{N}\frac{v_h^k}{k!} + \left(\frac{\lambda}{\lambda_{\mathrm{h}}}\right)^{N-g}\cdot\frac{N^N}{N!}\cdot\frac{\tilde{v}_{\mathrm{h}}^{N+1}}{1-\tilde{v}_{\mathrm{h}}}\right)^{-1};$$

$$\lambda := \lambda_{\mathrm{o}} + \lambda_{\mathrm{h}};\quad v := \frac{\lambda}{\mu};\quad \mu := \mu_{\mathrm{o}} = \mu_{\mathrm{h}};\ \tilde{v}_{\mathrm{h}} := \frac{v_{\mathrm{h}}}{N};\ A := \left(\frac{\lambda}{\lambda_{\mathrm{h}}}\right)^{N-g}\cdot\frac{N^N}{N!}.$$

Note that approximate values (AV) of QoS metrics are almost identical to their exact values when the accepted assumption about ratios of load parameters of o- and h-calls is valid. Some comparisons for the models with parameters $N = 10$,

$\lambda_0 = 0.3$, $\lambda_h = 2$, $\mu_o = \mu_h = 3$ and $N = 15$, $\lambda_0 = 2$, $\lambda_h = 4$, $\mu_o = \mu_h = 5$ are presented in Tables 2.1 and 2.2, respectively.

As indicated in Tables 2.1 and 2.2 the accuracy of approximate formulae is sufficiently high even when the accepted assumption is not valid, i.e. $\mu_o = \mu_h$. Similar results are obtained for any possible values of initial data of the model.

It is worth noting that sufficiently high accuracy exists for the initial data not satisfying the above-mentioned assumption concerning the ratios of traffic of o- and h-calls. In other words, the numerical experiments for the models with symmetric traffic (i.e. $\lambda_o = \lambda_h$) in addition to the ones in which the intensity of o-calls greatly exceeds the intensity of h-calls showed a sufficiently high accuracy for the suggested approximate formulae. Certain results of the comparison for the models

Table 2.1 Comparison of exact and approximate values of QoS metrics for the model with patient h-calls in the case where $N = 10$, $\lambda_o = 0.3$, $\lambda_h = 2$, $\mu_o = \mu_h = 3$

	P_o		$\tilde{N}$		L_h	
g	EV	AV	EV	AV	EV	AV
1	1.25516E-07	1.26890E-07	0.766666654	0.76666665	1.28659E-09	8.27322E-10
2	1.48437E-06	1.50207E-06	0.766666518	0.76666652	1.11877E-09	8.27185E-10
3	1..56409E-05	1.58504E-05	0.766665103	0.76666508	9.72847E-10	8.26463E-10
4	1.44624E-04	1.46806E-04	0.766652204	0.76665199	8.45954E-10	8.23177E-10
5	1.15118E-03	1.17021E-03	0.766551548	0.76654965	7.35612E-10	8.01069E-10
6	7.68970E-03	7.81343E-03	0.765897696	0.76588532	6.39663E-10	7.72305E-10
7	4.16251E-02	4.20410E-02	0.762504156	0.76246247	5.56228E-10	6.80938E-10
8	1.73829E-01	1.72958E-01	0.749283793	0.74937084	4.83677E-10	5.22265E-10
9	5.21507E-01	5.11655E-01	0.714515980	0.71550113	4.20589E-10	3.33886E-10

Table 2.2 Comparison of exact and approximate values of QoS metrics for the model with patient h-calls in the case where $N = 15$, $\lambda_o = 2$, $\lambda_h = 4$, $\mu_o = \mu_h = 5$

	P_o		$\tilde{N}$		L_h	
g	EV	AV	EV	AV	EV	AV
1	4.68574E-11	4.8229E-11	1.200000000	1.20000000	4.67447E-13	3.54354E-13
2	5.48753E-10	5.65712E-10	1.200000000	1.20000000	3.11631E-13	3.54052E-13
3	5.97224E-09	6.17134E-09	1.199999998	1.20000000	2.07754E-13	3.53091E-13
4	6.00456E-08	6.22177E-08	1.199999976	1.19999998	1.38503E-13	3.50246E-13
5	5.53951E-07	5.75773E-07	1.199999778	1.19999977	9.23352E-14	3.42914E-13
6	4.65197E-06	4.85165E-06	1.199998139	1.19999806	6.15568E-14	3.26731E-13
7	3.52214E-05	3.68591E-05	1.199985911	1.19998526	4.10379E-14	2.96472E-13
8	2.37618E-04	2.49355E-04	1.199904953	1.19990026	2.73586E-14	2.49023E-13
9	1.407653E-03	1.478063E-03	1.199436939	1.19940877	1.82391E-14	1.8732E-13
10	7.18795E-03	7.51375E-03	1.19712482	1.1969945	1.21594E-14	1.21736E-13
11	3.090382E-02	3.186036E-02	1.187638471	1.18725586	8.10625E-15	6.58789E-14
12	1.087237E-01	1.091789E-01	1.156510523	1.15632842	5.40416E-15	2.87429E-14
13	3.032389E-01	2.945699E-01	1.078704409	1.08217203	3.60278E-15	9.97493E-15
14	6.476778E-01	6.191261E-01	0.940928864	0.95234956	2.40185E-15	2.82974E-15

with parameters $N=10,\ \lambda_o=2,\ \lambda_h=0.3,\ \mu_o=\mu_h=3$ and $N=15,\ \lambda_o=\lambda_h=4,$ $\mu_o=\mu_h=5$ are shown in Tables 2.3 and 2.4, respectively.

As seen from Tables 2.3 and 2.4 small deviations are observed upon calculation of average length of h-calls. Therewith the approximate values of the QoS metrics are always larger than their exact values. The last circumstance suggests that to increase the reliability of network performance the obtained approximate formulae can be applied at the initial stages of its design.

Analogous results are obtained for the models with a limited queue of h-calls. For the model with a limited queue of patient h-calls the approximate results obtained were compared with results in [11], wherein accurate formulae for the model with identical (with respect to channel occupation times) calls were developed. The

Table 2.3 Comparison of exact and approximate values of QoS metrics for the model with patient h-calls in the case where $N=10,\ \lambda_o=2,\ \lambda_h=0.3,\ \mu_o=\mu_h=3$

	P_o		$\tilde{N}$		L_h	
g	EV	AV	EV	AV	EV	AV
1	1.18332E-07	1.20335E-07	0.766666588	0.76666659	2.57292E-11	5.31368E-10
2	1.39066E-06	1.41037E-06	0.766665740	0.76666573	3.35598E-12	1.51235E-10
3	1.45316E-05	1.46932E-05	0.766656979	0.76665687	4.37736E-13	3.60410E-11
4	1.32920E-04	1.33885E-04	0.766578053	0.76657741	5.70961E-14	6.56823E-12
5	1.04287E-03	1.04528E-03	0.765971420	0.76596981	7.44731E-15	8.94899E-13
6	6.83047E-03	6.80289E-03	0.762113022	0.76213141	9.71388E-16	8.95273E-14
7	3.60318E-02	3.56001E-02	0.742645458	0.74293325	1.26703E-16	6.41398E-15
8	1.46785E-01	1.43779E-02	0.668809421	0.67081413	1.65265E-17	3.19136E-16
9	4.46385E-01	4.35614E-01	0.469076476	0.47625721	2.15562E-18	1.08186E-17

Table 2.4 Comparison of exact and approximate values of QoS metrics for the model with patient h-calls in the case where $N=15,\ \lambda_o=\lambda_h=4,\ \mu_o=\mu_h=5$

	P_o		$\tilde{N}$		L_h	
g	EV	AV	EV	AV	EV	AV
1	1.76280E-09	1.86813E-09	1.599999999	1.6000000	2.62346E-11	2.73750E-11
2	1.54833E-08	1.64400E-08	1.599999988	1.59999999	1.31173E-11	2.65173E-11
3	1.26382E-07	1.34674E-07	1.599999899	1.59999989	6.55865E-12	2.51571E-11
4	9.52992E-07	1.01936E-06	1.599999238	1.59999918	3.27933E-12	2.28218E-11
5	6.59388E-06	7.07790E-06	1.599994725	1.59999434	1.63966E-12	1.93241E-11
6	4.15307E-05	4.46986E-05	1.599966775	1.59996424	8.19832E-13	1.48828E-11
7	2.35835E-04	2.54048E-04	1.599811332	1.59979676	4.09916E-13	1.01637E-11
8	1.19341E-03	1.28238E-03	1.599045275	1.59897409	2.04958E-13	6.00668E-12
9	5.30507E-03	5.65419E-03	1.595755945	1.59547665	1.02479E-13	3.00061E-12
10	2.03616E-02	2.13408E-02	1.583710735	1.58292732	5.12395E-14	1.23831E-12
11	6.61732E-02	6.74847E-02	1.547061432	1.54601217	2.56197E-14	4.13800E-13
12	1.78719E-01	1.75864E-01	1.457024740	1.45930802	1.28099E-14	1.10834E-13
13	3.95653E-01	3.76449E-01	1.283477649	1.29884051	6.40494E-15	2.40351E-14
14	7.10236E-01	6.69481E-01	1.031811366	1.06441553	3.20247E-15	4.38201E-15

comparative analysis was performed over a wide range of variance in the initial data of the model. It is worth noting that the approximate and exact results from calculating the loss probability of h-calls were identical. An insignificant difference is observed in calculating the blocking probability of o-calls. It can be seen that the maximal value of error in all executed experiments was negligible and in most cases equals zero even for data not satisfying the above-mentioned assumption concerning the ratios of traffic of o- and h-calls. So, for example, at $N = 50$, $B = 10$, $\lambda_0 = 2$, $\lambda_h = 1$, $\mu = 10$ the maximal error holds for $g = 49$ and is 1.6%, i.e. for these initial data the exact value of P_o equals 0.233 while its approximate value equals 0.249.

2.2 Models with Queues for o-Calls

In order to compensate for the chance of o-calls a queue (limited or unlimited) is required for them, while maintaining a high chance for h-calls to access the system via reservation of channels. Here we consider four types of models with queues for o-calls whereas h-calls are treated according to a lost model [2, 9]. Note that in this section it is assumed that the required handling time is independent of call type and exponentially distributed with the same average μ^{-1}.

In all schemes it is assumed that all m+n channels are divided into two groups: a Primary group with m channels and a Secondary group with n channels. If all channels in both groups are busy the h-call is lost. New calls are only allowed to the Primary group; therefore if all m channels are busy this call is placed into a queue.

In two schemes reallocation of channels from one group to another is not allowed, i.e. isolated reservation is considered. In the HOPS (Handoff calls Overflow from Primary to Secondary) scheme, for handling of an h-call, the Primary group channels are used first and upon absence of an empty channel in the Primary group (all m channels are busy) the Secondary group channels are used. In the HOSP (Handoff calls Overflow from Secondary to Primary) scheme the search for a free channel for handling of an h-call is realized in the Secondary group first.

