

Chapter 2

Single Phase Flow in Pneumatic Conveying Systems

Abstract This Chapter on single phase flow provides the basis for pneumatic conveying of solids in laying out the fundamental premises of single phase gas flow with both thermodynamic and fluid dynamic expressions. Often times in dilute pneumatic conveying the energy loss can be dominated by the gas phase flow with a small contribution due to the presence of solids.

Keywords Gas phase · Thermodynamics · Isothermal · Adiabatic · Viscosity · Frictional loss

2.1 Introduction

The design of any pneumatic conveying system requires a basic understanding of air flow in pipes and ducts as well as an appreciation of the various prime movers used to supply air.

2.2 Definitions

The following definitions will be used in this text.

2.2.1 Free Air

Free air is defined as air at the prevailing atmospheric conditions at the particular site in question. The density of air will vary according to site conditions and locality. As such, free air is **not** a standard condition. In Table 2.1 the variation of atmospheric pressure with altitude is provided.

Table 2.1 Variation of standard atmospheric pressure according to NASA [1]. Values below sea level have been extrapolated

Altitude (m)	Pressure (bar)	Temperature (°C)	Density (kg/m ³)
−1,000	1.138	21.5	1.345
−800	1.109	20.2	1.317
−600	1.080	18.9	1.288
−400	1.062	17.6	1.272
−200	1.038	16.3	1.249
0	1.013	15.0	1.225
100	1.001	14.4	1.213
200	0.989	13.7	1.202
300	0.978	13.1	1.190
400	0.966	12.4	1.179
500	0.955	11.8	1.167
600	0.943	11.1	1.156
800	0.921	9.8	1.134
1,000	0.899	8.5	1.112
1,200	0.877	7.2	1.090
1,400	0.856	5.9	1.069
1,600	0.835	4.6	1.048
1,800	0.815	3.3	1.027
2,000	0.795	2.0	1.007
2,200	0.775	0.7	0.986
2,400	0.756	−0.6	0.966
2,600	0.737	−1.9	0.947
2,800	0.719	−3.2	0.928
3,000	0.701	−4.5	0.909
3,200	0.683	−5.8	0.891
3,400	0.666	−7.1	0.872
3,600	0.649	−8.4	0.854
3,800	0.633	−9.7	0.837
4,000	0.616	−11.0	0.819
5,000	0.540	−17.5	0.736
6,000	0.472	−24.0	0.660
7,000	0.411	−30.5	0.590
8,000	0.356	−37.0	0.525

2.2.2 Standard Temperature and Pressure (STP)

This is defined as air at a pressure of 101.325 kPa (abs) and a temperature of 0°C. It is customary to assume that 1 bar = 100 kPa (error = 1.3%).

2.2.3 Standard Reference Conditions (SRC)

This is defined as dry air at a pressure of 101.325 kPa (abs) and a temperature of 15°C.

2.2.4 Free Air Delivered (FAD)

This is the volume, rated at the site conditions, which the blower or compressor takes in, compresses and delivers at the specified discharge pressure. The compressor will deliver compressed air having a smaller volume than that at intake.

2.3 Perfect Gas Laws

An ideal or perfect gas is one which obeys the laws of Boyle, Charles and Gay-Lussac. In practice no gases behave in such a manner and the laws are corrected by taking into consideration *compressibility* factors.

	Boyle's law	Charles's law	Gay-Lussac's law
Condition	Constant temperature	Constant pressure	Constant volume
Law	$\frac{V_2}{V_1} = \frac{p_1}{p_2} \quad (2.1)$	$\frac{V_2}{V_1} = \frac{T_2}{T_1} \quad (2.2)$	$\frac{p_2}{p_1} = \frac{T_2}{T_1} \quad (2.3)$

From the above we obtain the following equation:

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \quad (2.4)$$

For an ideal gas we have

$$pV = MRT \quad (2.5a)$$

where R (the gas constant) = $p_o V_o / T_o M$ and

$$\rho = \frac{p}{RT} \quad (2.5b)$$

and the subscript_o stands for STP. Note the following:

- (a) 1 kg air at STP has a volume $V_o = 1.292 \text{ m}^3$.
- (b) For air $R = 0.287 \text{ kJ/(kgK)}$.

2.4 Drying of Compressed Air

All air contains an amount of moisture which is dependent upon the 'site' pressure and temperature conditions. Many pneumatic conveying systems require 'dry' air and as such the designer must ensure that appropriate steps are taken to remove excess moisture.