The last two schemes involve shared reservation of channels. They maybe described as follows. In one scheme upon release of a channel in the Primary group (either by an o-call or h-call) an h-call from the Secondary group is reallocated to the Primary group regardless of the length of the o-calls in the queue. However, o-calls from the queue are admitted to channels in accordance with the FIFO discipline only if the number of total empty channels exceeds n. The given channel reallocation scheme is called Handoff Reserve Margin Algorithm (HRMA). The main difference of the final scheme from the previous one is in the reallocation of the h-call from the Secondary group to the Primary group. Upon release of a channel in the Primary group (either by an o-call or h-call) an h-call from the Secondary group is reallocated to the Primary group if and only if there are no o-calls in the queue. In other words reallocation of h-calls from the Secondary group to the Primary group is not allowed until the queue does not contain any o-calls. This reservation scheme is called Handoff calls Overflow from Primary to Secondary with Rearrangement (HOPSWR).

2.2.1 Models Without Reassignment of Channels

Here we consider two isolated reservation schemes in which reallocation (reassignment) of channels from one group to another is not allowed. In the HOPS (Handoff calls Overflow from Primary to Secondary) scheme the initial search for a free channel for service of an h-call is carried out in the Primary Group and upon the absence of an empty channel in the Primary group (all m channels are busy) Secondary group channels are used (see Fig. 2.8a). In the HOSP (Handoff calls Overflow from Secondary to Primary) scheme the initial search for a free channel to service an h-call is carried out in the Secondary Group (see Fig. 2.8b).

In both schemes o-calls can be served only in the Primary Group of channels and any conservative discipline of service which does not admit to idle times of channels in the presence of a queue can be used for service of the o-calls queue. In both schemes in cases of occupancy of all $m+n$ channels the h-call is lost and any reassignment of the h-call from one group to another is not permitted.

First of all we shall consider the HOPS scheme in the model with an infinite queue of o-calls. The QoS metrics include probability of loss of h-calls (P_h), average length of o-calls queue (L_o), and the average latency period in the queue (W_o).

The following hierarchical approach can be used for the analysis of this model as in this scheme received h-calls initially go to the Primary Group of channels, and

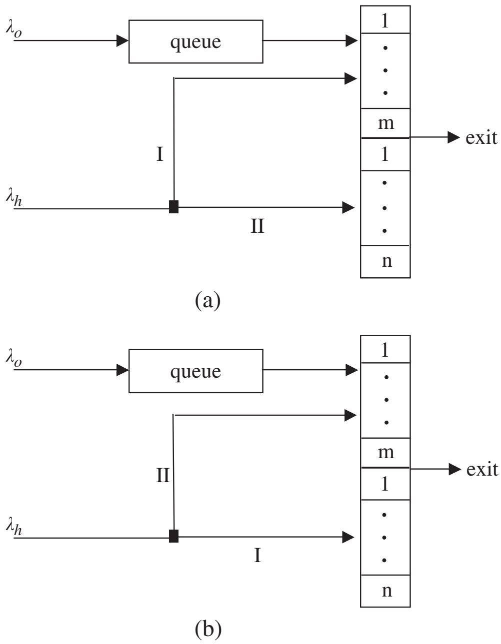

Fig. 2.8 Diagram of the HOPS (**a**) and HOSP (**b**) model

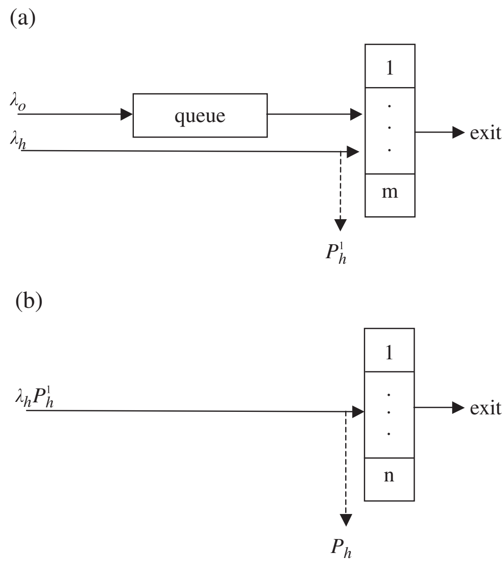

Fig. 2.9 Hierarchical approach for study of the HOPS model

only missed calls of the given type are further received in the Secondary Group of channels. In the first step of the hierarchy (see Fig. 2.9a) we shall consider a system with m channels which serve calls of two types with rates λ_o and λ_h, thus the holding time of any type of call has an exponential distribution with a common mean μ^{-1}. New calls are buffered in an infinite queue and h-calls are lost in case of occupancy of all channels. Missed h-calls are forwarded to the Secondary Group of channels for service.

Consider that the probability of loss of h-calls is equal to P_h^1 in the Primary Group of channels. From Poisson flow property we deduce that the input to the Secondary Group of channels forms Poisson flow with rate $\tilde{\lambda}_h := \lambda_h P_h^1$. Hence on the second stage of hierarchy (see Fig. 2.9b) we examine classic Erlang model *M/M/n/0*. Loss probability of calls of this model then will be considered as the desired P_h. And the desired L_o and W_o are calculated as QoS parameters of queuing system described at the first step of hierarchy.

Now we shall consider the problem of calculation of the above-specified QoS metrics. The state of the queuing system with two types of calls described in the first step of the hierarchy is given by scalar parameter k which specifies the total number of calls in the system, $k = 0,1,2, \ldots$. Stationary distribution of the corresponding one-dimensional birth-death process (1-D BDP) is calculated as (Fig. 2.10):

$$\rho_k = \begin{cases} \dfrac{v^k}{k!}\rho_0 & \text{if } 1 \le k \le m, \\ \dfrac{v^m}{m!} \cdot \dfrac{v_o^{k-m}}{m^{k-m}}\rho_0 & \text{if } k \ge m+1, \end{cases} \tag{2.35}$$

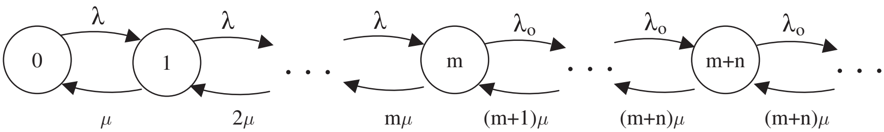

Fig. 2.10 State transition diagram for the HOPS model

where

$$v := \lambda/\mu,\ \lambda := \lambda_{\mathrm{o}} + \lambda_{\mathrm{h}}\,,\ v_{\mathrm{o}} := \lambda_{\mathrm{o}}/\mu,\ \rho_0 = \left(\sum_{k=0}^{m} \frac{v^k}{k!} + \frac{v^m}{m!} \cdot \frac{v_{\mathrm{o}}}{m - v_o}\right)^{-1}.$$

On derivation of formulae (2.35) an intuitively clear and simple condition of ergodicity of the models becomes apparent: $v_{\mathrm{o}} < m$. Hence we can see that the condition of ergodicity of the models does not depend on the loading of handover calls. From (2.35) it is concluded that the probability of h-call loss in the Primary Group (P_{h}^1) is defined as:

$$P_{\mathrm{h}}^1 = 1 - \sum_{k=0}^{m-1} \rho_k\,. \tag{2.36}$$

Hence, required QoS metric P_{h} is calculated by means of Erlang's classical B-formula for the *M/M/n/*0 c model with a load of $\tilde{v}_{\mathrm{h}} := \tilde{\lambda}_{\mathrm{h}}/\mu$ Erl. In other words, $P_{\mathrm{h}} = E_{\mathrm{B}}\left(\tilde{v}_{\mathrm{h}}, n\right)$.

After certain transformations we obtain the following formula for calculation of QoS metric L_{o}:

$$L_{\mathrm{o}} = \sum_{k=1}^{\infty} k\rho_{k+m} = \frac{v^m}{(m-1)!} \cdot \frac{v_{\mathrm{o}}}{(m - v_o)^2} \cdot \rho_0\,. \tag{2.37}$$

QoS metric W_{o} is obtained from the Little formulae, i.e. $W_{\mathrm{o}} = L_{\mathrm{o}}/\lambda$.

The developed approach allows the definition of QoS metrics of the model also in the presence of a limited buffer for waiting in a queue of o-calls. We should note that in these models at any loading and structural parameter values in the system there is a stationary mode, i.e. ergodicity performance is not required, $v_{\mathrm{o}} < m$.

Let the maximum size of the buffer be equal to R, $R<\infty$. On the basis of formulae for finite BDP we conclude that stationary distribution of the appropriate system is calculated as follows:

$$\rho_k = \begin{cases} \dfrac{\nu^k}{k!}\rho_0 & \text{если} \quad 1 \le k \le m, \\ \dfrac{\nu^m}{m!} \cdot \dfrac{\nu_o^{k-m}}{m^{k-m}}\rho_0 & \text{если} \quad m+1 \le k \le m+R, \end{cases} \tag{2.38}$$

where

$$\rho_0 = \left(\sum_{k=0}^{m} \frac{v^k}{k!} + \frac{v^m}{m!} \cdot \frac{m^m}{v_o^m} \sum_{k=m+1}^{R} \left(\frac{v_o}{m} \right)^k \right)^{-1} .$$

Then by using (2.38) from (2.36) and Erlang's B-formula QoS metric P_h is calculated. And from (2.38) the following formula for calculation of the QoS metric $L_o(R)$ for the model with a limited queue of o-calls finds:

$$L_o(R) = \sum_{k=1}^{R} k\rho_{k+m} . \tag{2.39}$$

From (2.39) quantity $W_o(R)$ in the given model is calculated as follows

$$W_o(R) = \frac{L_o(R)}{\lambda_o (1 - P_o(R))} ,$$

where $P_o(R)$ denotes the loss probability of o-calls in the given model, i.e. $P_o(R) = \rho_{m+R}$.

Now consider the HOSP scheme in the model with an infinite queue of o-calls. As in the previous scheme it is possible to use the hierarchical approach. Here in the first step of the hierarchy (see Fig. 2.11a) the classical Erlang model *M/M/n/0* with

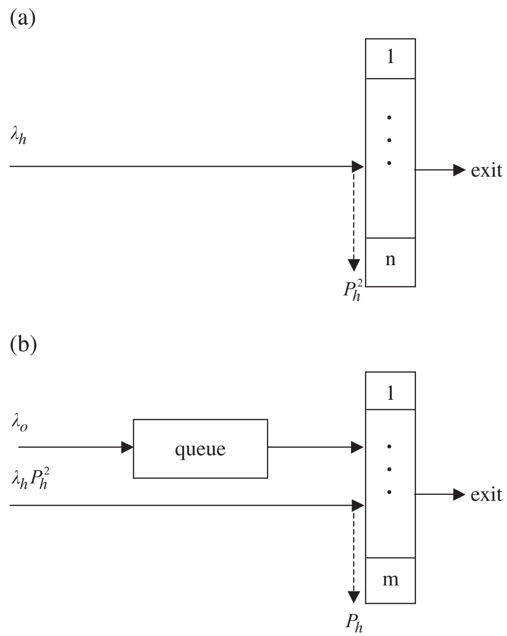

Fig. 2.11 Hierarchical approach for study of the HOSP model

a load $v_{\mathrm{h}} := \lambda_{\mathrm{h}}/\mu$ Erl is considered. The probability of h-call loss in this model will be denoted by P_{h}^2, i.e. $P_{\mathrm{h}}^2 = E_{\mathrm{B}}(v_{\mathrm{h}}, n)$.

Missed h-calls in this system are forwarded to the Primary Group for reception of service. Hence, in the second step of the hierarchy (see Fig. 2.11b) the system with m channels which serves calls of two types with rates λ_{o} and $\hat{\lambda}_{\mathrm{h}} := \lambda_{\mathrm{h}} P_{\mathrm{h}}^2$, thus the holding time of a call of any type that has an exponential distribution with the general average μ^{-1}, is considered. New calls are buffered in an infinite queue and in the case of occupancy of all channels h-calls are lost. Missed h-calls in this system are finally lost. Thus, stationary distributions of the queuing system described in the second step of the hierarchy are calculated by means of the formula (2.35). However, in this case the specified formula parameter λ is determined as $\lambda = \lambda_{\mathrm{o}} + \hat{\lambda}$.

Stationary distribution of the last system will be denoted through $\sigma_{k,}$ $k = 0, 1, 2, \ldots$. The condition of ergodicity of models in the given scheme is also $v_o < m$. Then in view of the above-stated, it is concluded that the required QoS metric P_{h} in the HOSP scheme of channel distribution is calculated as:

$$P_{\mathrm{h}} = 1 - \sum_{k=0}^{m-1} \sigma_k \ . \tag{2.40}$$

Other QoS metrics L_{o} and W_{o} in the given scheme of channel allocation are also calculated from (2.37) and Little's formula, accordingly. Thus, it is necessary to consider that in this case $\lambda = \lambda_{\mathrm{o}} + \hat{\lambda}$.

The above-described approach might also be used for calculation of the QoS metrics in the HOSP scheme with a limited queue of o-calls. As this procedure is almost the same as the analogous procedure for the HOPS scheme, it is not presented here.

2.2.2 Models with Reassignment of Channels

In this section we consider two schemes with reassignment of channels. First we consider the HRMA (Handoff Reserve Margin Algorithm) scheme for channel assignment. In this case for the handling of an h-call a channel from the Primary group is used first and upon absence of an empty channel in this group (all m channels are busy) Secondary group channels are used. If all channels in both groups are busy the h-call is lost. New calls are only allowed to the Primary group; therefore if all m channels are busy this call is placed into a queue. Upon release of a channel in the Primary group (either by an o-call or h-call) an h-call from the Secondary group is reallocated to the Primary group regardless of the length of the o-calls in the queue. However, o-calls from the queue are admitted to channels in accordance with FIFO discipline only if the number of total empty channels exceeds n.

The system's state at any time is described by the two-dimensional vector $\mathbf{k} = (k_1, k_2)$, where k_1 is the total number of o-calls in the system $k_1 = 0, 1, 2, \ldots$, and k_2

is the total number of busy channels, $k_2 = 0, 1, \ldots, m+n$. Note that state space S of the given model does not include vectors $\mathbf{k}$ with components $k_1 > 0$, $k_2 < m$.

On the basis of the adopted channel allocation scheme, we can conclude that elements of the generating matrix $q(\mathbf{k}, \mathbf{k}')$, $\mathbf{k}, \mathbf{k}' \in S$ of the appropriate 2-D MC are defined from the following relations (see Fig. 2.12):

$$q(\mathbf{k}, \mathbf{k}') = \begin{cases} \lambda_{\mathrm{o}} + \lambda_{\mathrm{h}}, & \text{if } k_1 = 0,\ k_2 \le m-1,\ \mathbf{k}' = \mathbf{k} + \mathbf{e_2}, \\ \lambda_{\mathrm{h}}, & \text{if } k_2 \ge m,\ \mathbf{k}' = \mathbf{k} + \mathbf{e_2}, \\ \lambda_{\mathrm{o}}, & \text{if } k_2 \ge m,\ \mathbf{k}' = \mathbf{k} + \mathbf{e_1}, \\ k_2 \mu, & \text{if } k_2 \ne m,\ \mathbf{k}' = \mathbf{k} - \mathbf{e_2}, \\ m\mu, & \text{if } k_2 = m,\ \mathbf{k}' = \mathbf{k} - \mathbf{e_1}, \\ 0 & \text{in other cases.} \end{cases} \tag{2.41}$$

The desired QoS metrics for the model with an infinite queue of o-calls are calculated by stationary distribution of the model as follows:

$$P_{\mathrm{h}} = \sum_{k_1=0}^{\infty} p\,(k_1, m+n), \tag{2.42}$$

$$L_{\mathrm{o}} = \sum_{k_1=1}^{\infty} \sum_{k_2=m}^{m+n} k_1 p\,(k_1, k_2), \tag{2.43}$$

Here we develop simple approximate computational procedures to calculate these QoS metrics. For correct application of the approximate approach condition

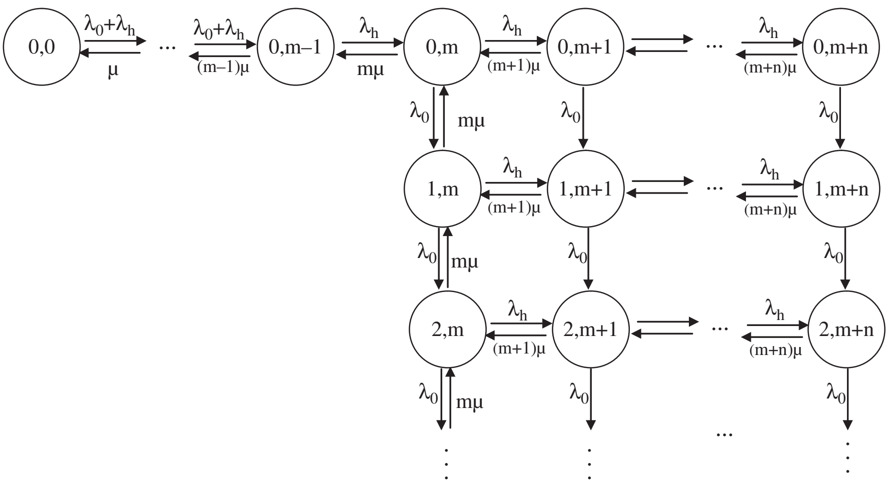

Fig. 2.12 State transition diagram for the HRMA model with an unlimited queue of patient o-calls

$\lambda_h >> \lambda_o$is required. As was mentioned in previous sections this condition is true for micro- and picocells. In addition, we assume that both types of calls are characterized by very short duration with respect to their frequency of arrivals. The last assumption is valid for many real system models that are quite close to those in this work (see Sect. 1.1).

The following splitting of state space of the given model is considered:

$$S = \bigcup_{i=0}^{\infty} S_i, S_i \bigcap S_j = \varnothing, \, i \neq j, \tag{2.44}$$

where

$$S_i := \{k \in S : k_1 = i\}.$$

Furthermore, the class of microstates S_i is merged into the isolated merged state $<i>$ and an appropriate merged model with state space $\tilde{S} := \{< i >: i = 0, 1, 2, \ldots\}$ is constructed.

The elements of the generated matrix of splitting models with state space S_i that is denoted by $q_i(\mathbf{k}, \mathbf{k}')$, $\mathbf{k}, \mathbf{k}' \in S_i$ are calculated as follows [see (2.41)]:

For the model with state space S_0:

$$q_0\left(\mathbf{k}, \mathbf{k}'\right) = \begin{cases} \lambda_o + \lambda_h, & \text{if } k_2 \leq m-1, \quad k_2{}' = k_2 + 1, \\ \lambda_h, & \text{if } m \leq k_2 \leq m+n-1, \quad k_2{}' = k_2 + 1, \\ k_2\mu, & \text{if } k_2{}' = k_2 - 1, \\ 0 & \text{in other cases,} \end{cases} \tag{2.45}$$

For models with state spaces S_i, $i \geq 1$:

$$q_i\left(\mathbf{k}, \mathbf{k}'\right) = \begin{cases} \lambda_h & \text{if } k_2{}' = k_2 + 1, \\ k_2\mu & \text{if } k_2{}' = k_2 - 1, \\ 0 & \text{in other cases.} \end{cases} \tag{2.46}$$

By using (2.45) and (2.46) the stationary distribution of the splitting models are calculated from the following expressions:

For $i = 0$:

$$\rho^0(j) = \begin{cases} \frac{\nu^j}{j!}\rho_0 & \text{if } j = \overline{1, m}, \\ \left(\frac{\nu}{\nu_h}\right)^m \frac{\nu_h{}^j}{j!}\rho_0 & \text{if } j = \overline{m+1, m+n}. \end{cases} \tag{2.47}$$

For $i > 0$:

$$\rho^i(j) = \frac{m!}{\nu_h{}^m} \frac{\nu_h{}^j}{j!}\rho_1, \; j = \overline{m+1, m+n}, \tag{2.48}$$

where

$$\rho_0 = \left(\sum_{i=0}^{m} \frac{v^i}{i!} + \left(\frac{v}{v_{\mathrm{h}}}\right)^m \sum_{i=m+1}^{m+n} \frac{v_{\mathrm{h}}^i}{i!} \right)^{-1}, \quad \rho_1 = \left(\frac{m!}{v_{\mathrm{h}}{}^m} \sum_{i=m}^{m+n} \frac{v_{\mathrm{h}}^i}{i!} \right)^{-1}, v := v_{\mathrm{o}} + v_{\mathrm{h}}.$$

By using (2.45), (2.46), (2.47), and (2.48) we conclude that the elements of the generating matrix of merged model $q\left(< i >, < i' >\right)$, $< i >, < i' > \in \tilde{S}$ are calculated as follows:

$$q\left(< i >,\ < i' >\right) = \begin{cases} \lambda_{\mathrm{o}} \sum\limits_{j=m}^{m+n} \rho^0\,(j) & \text{if } i = 0,\ i' = i+1, \\ \lambda_o & \text{if } i \geq 0,\ i' = i+1, \\ m\mu\rho_1 & \text{if } i \geq 0,\ i' = i-1, \\ 0 & \text{in other cases.} \end{cases} \tag{2.49}$$

From (2.49) we find the following ergodicity condition of the merged model:

$$a := \frac{v_{\mathrm{o}}}{m\rho^1(m)} < 1$$

or in explicit form

$$\frac{v_{\mathrm{o}}}{m} \cdot \frac{m!}{v_{\mathrm{h}}^m} \left(\sum_{i=m}^{m+n} \frac{v_{\mathrm{h}}^i}{i!} \right) < 1. \tag{2.50}$$

Note 2.3. It is important to note that ergodicity condition (2.50) is exactly the stability condition of the system established in [2], i.e. here we easily obtain the stability condition of the investigated model.

By fulfilling the condition (2.50) the stationary distribution of the merged model $(\pi(< i >) :< i > \in \tilde{S})$ is calculated as

$$\pi(< i >) = a^i b\pi(< 0 >), \qquad i = 1, 2, \ldots, \tag{2.51}$$

where

$$b := \sum_{i=m}^{m+n} \rho^0(i), \quad \pi(< 0 >) = \frac{1-a}{1-a+ab}.$$

In summary the following simple formulae for calculation of the desired QoS metrics (2.42) and (2.43) can be suggested:

$$P_{\mathrm{h}} \approx \frac{1}{1-a+ab}((1-a)E_{\mathrm{B}}(v_{\mathrm{h}}, m+n) + abE_{\mathrm{B}}^{\mathrm{T}}(v_{\mathrm{h}}, m+n)), \tag{2.52}$$

$$L_o \approx \frac{ab}{(1-a+b)(1-a)}, \tag{2.53}$$

where $E_{\mathrm{B}}^{\mathrm{T}}(\nu_{\mathrm{h}}, m+n)$ – the truncated Erlang's B-formula, i.e.

$$E_{\mathrm{B}}{}^{\mathrm{T}}(\nu_{\mathrm{h}}, m+n) := \frac{\nu_{\mathrm{h}}{}^{m+n}}{(m+n)!}\left(\sum_{i=m}^{m+n}\frac{\nu_{\mathrm{h}}{}^{i}}{i!}\right)^{-1}.$$

From (2.52) we conclude that P_{h} is the convex combination of two functions $E_{\mathrm{B}}(\nu_{\mathrm{h}}, m{+}n)$ and $E_{\mathrm{B}}^{\mathrm{T}}(\nu_{\mathrm{h}}, m{+}n)$. In other words, for any admissible values of number of channels and traffic loads the following limits for P_{h} may be proposed:

$$E_{\mathrm{B}}(\nu_{\mathrm{h}}, m+n) \le P_{\mathrm{h}} \le E_{\mathrm{B}}^{\mathrm{T}}(\nu_{\mathrm{h}}, m+n). \tag{2.54}$$

The limits in (2.54) will be achieved and are the same only in the special case $m=0$, i.e. when only h-calls arrive in the system.

The proposed approach also allows calculation of QoS metrics for the model with a limited queue of o-calls. Indeed, let the maximal length of the queue of o-calls be R, $R<\propto$. Then for any admissible values of number of channels and traffic loads in this system the stationary mode exists, i.e. in this case fulfilling of the ergodicity condition (2.50) is not required.

For the given model the number of splitting models is R+1 and their stationary distribution are calculated by (2.47) and (2.48). Making use of the above-described approach and omitting the known transformation the following expressions are determined to calculate the stationary distribution of the merged model:

$$\pi(< i >) = a^{i} b\pi(< 0 >),\ i = \overline{1, R}, \tag{2.55}$$

where

$$\pi(< 0 >) = \left(1 + ab\frac{1 - a^{R}}{1 - a}\right)^{-1}.$$

Since the approximate values of the QoS metrics for the model with a limited queue of o-calls are calculated as follows:

$$P_{\mathrm{h}}(R) \approx \pi(< 0 >)E_{\mathrm{B}}(\nu_{\mathrm{h}}, m+n) + (1 - \pi(< 0 >))E_{\mathrm{B}}^{\mathrm{T}}(\nu_{\mathrm{h}}, m), \tag{2.56}$$

$$L_{\mathrm{o}}(R) \approx ab\frac{1 - a^{R}(R + 1 + Ra)}{(1 - a)^{2}}\pi\,(< 0 >)\,, \tag{2.57}$$

$$W_{\mathrm{o}}(R) \approx \frac{L_{\mathrm{o}}\,(R)}{\lambda_{\mathrm{o}}\,(1 - P_{\mathrm{o}}\,(R))}\ , \tag{2.58}$$

where $P_{\mathrm{o}}(R)$ is the probability of loss of o-calls which is calculated by

$$P_{\mathrm{o}}(R) \approx \pi(R) or P_{\mathrm{o}}(R) \approx a^{R} b\pi(< 0 >). \tag{2.59}$$

Now we consider another shared reservation scheme of channels for h-calls in the model with a queue of o-calls. The main difference between this scheme and the previous one is reallocation of h-calls from the Secondary group to the Primary group. Upon release of a channel in the Primary group (either by an o-call or h-call) an h-call from the Secondary group is reallocated to the Primary group if and only if there are no o-calls in the queue. In other words reallocation of h-calls from the Secondary group to the Primary group is not allowed until the queue does not contain any o-calls. This reservation scheme is called Handoff calls Overflow from Primary to Secondary with Rearrangement (HOPSWR).

The system's state at any time is described by the two-dimensional vector $\mathbf{k} = (k_1, k_2)$, where k_1 is the total number of busy channels $k_1 = 0,1,\ldots,m+n$, and k_2 is the number of o-calls in the queue, $k_2 = 0,1,2,\ldots$. The model's state space S has the following view:

$$S = \bigcup_{i=0}^{n} S_i \, , S_i \bigcap S_j = \varnothing, \; i \neq j, \tag{2.60}$$

where

$$S_0 = \{(j,0) : j = 0, 1, \ldots, m\} \cup \{(m,j) : j = 1, 2, \ldots\},$$
$$S_i = \{(m+i, j) : j = 0, 1, 2, \ldots\}, \; i \geq 0.$$

On the basis of the adopted channel allocation scheme, we can conclude, that elements of the generating matrix $q(\mathbf{k}, \mathbf{k}')$, $\mathbf{k}, \mathbf{k}' \in S$ of appropriate 2-D MC are defined from the following relations (see Fig. 2.13):

$$q\left(\mathbf{k}, \mathbf{k}'\right) = \begin{cases} \lambda_o + \lambda_h & \text{if } k_1 \leq m-1, \; \mathbf{k}' = \mathbf{k} + \mathbf{e_1}, \\ \lambda_h & \text{if } k_1 \geq m, \; \mathbf{k}' = \mathbf{k} + \mathbf{e_1}, \\ \lambda_o & \text{if } k_1 \geq m, \; \mathbf{k}' = \mathbf{k} + \mathbf{e_2}, \\ k_1 \mu & \text{if } k_1 \leq m-1, \; \mathbf{k}' = \mathbf{k} - \mathbf{e_2}, \\ m\mu & \text{if } k_1 \geq m, \; \mathbf{k}' = \mathbf{k} - \mathbf{e_2}, \\ k_1 \mu & \text{if } k_1 \geq m, \; k_2 = 0, \; \mathbf{k}' = \mathbf{k} - \mathbf{e_1}, \\ (k_1 - m)\,\mu & \text{if } k_1 \geq m, \; k_2 \geq 0, \; \mathbf{k}' = \mathbf{k} - \mathbf{e_1}, \\ 0 & \text{in other cases.} \end{cases} \tag{2.61}$$

The desired QoS metrics of the system in this case are defined via stationary distribution of the model as follows:

$$P_h = \sum_{i=0}^{\infty} p\,(m+n, i), \tag{2.62}$$

$$L_o = \sum_{i=1}^{\infty} ip\,(i), \tag{2.63}$$

where $p\,(i) := \sum_{k \in S} p\,(\mathbf{k})\delta(k_2, i)$ are marginal probability mass functions.

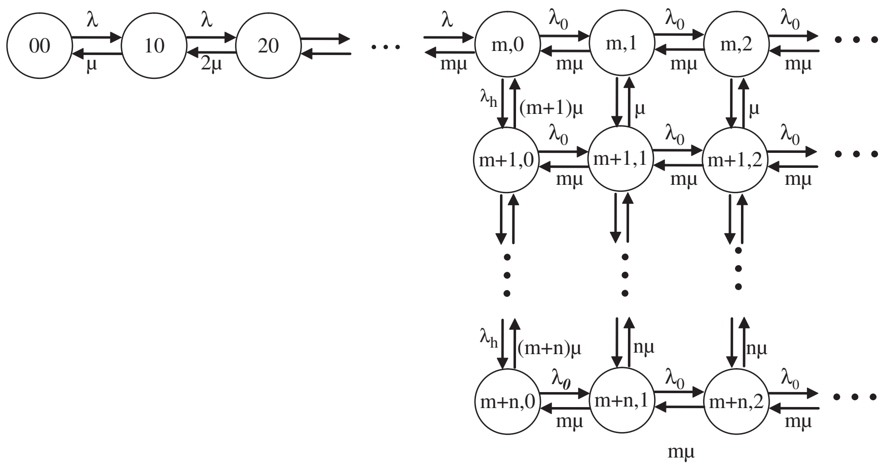

Fig. 2.13 State transition diagram for the HOPSWR model with an unlimited queue of patient o-calls

First, we will analyze the model of a macrocell with an infinite queue for o-calls. It is clear that the following condition holds true in macrocells $\lambda_o >> \lambda_h$.

The above-mentioned condition on relations of intensities of different types of calls allow the conclusion that transitions from state $\mathbf{k} \in S$ into state $\mathbf{k} + \mathbf{e_2} \in S$ occur more often than into state $\mathbf{k} + \mathbf{e_1} \in S$. In other words transition between states (microstates) inside classes S_i occurs more often than transitions between states from different classes. Because of this fact, classes of microstates S_i in (2.60) are depicted as isolated merged state $<i>$, and in initial state space S a known merging function is introduced. Thus, an appropriate 1-D MC with state space $\tilde{S} := \{< i >: i = 0, 1, 2, \ldots, n\}$ is constructed.

Elements of the generating matrix of split models with state space S_i, denoted as $q_i(\mathbf{k}, \mathbf{k}'),\ \mathbf{k}, \mathbf{k}' \in S_i$, are found through relations (2.61):

For the model with state space S_0:

$$q_0\left(\mathbf{k}, \mathbf{k}'\right) = \begin{cases} \lambda_o + \lambda_h & \text{if } k_1 \le m-1,\ \mathbf{k}' = \mathbf{k} + \mathbf{e_1}, \\ \lambda_o & \text{if } k_1 \ge m,\ \mathbf{k}' = \mathbf{k} + \mathbf{e_2}, \\ k_1\mu & \text{if } k_1 \le m-1,\ \mathbf{k}' = \mathbf{k} - \mathbf{e_1}, \\ m\mu & \text{if } k_1 \ge m,\ \mathbf{k}' = \mathbf{k} - \mathbf{e_2}, \\ 0 & \text{in other cases;} \end{cases} \tag{2.64}$$

For models with state space S_i, $i \ge 1$:

$$q_i\left(\mathbf{k}, \mathbf{k}'\right) = \begin{cases} \lambda_o, & \text{if } \mathbf{k}' = \mathbf{k} + \mathbf{e_2}, \\ m\mu, & \text{if } \mathbf{k}' = \mathbf{k} - \mathbf{e_2}, \\ 0 & \text{in other cases.} \end{cases} \tag{2.65}$$

The stationary probability of state $\mathbf{k} \in S$ inside the split model with state space S_i is denoted $\rho(\mathbf{k})$. Then with the aid of (2.64) and (2.65) the stationary distribution of split models can be found:

For the model with state space S_0:

$$\rho(i,j) = \begin{cases} \dfrac{v^i}{i!}\rho_0 & \text{if } 1 \le i \le m,\ j = 0, \\ \dfrac{v^m}{m!}\tilde{v}_{\mathrm{o}}{}^j \rho_0 & \text{if } i = m,\ j \ge 1, \end{cases} \tag{2.66}$$

where

$$\rho_0 = \left(\sum_{i=0}^{m} \frac{v^i}{i!} + \frac{v^m}{m!} \cdot \frac{\tilde{v}_{\mathrm{o}}}{1 - \tilde{v}_{\mathrm{o}}} \right)^{-1}, \ v = v_{\mathrm{o}} + v_{\mathrm{h}},\ \tilde{v}_{\mathrm{o}} = v_{\mathrm{o}}/m;$$

For models with state space $S_i,\ i = 1, 2, \ldots, n$:

$$\rho\,(m+i,j) = \tilde{v}_{\mathrm{o}}{}^j\,(1 - \tilde{v}_{\mathrm{o}})\,,\ j = 0, 1, 2, \ldots\ . \tag{2.67}$$

Upon derivation of formula (2.67) we obtain an intuitively clear and simple ergodicity condition of the model: $\tilde{v}_{\mathrm{o}} < 1$. It is seen that this condition does not depend on h-call load.

Transition intensities between merged states $< i >,\ < i' > \in \tilde{S}$ that are denoted $q\left(< i >, < i' >\right) \in \tilde{S}$ are found with the aid of (2.61), (2.66), and (2.67), i.e.

$$q\left(< i >,\ < i' >\right) = \begin{cases} \lambda_{\mathrm{h}} c & \text{if } i = 0,\ i' = 1, \\ \lambda_{\mathrm{h}} & \text{if } 1 \le i \le n-1,\ i' = i+1, \\ (m+i)\,(1 - \tilde{v}_o) + i\mu\tilde{v}_o & \text{if } 1 \le i \le n,\ i' = i-1, \\ 0 & \text{in other cases} \end{cases} \tag{2.68}$$

where

$$c := 1 - \rho_0 \sum_{j=0}^{m-1} \frac{v^j}{j!} \quad .$$

With the ergodicity condition of the system holding true, from relation (2.68) the stationary distribution of a merged model is defined as follows:

$$\pi(< i >) = c \prod_{j=1}^{i} \frac{\lambda_{\mathrm{h}}{}^j}{(m+j)\,(1 - \tilde{v}_{\mathrm{o}}) + j\mu\tilde{v}_{\mathrm{o}}} \pi(< 0 >), \qquad i = 1, 2, \ldots, n, \tag{2.69}$$

where

$$\pi\,(<0>) = \left(1 + c\sum_{i=1}^{n}\prod_{j=1}^{i}\frac{\lambda_{\mathrm{h}}{}^{j}}{(m+j)\,(1-\tilde{\nu}_{\mathrm{o}}) + j\mu\tilde{\nu}_{\mathrm{o}}}\right)^{-1} .$$

After required mathematical transformations we obtain the following approximate formulae for calculating the QoS metrics of the initial model:

$$P_{\mathrm{h}} \approx \pi\,(<n>)\ , \tag{2.70}$$

$$L_{\mathrm{o}} \approx \frac{\tilde{\nu}_{\mathrm{o}}}{1-\tilde{\nu}_{\mathrm{o}}}\ , \tag{2.71}$$

Note 2.4. From formula (2.71) it can be seen that the average queue length of o-calls and appropriate average queue waiting time does not depend on the load of h-calls. This has a simple explanation in macrocells, since in such cells the intensity of o-calls is much higher than the intensity of h-calls, hence the load of h-calls on the Primary Group of channels is negligible.

The supposed method allows calculation of QoS metrics of a macrocell with a finite buffer for o-calls as well. Let $R, R < \infty$ be the maximum allowable length of the queue for o-calls. Then at any structural and load parameter values there is a stationary regime in the system, which has no need for ergodicity condition $\tilde{\nu}_{\mathrm{o}} < 1$ to be true.

In this case the state space of the initial model $S(R)$ is defined as:

$$S(R) = \bigcup_{i=0}^{n} S_i(R)\ , S_i(R) \cap S_j(R) = \emptyset,\ i \neq j, \tag{2.72}$$

where

$$\begin{aligned} S_0(R) &= \{(j,0) : j = 0,1,\ldots,m\} \cup \{(m,j) : j = 1,2,\ldots,R\}\,, \\ S_i(R) &= \ \{(m+i,j) : j = 0,1,2,\ldots,R\}\,. \end{aligned}$$

Using the above-described method and dropping in-between mathematical transformations, we determine that for the given model the stationary distribution of the merged model is defined as:

$$\pi_R(<i>) = c\prod_{j=1}^{i}\frac{\lambda_{\mathrm{h}}{}^{j}}{(m+j)\,d + j\mu\,(1-d)}\,\pi_R(<0>),\ i = 1,2,\ldots,n, \tag{2.73}$$

where

$$\pi_R\left(<0>\right)=\left(1+c\sum_{i=1}^{n}\prod_{j=1}^{i}\frac{\lambda_h{}^j}{(m+j)\,d+j\mu\,(1-d)}\right)^{-1}\;,\;d:=\frac{1-\tilde{v}_{\mathrm{o}}}{1-\tilde{v}_{\mathrm{o}}^{R+1}}\;.$$

Consequently, estimated QoS values for the model with a finite queue are calculated as follows:

$$P_{\mathrm{h}}(R)\approx\pi_R(<n>),\tag{2.74}$$

$$L_{\mathrm{o}}\left(R\right)\approx d\sum_{i=1}^{R}i\tilde{v}_{\mathrm{o}}^{i}\;,\tag{2.75}$$

$$W_{\mathrm{o}}\left(R\right)\approx\frac{L_{\mathrm{o}}\left(R\right)}{\lambda_0\left(1-P_{\mathrm{o}}\left(R\right)\right)}\;,\tag{2.76}$$

where $P_{\mathrm{o}}(R)$ is the loss probability of o-calls, which, for this model, is calculated as shown below:

$$P_{\mathrm{o}}\left(R\right)\approx\tilde{v}_{\mathrm{o}}^{R}\left(\frac{v^m}{m!}\pi_R\left(<0>\right)+d\left(1-\pi_R\left(<0>\right)\right)\right)\;.\tag{2.77}$$

Note 2.5. From formula (2.75) it can be seen that the average queue length of o-calls for the model with a finite queue also does not depend on h-call load. This is obvious in macrocells. However, in this model average queue waiting of o-calls depends on h-call load [see formula (2.76)]. But on the other hand, this dependency occurs only at small values of buffer for o-calls and disappears with growing buffer size.

Now we will consider calculation of QoS metrics for micro-cell models. As was noted above the following condition $\lambda_{\mathrm{o}} << \lambda_{\mathrm{h}}$ holds true in microcells. This means that transition from state $\mathbf{k}\in S$ into state $\mathbf{k}+\mathbf{e_1}\in S$ occurs more often than into state $\mathbf{k}+\mathbf{e_2}\in S$. In this case the following splitting of state space S is studied:

$$S=\bigcup_{i=0}^{\infty}\tilde{S}_i\;,\;\tilde{S}_i\bigcap\tilde{S}_{i'}=\emptyset,\;i\neq i'\;,\tag{2.78}$$

where

$$\tilde{S}_0=\{(j,0):j=0,1,\ldots,m+n\}\,,\;\tilde{S}_i=\{(j,i):j=m,m+1,\ldots,m+n\}\,,i\geq 1.$$

According to the above-mentioned condition on relations of intensities for different types of calls, in (2.78) transitions between microstates inside $\tilde{S}_i$ classes occur more often than between states from different classes.

Since the selected scheme of splitting of the initial state space completely defines the structure of the split and merged models, then further procedures for approximate calculation of the stationary distribution of the initial model are obvious. That is why below we drop some known interim steps for solution of this problem.

The stationary distribution of the split model with state space $\tilde{S}_0$ coincides with the appropriate stationary distribution of the Erlang model *M/M/m+n/*0 with state-dependent rate $\lambda(j)$, i.e.

$$\lambda\,(j) = \begin{cases} \lambda_{\mathrm{o}} + \lambda_{\mathrm{h}} & \text{if } j \le m \\ \lambda_{\mathrm{h}} & \text{if } j \ge m. \end{cases}$$

Thus, the stationary distribution of the split model with state space $\tilde{S}_0$ is calculated as follows:

$$\rho_0\,(j) = \begin{cases} \dfrac{v^j}{j!}\rho_0\,(0) & \text{if } j = 1, \dots, m, \\ \left(\dfrac{v}{v_{\mathrm{h}}}\right)^m \cdot \dfrac{{v_{\mathrm{h}}}^j}{j!}\rho_0\,(0) & \text{if } j = m+1, \dots, m+n, \end{cases} \tag{2.79}$$

where

$$\rho_0\,(0) = \left(\sum_{j=0}^{m} \frac{v^j}{j!} + \left(\frac{v}{v_{\mathrm{h}}}\right)^m \sum_{j=m+1}^{m+n} \frac{{v_{\mathrm{h}}}^j}{j!} \right)^{-1} .$$

Stationary distributions of split models with state spaces $\tilde{S}_i,\ i \ge 1$, are equal and coincide with the appropriate distribution of the classical Erlang model *M/M/n/*0 with load v_{h}.

Since the number of micro-state classes in (2.78) is infinite, then in this case the merged model represents 1-D MC with infinite state space $S' = \{< i >: i = 0, 1, 2, \dots\}$. Here merged state $< i >$ comprises all microstates from class $\tilde{S}_i$. Then, considering (2.61) and the above-mentioned facts about stationary distributions inside split models, elements of the generating matrix of this given merged model are found:

$$q\left(< i >, < i' >\right) = \begin{cases} \lambda_{\mathrm{o}} f & \text{if } i = 0,\ i' = 1 \\ \lambda_{\mathrm{o}} & \text{if } i \ge 1,\ i' = i+1 \\ m\mu & \text{if } i' = i-1 \\ 0 & \text{in other cases,} \end{cases} \tag{2.80}$$

where

$$f := \sum_{k=m}^{m+n} \rho_0\,(k)\ .$$

Then from relation (2.80) the ergodicity condition $\tilde{v}_o < 1$ of the model is found which corresponds exactly to the similar condition found for the macro-cell model. Upon meeting the ergodicity condition from (2.80) the stationary distribution of the merged model is found:

$$\pi\left(< i >\right) = f\tilde{v}_o^i \pi\left(< 0 >\right),\ i \geq 1, \tag{2.81}$$

where

$$\pi\left(< 0 >\right) = \frac{1 - \tilde{v}_o}{1 - \tilde{v}_o + f\tilde{v}_o}\ .$$

After some mathematical transformation the following relations for approximate calculation of the QoS metrics of the micro-cell model with an unlimited queue for o-calls are found:

$$P_h \approx \rho_0\left(m+n\right)\pi\left(< 0 >\right) + E_B\left(v_h, n\right)\left(1 - \pi\left(< 0 >\right)\right), \tag{2.82}$$

$$L_o \approx \frac{1 - \pi\left(< 0 >\right)}{1 - \tilde{v}_o}\ . \tag{2.83}$$

As in the previous case, for the micro-cell model QoS metrics can also be found when there is finite buffer $R, R < \infty$ for o-calls. Then at any structural and load parameter values there is a stationary regime in the system. Dropping well-known steps in solution of this problem, below are given the final formulae for approximate calculation of QoS metrics of the micro-cell model with a finite queue:

$$P_h\left(R\right) \approx \rho_0\left(m+n\right)\pi_R\left(< 0 >\right) + E_B\left(v_h, n\right)\left(1 - \pi_R\left(< 0 >\right)\right), \tag{2.84}$$

$$L_o \approx f\pi_R\left(< 0 >\right)\sum_{i=1}^{R} i\tilde{v}_o^i\ , \tag{2.85}$$

$$P_o\left(R\right) \approx \pi_R\left(< R >\right)\ , \tag{2.86}$$

$$W_o \approx \frac{L_o\left(R\right)}{\lambda_o\left(1 - P_o\left(R\right)\right)}\ . \tag{2.87}$$

Here stationary distribution of the merged model is calculated from the following:

$$\pi_R\left(< i >\right) = f\tilde{v}_o^i \pi_R\left(< 0 >\right),\ i = 1, 2, \ldots, R, \tag{2.88}$$

where

$$\pi_R\left(< 0 >\right) = \frac{1 - \tilde{v}_o}{1 - \left(1 - f\right)\tilde{v}_o - f\tilde{v}_o^{R+1}}\ .$$

2.2.3 Numerical Results

Let us first examine the results of numerical experiments for both schemes with isolated channel reservation. To keep it brief only models for the microcell are shown (in which $\lambda_o << \lambda_h$) in two series of experiments. In both series the input data is chosen as follows: $m+n=20$, $\lambda_o = 0.5$, $\lambda_h = 10$. Also in one series it is assumed that $\mu = 0.8$, in another – $\mu = 1$.

In Fig. 2.14 the dependency of function P_h on the number of reserved channels is shown in the HOPS scheme. It is seen from this figure that for given input data this function decreases in a particular range of value n, thereafter it increases. This fact has the following explanation. With the increase of n (i.e. decrease of m) loss intensity of h-calls from the Primary Group increases and consequently the intensity of these calls in the Secondary Group of channels increases. Thus, in Erlang's B-formula load ($\tilde{v}_h$) and number of channels (n) increases simultaneously and therefore, it is impossible to foresee the P_h function's behavior from the number of reserved channels in this scheme. In other words determination of the type of P_h function for concrete values of the model's parameters requires appropriate numerical experiments.

Dependency of L_o on the number of reserved channels is shown in Fig. 2.15. This function is an increasing function at constant intensity of input traffic, since increase of the number of reserved channels leads to decrease of the number of channels in Primary Group, i.e. average queue length of o-calls increases. The same type of dependency on the number of reserved channels is demonstrated by W_o.

Figures 2.16 and 2.17 demonstrate the dependency of QoS parameters of the model on the number of reserved channels in the HOSP scheme. It is seen from

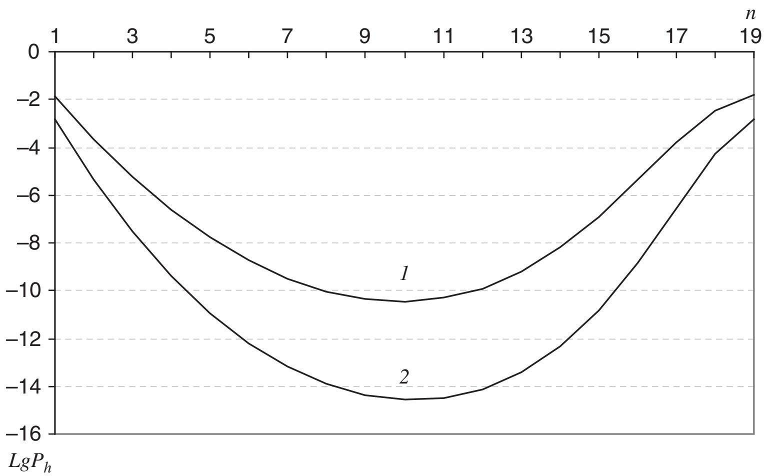

Fig. 2.14 P_h versus n for the HOPS model in the case where $m+n=20$, $\lambda_o=0.5$, $\lambda_h=10$; $1\mu=0.8$; $2\text{–}\mu=1$

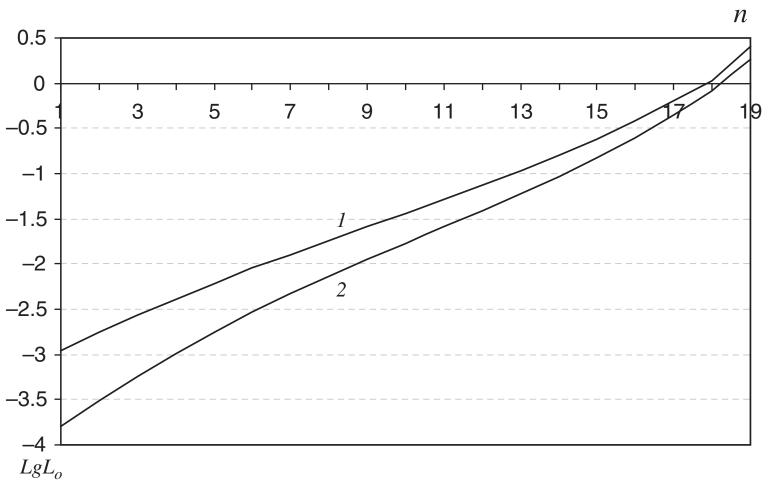

Fig. 2.15 L_o versus n for the HOPS model in the case where $m+n=20$, $\lambda_o=0.5$, $\lambda_h=10$; 1–$\mu=0.8$; 2–$\mu=1$

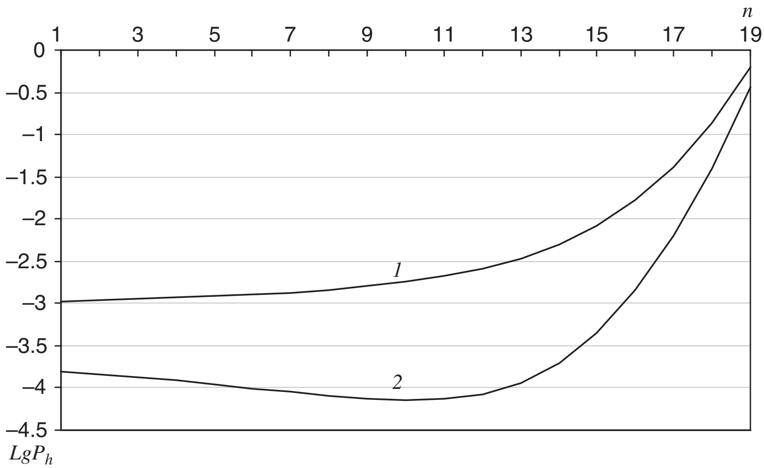

Fig. 2.16 P_h versus n for the HOSP model in the case where $m+n=20$, $\lambda_o=0.5$, $\lambda_h=10$; 1–$\mu=0.8$; 2–$\mu=1$

Fig. 2.16 that at $\mu=0.8$ the function P_h strictly increases in the whole range of the reserved channels value, whereas at $\mu=1$ it decreases within an interval [1, 10] and then increases again. Note that such behavior is specific to the given data and for other input data it will be different. As a matter of fact, in this scheme, the intensity of h-calls to the Primary Group of channels ($\hat{\lambda}_h$) decreases upon the increase of the number of reserved channels. At the same time, the number of channels in this group also decreases, i.e. in the queuing system *M/M/m/*∞ both input traffic intensity and

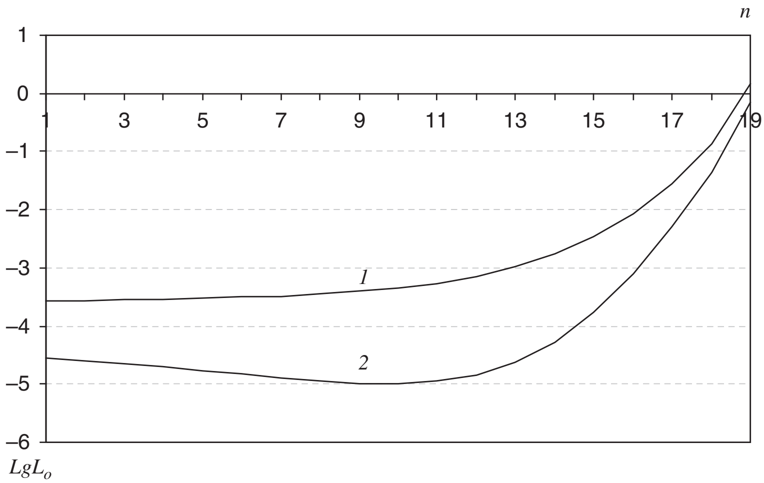

Fig. 2.17 L_o versus n for the HOSP model in the case where $m+n=20$, $\lambda_o=0.5$, $\lambda_h=10$; $1–\mu=0.8$; $2–\mu=1$

number of channels decrease simultaneously. In other words it is difficult to predict P_h behavior since it essentially depends on concrete values of number of channels as well as load parameters. Consequently, as in the case with the HOPS scheme the definition of P_h behavior at concrete structural and load parameter values requires appropriate numerical experiments.

Dependency of L_o on the number of reserved channels for the HOSP scheme is shown in Fig. 2.17. Unlike the HOPS scheme, in this scheme the monotony of this QoS parameter (as well as QoS parameter W_o) is not guaranteed at any load and structural parameter values of the model. This is explained also by the fact that the increase of the number of reserved channels simultaneously decreases the intensity of input traffic and number of channels in the queuing system $M/M/m/\infty$.

Note that in both schemes an increase of handling intensity has a beneficial effect on all QoS parameters, i.e. loss probabilities of h-calls and queue length and hence waiting time of o-calls (see Figs. 2.14, 2.15, 2.16, and 2.17).

Remarkably, from theoretical considerations we can deduce that loss probability of h-calls is less in the HOPS scheme than in the HOSP scheme, whereas queue length for o-calls is less in the HOSP scheme hence average waiting time is less than in HOPS. The first part of this clause is explained by the fact that the initial search of an empty channel in an "alien" group of channels increases the chances for h-calls to be accepted. The second part of the clause is explained by the fact that total load of incoming traffic in the Primary Group of channels in the HOPS scheme $(\lambda_o+\lambda_h)$ is higher than that of the HOSP scheme $(\lambda_o+\hat{\lambda})$. These are clearly demonstrated in Table 2.5.

From the table we can deduce that if the research aims at decreasing of loss probability of h-calls, then HOPS scheme has greater advantages compared to the

Table 2.5 Comparison of QoS metrics for the models HOPS and HOSP in the case where $m+n=20$, $\lambda_o=0.5$, $\lambda_h=10$, $\mu=0.8$

	P_h		L_o		W_o	
n	HOPS	HOSP	HOPS	HOSP	HOPS	HOSP
1	2.18945E-04	1.05607E-03	1.09198E-03	2.71982E-04	2.18397E-03	5.43963E-04
2	2.18945E-04	1.08889E-03	1.75272E-03	2.76861E-04	3.50543E-03	5.53721E-04
3	5.72726E-06	1.12561E-03	2.72991E-03	2.82620E-04	5.45982E-03	5.65239E-04
4	2.42487E-07	1.16770E-03	4.14018E-03	2.89725E-04	8.28036E-03	5.79451E-04
5	1.66361E-08	1.21737E-03	6.13771E-03	2.98889E-04	1.22754E-02	5.97777E-04
6	1.83783E-09	1.27779E-03	8.93135E-03	3.11207E-04	1.78627E-02	6.22414E-04
7	3.26134E-10	1.35421E-03	1.28126E-02	3.28418E-04	2.56252E-02	6.56836E-04
8	9.35392E-11	1.45476E-03	1.82008E-02	3.53361E-04	3.64017E-02	7.06723E-04
9	4.40913E-11	1.59290E-03	2.57188E-02	3.90880E-04	5.14376E-02	7.81759E-04
10	3.51240E-11	1.79172E-03	3.63195E-02	4.49701E-04	7.26389E-02	8.99401E-04
11	4.91568E-11	2.09295E-03	5.15097E-02	5.46673E-04	1.03019E-01	1.09335E-03
12	1.26751E-10	2.57717E-03	7.37537E-02	7.17194E-04	1.47507E-01	1.43439E-03
13	6.33316E-10	3.41292E-03	1.07233E-01	1.04361E-03	2.14466E-01	2.08721E-03
14	6.36998E-09	4.98949E-03	1.59309E-01	1.74254E-03	3.18619E-01	3.48508E-03
15	1.27549E-07	8.32246E-03	2.43389E-01	3.47529E-03	4.86779E-01	6.95058E-03
16	4.41134E-06	1.64976E-02	3.84356E-01	8.66709E-03	7.68711E-01	1.73342E-02
17	1.73348E-04	4.09669E-02	6.28121E-01	2.86297E-02	1.25624E+00	5.72594E-02
18	3.48161E-03	1.37682E-01	1.06891E+00	1.39320E-01	2.13781E+00	2.78640E-01
19	1.60324E-02	6.36581E-01	2.55009E+00	1.45224E+00	5.10018E+00	2.90447E+00

HOSP scheme, since in some individual cases this parameter is almost 10^8-times better. Likewise using the HOSP scheme allows decreasing average queue length for o-calls and hence the average waiting time of o-calls is almost 10^2-times shorter compared to the HOPS scheme.

This table also suggests a choice of appropriate scheme dependent on load and structural parameters of a model. The solution of such problems implies defining requirements for QoS parameters and thus finding a scheme that will meet these requirements. For instance, if for given selected input data one needs to meet the following requirement $P_h \leq 10^{-4}$, then the HOSP scheme will not allow doing this at any values of n, whereas in the HOPS scheme this requirement is met at $n=3,4,\ldots,16$.

It is important to note that the obtained results correspond exactly to those from [9], where the values were calculated using the fairly complex theory of a multidimensional generating function. Moreover, this work only suggests a solution for models with infinite queues of o-calls, whereas the suggested approach works with finite queues as well.

Numerical experiments were also conducted separately for macro-cell models (where $\lambda_o >> \lambda_h$) and for models with a symmetrical load of different types of calls (where $\lambda_o = \lambda_h$) with different schemes of channel assignment. For brevity these results are not given here. We will just note that the above-mentioned behavior of QoS parameters is demonstrated for any type of cell.

Now we will consider the results of numerical experiments for schemes with reassignment of channels. First we will consider the HRMA model. In numerical experiments for the model with an unlimited queue of o-calls initial data are chosen as follows: $N = 40$, $\lambda_h = 15$, $\mu = 1$. As expected the function P_h (Fig. 2.18) decreases while both functions L_o (Fig. 2.19) and W_o (Fig. 2.20) increase versus

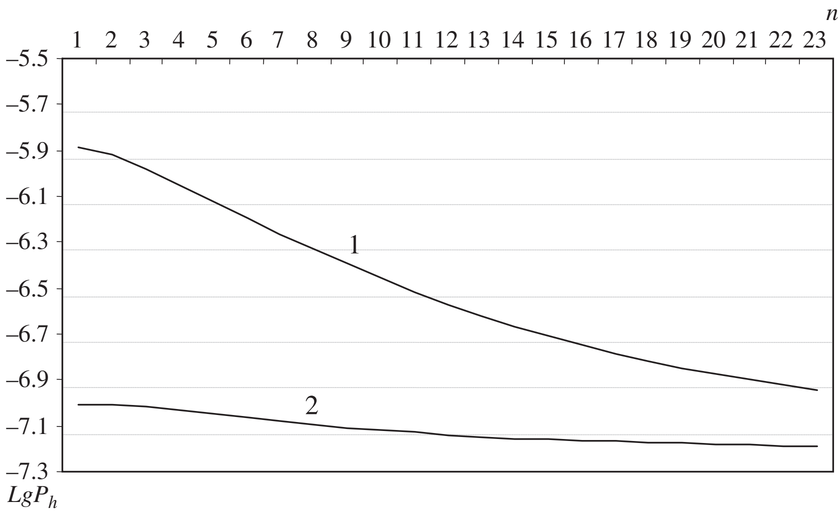

Fig. 2.18 P_h versus n for the HRMA model in the case where $N = 40$, $\lambda_h = 15$, $\mu = 1$; 1–$\lambda_o = 4$, 2–$\lambda_o = 2$

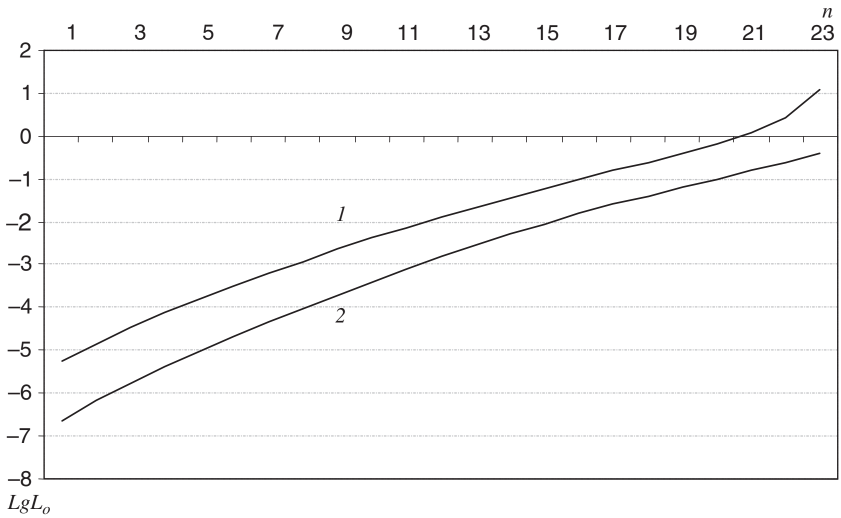

Fig. 2.19 L_o versus n for the HRMA model in the case where $N = 40$, $\lambda_h = 15$, $\mu = 1$; 1–$\lambda_o = 4$, 2–$\lambda_o = 2$

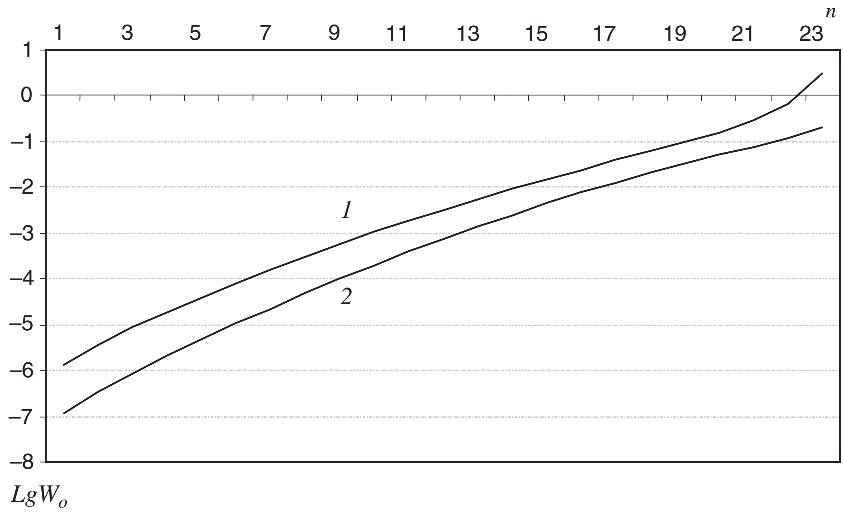

Fig. 2.20 W_o versus n for the HRMA model in the case where $N=40$, $\lambda_h=15$, $\mu=1$; 1–$\lambda_o=4$, 2–$\lambda_o=2$

the number of guard channels. Therewith all functions are increasing versus o-call load. Note that for the indicated initial data the ergodicity property of the model is violated at $n \geq 24$ therefore in graphs the values of parameter n are shown in interval [1, 23]. Numerical experiments were executed for the model with a limited queue of o-calls also and analogous results were found.

The accuracy of the proposed approximate formulae for the given model was also estimated. Exact values of QoS are considered to be those calculated by formulae which were proposed in [2]. Note that both the approximate and exact results for P_h are almost identical. Some results of the comparison are given in Table 2.6.

It is worth noting that sufficiently high accuracy exists even for the initial data not satisfying the known assumption concerning the ratios of traffic loads of o- and

Table 2.6 Comparison of exact and approximate values of QoS metrics for the model HRMA with patient o-calls in the case where $\lambda_o=1$, $\lambda_h=10$, $\mu=2.0$

		P_h	
$m+n$	n	EV	AV
20	3	8.43E-07	2.98E-07
20	5	7.02E-07	2.97E-07
20	7	5.88E-07	2.96E-07
30	3	1.92E-13	2.79E-14
30	5	1.59E-13	2.75E-14
30	7	1.32E-13	2.72E-14
30	20	4.07E-14	2.71E-14
40	7	1.07E-21	9.42E-23
40	20	3.17E-22	8.54E-23

h-calls. So, for example, for the model with 20 channels and load parameters $\lambda_o = \lambda_h = 7$, $\mu = 2$ maximal difference between EV and AV occur at $n = 1$, i.e. in this case the exact value of P_h equals 1.91E-05 while its approximate value equals 4.13E-06. Similar results were obtained for other initial data.

In Figs. 2.21, 2.22, 2.23, 2.24, and 2.25 the dependency of QoS metrics on number of guard channels for h-calls (i.e. number of channels in the Secondary group)

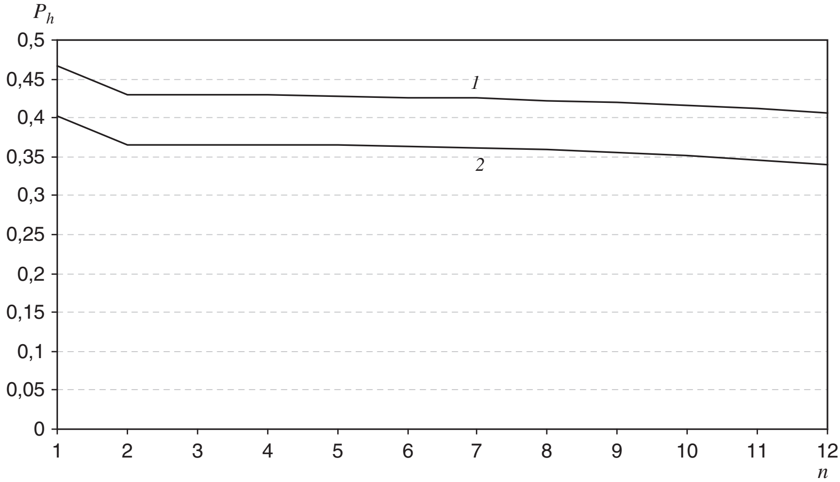

Fig. 2.21 P_h versus n for the HOPSWR model of the microcell in the case where $m+n = 15$, $\lambda_o = 5$, $\mu = 2$; 1–$\lambda_h = 40$, 2–$\lambda_h = 35$

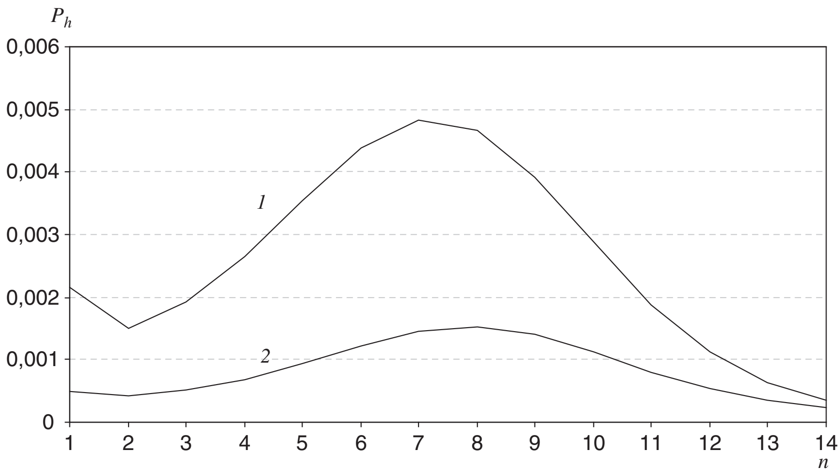

Fig. 2.22 P_h versus n for the HOPSWR model of the microcell in the case where $m+n = 15$, $\lambda_h = 20$, $\mu = 4$; 1–$\lambda_o = 5$, 2–$\lambda_o = 2$

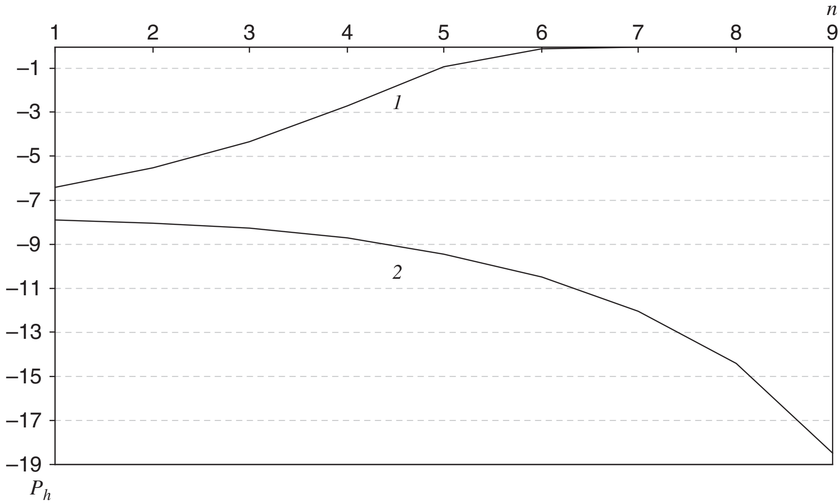

Fig. 2.23 P_{h} versus n for the HOPSWR model of the macrocell in the case where $m+n=10$, $\lambda_0=15$, $\mu=20$; $1–\lambda_{\mathrm{h}}=3$, $2–\lambda_{\mathrm{h}}=0.9$

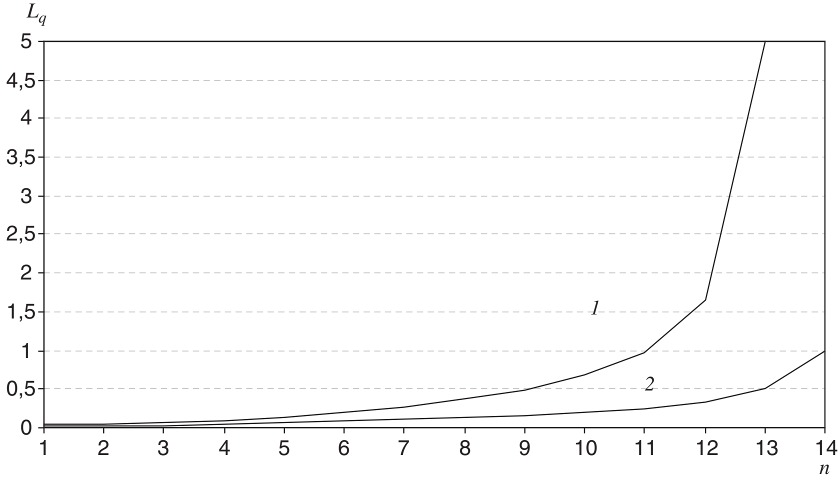

Fig. 2.24 L_q versus n for the HOPSWR model of the macrocell in the case where $m+n=15$, $\mu=2$; $1–\lambda_{\mathrm{o}}=5$, $\lambda_{\mathrm{h}}=15$, $2–\lambda_{\mathrm{o}}=2$, $\lambda_{\mathrm{h}}=20$

in the model HOPSWR are given. They completely meet theoretical expectations. So, the P_{h} function's shape in the microcell is given in Figs. 2.21 and 2.22. It can be seen from Fig. 2.21 that at fixed initial values of the model, this function systematically decreases. This is explained by the fact that at these values of cell parameters h-calls use channels of the Primary group poorly, i.e. they in fact use

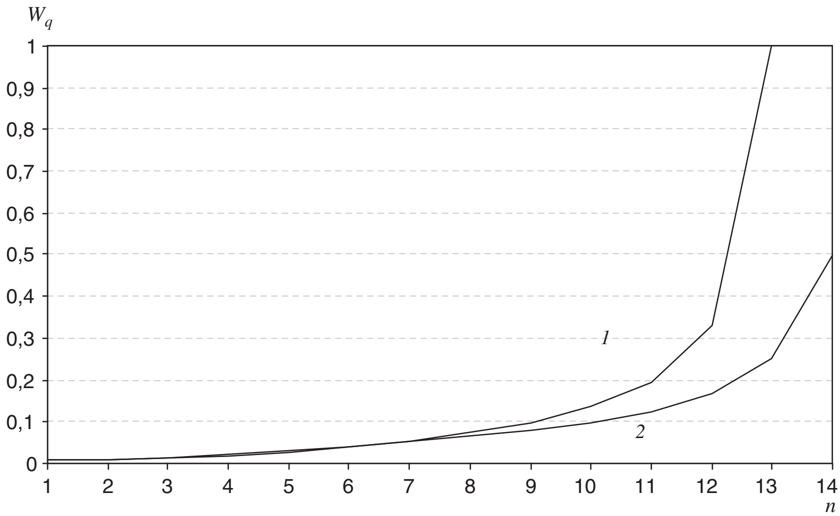

Fig. 2.25 W_q versus n for the HOPSWR model of the macrocell in the case where $m+n=15$, $\mu=2$; 1–$\lambda_o=5$, $\lambda_h=15$, 2–$\lambda_o=2$, $\lambda_h=20$

channels from the Secondary group, and, therefore, with the increase of the latter the given function decreases. The picture is different in Fig. 2.22. Here at low values of the number of channels in the Secondary group, h-calls use all available channels poorly, however with an increase of the number of channels in the Secondary group, total channel usage rate improves, and consequently, P_h has the shape we see in Fig. 2.22. Noticeably, as was expected, the function increases in a monotonic fashion depending on traffic loads of both types of calls.

Figure 2.23 depicts P_h function shape in the macrocell. Here both cases of function decrease and increase upon variation of the number of channels in Secondary group are shown. The load of new calls is constant. In other words, at low load h-calls mainly occupy channels in the Secondary group, therefore, with an increase of the number of such channels, P_h decreases. And at higher load h-calls occupy channels from both groups, but at the given initial values the total channel occupation rate of h-calls worsens, hence with an increase of the number of channels in the Secondary group P_h increases.

In both types of cells functions L_o and W_o increase with respect to the number of channels in the Secondary group regardless of traffic loads and total number of channels. Figures 2.24 and 2.25 depict these functions' shape for the macrocell. They have similar shapes for the microcell as well.

Analysis of numerical experiments reveals that the problem of optimal distribution of channels with the aim of compliance with QoS parameters of different types of calls is not a trivial one, consequently, its solution in each concrete case will require special investigation. This is caused by P_h behavior at various relations of traffic intensities (see Figs. 2.21, 2.22, and 2.23).

Table 2.7 Comparison of exact and approximate values of QoS metrics for the model HOPSWR with patient o-calls in the case where $m+n=15$, $\lambda_o=5$, $\lambda_h=15$, $\mu=2$

	P_h		L_q		W_q	
n	EV	AV	EV	AV	EV	AV
1	0.0509	0.0506	0.0401	0.0358	0.0098	0.0072
2	0.0445	0.0436	0.0179	0.0366	0.0088	0.0073
3	0.0498	0.048	0.0625	0.0609	0.0156	0.0122
4	0.0588	0.0534	0.0989	0.0945	0.0201	0.0189
5	0.0601	0.0583	0.1542	0.1395	0.0302	0.0279
6	0.0626	0.0612	0.2002	0.1978	0.0411	0.0396
7	0.0655	0.0611	0.2823	0.2724	0.0600	0.0545
8	0.0599	0.0576	0.3987	0.3678	0.0765	0.0736
9	0.0545	0.0511	0.5002	0.4943	0.0856	0.0989
10	0.0478	0.0428	0.6987	0.6757	0.1246	0.1351
11	0.0352	0.0341	0.9899	0.9792	0.2003	0.1958
12	0.0279	0.0259	1.6803	1.6576	0.2998	0.3315

Another aim of numerical experiments is to measure the accuracy of suggested formulae. Thus, the approximate results for macro- and microcells are almost completely identical to the results of [10] which are considered exact when $\mu_o=\mu_h$. Some comparisons for the microcell are given in Table 2.7. Noticeably, the accuracy of given formulae increases with the increase of intensity ratios of different types of calls. Similar results are also achieved for other parameters of the models studied.

It is important to note that the given approximate formulae have low accuracy at close values of load parameters of heterogeneous calls (i.e. when $\lambda_o \approx \lambda_h$ and $\mu_o \approx \mu_h$) and therefore cannot be used in QoS research for cells where load parameters of original and handover calls do not differ substantially.

2.3 Conclusion

In this chapter simple numerical procedures for calculation of QoS metrics in wireless networks are proposed, where well-known shared channel reservation schemes for prioritized h-calls and either limited or unlimited queues for homogenous calls are used. It is important to note that unlike the classical models of the above communication networks, here new and handover calls are assumed not to be identical in time of radio channel occupancy. The works [3, 5] applied the analytical models of a cell with an unlimited queue of h-calls under the assumption that the duration of the degradation interval has an exponential distribution. The analogous model with a limited queue of h-calls and infinite degradation interval was studied in [11]. The works [6, 7] suggested numerical algorithms for studying models with a limited length of queue for h-calls. Therewith consideration was given to models with patient [6] and impatient calls [7]. In [1] signal-flow graphs and Mason's formula were used to obtain the blocking probabilities of o- and h-calls and mean waiting times in the model with a limited queue of both kinds of calls and reneging/dropping of waiting calls. In all the above works o- and h-calls were assumed to be identical in terms of channel occupancy time. Computational procedures to calculate QoS metrics of investigated networks with an unlimited queue of patient or impatient h-calls were proposed in [4]. In the latter work it was assumed that o- and h-calls were nonidentical.

Models of investigated networks with unlimited queues of o-calls were studied in [2, 9]. For calculation of QoS metrics of the HRMA model, a matrix-geometric approach was used in [2], while generation of function method in conjunction with matrix spectral tools for the model HOPSWR was used in [9]. Note that these methods allow for calculation of QoS metrics in the case of an unlimited queue of o-calls only. Approximate methods to calculate QoS metrics for the model HOPSWR were proposed in [10]. It is important to note that unlike the methods that were proposed in [2, 9] the approximate method allows one to investigate models with limited queues of o-calls as well. The proposed method can also be used for research into two-dimensional models where more sophisticated channel reservation schemes are used as well as for models with a finite buffer for both kinds of impatient calls, for example see [1, 12]. Simple algorithms for computing the QoS metrics of HOPS and HOSP schemes for channel assignment were proposed in [8].

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