There are two stages of air drying in a compressed air system: 1. after-cooling; 2. drying. After-cooling the air results in the air temperature being dropped below

Table 2.2 Cooling systems

Liquid cooled	Air cooled
Air flows through the tubes or	Forced draught
Cooling water flows through the tubes	Induced draught
Each of the above systems has specific advantages and disadvantages	Selection is made on an economic basis taking into account both capital and running costs

Table 2.3 Drying systems

Chemical drying	Absorption drying	Refrigeration drying
Deliquescent-chemical dissolves as vapour, is absorbed and must be replaced	Activated alumina or silica gel is used – normally a dual system is used in which alternative drying and re-activation processes take place	Direct expansion or non-cyclic type – uses cold refrigerant gas to produce a low temperature
Salt matrix – on exposure to hot air the crystal can be returned to its original form		Indirect refrigeration or cyclic type – uses a secondary medium such as chilled water to cool the air
Ethylene glycol liquid – can be regenerated using fuel gas or steam		

the dew point (the saturation vapour temperature at which any further decrease in temperature will cause condensation) and collecting the condensate in a trap.

After-coolers can be categorized into two main types: 1. direct contact 2. surface. Surface after-coolers form the bulk of the systems available and include the types shown in Table 2.2.

Drying of compressed air involves the next stage of processing following the after-cooler. The use of a dryer is to further reduce the moisture content and selection is based solely on the final quality of the air required for the process. There are three basic methods of drying compressed air as detailed in Table 2.3.

After-coolers are least effective with air at low pressure. For large volumetric flow rates at low pressure, driers may have to be used if water is likely to be a hazard.

The interplay between compressing a gas, cooling and expansion is illustrated by means of an example in Section 14.2. The example also demonstrates the use of temperature/mass concentration of water in air curves (Figs. 14.1, 14.2).

2.5 The Compression Process

In most modern pneumatic conveying systems it is generally accepted that either reciprocating or screw compressors, or a positive displacement Roots type blower is used as the prime mover. (There are still many low-pressure fan systems being installed, but pressures are so low that isothermal criteria can be applied.)

Prime mover	Gas process
Reciprocating compressor (ideal) with water cooling	Isothermal
Positive displacement blower	Adiabatic

Essentially the water-cooled reciprocating compressor compresses air at constant temperature – isothermal. The positive displacement blower compresses the air ‘explosively’ without time for cooling and compresses the air ‘adiabatically’, i.e. without the transfer of heat, the energy input from the shaft work is retained in the air.

2.5.1 Isothermal Compression

In isothermal compression the product of the pressure and volume remains a constant:

$$pV = \text{constant} = MRT \quad (2.6)$$

from which

$$V = MRT/p \quad (2.7)$$

$$W_t = - \int_1^2 MR(T/p)dp \quad (2.8)$$

$$= -MRT \ln (p_2/p_1) \quad (2.9)$$

where W_t is the compression work done in joules (j). The amount of heat to be removed from the process is equal to the compression work.

For *fans* and *low-pressure blowers*, the term for a pressure range

$$\ln (p_2/p_1) = \int_{p_1}^{p_2} \frac{dp}{p} \quad (2.10)$$

In this case $p_2 - p_1$ will be small, and the logarithm term can be replaced by the term $(p_2 - p_1)/p_1$ **as long as this ratio does not exceed 1%**. Thus for *fans* and *low-pressure blowers*

$$\text{Work} = p_1 V_1 \left(\frac{p_2 - p_1}{p_1} \right) = (p_2 - p_1) V_1 \quad (2.11)$$

In these low-pressure devices the kinetic energy required to accelerate the air cannot be omitted. In this case the total work per kilogram of gas W_T can be written as

$$\begin{aligned} W_T &= \frac{v_2^2 - v_1^2}{2} + (p_2 - p_1) V_1 \\ &= \text{velocity head} + \text{static head} \end{aligned} \quad (2.12)$$

where v_1 is the initial velocity, v_2 the terminal velocity and V the specific volume.

2.5.2 Adiabatic Compression [2]

In considering the gas law $pV = RT$, if the pressure remains constant whilst the volume changes by dV , the temperature will change by pdV/R . The quantity of heat absorbed for the constant pressure process is the specific heat C_p multiplied by the change in temperature ($C_p pdV/R$).

Similarly if the pressure changes whilst the volume remains constant, the heat absorbed will be ($C_v Vdp/R$) where C_v is the specific heat at constant volume.

For both a pressure and a volume change the quantity of heat received by the gas is

$$dQ = \frac{1}{R}(C_p pdV + C_v Vdp) \quad (2.13)$$

For an **adiabatic** process $dQ = 0$ and so

$$C_p pdV + C_v Vdp = 0 \quad (2.14)$$

Therefore

$$\frac{dp}{p} + \frac{C_p}{C_v} \frac{dV}{V} = 0 \quad (2.15)$$

Integrating this equation gives

$$\ln p = \frac{C_p}{C_v} \ln V = \text{constant} \quad (2.16)$$

Now

$$\begin{aligned} C_p/C_v &= \kappa \quad (\text{ratio of specific heats}) \\ &= 1.4 \quad \text{for air} \end{aligned} \quad (2.17)$$

from which

$$pV^\kappa = \text{constant} = p_1 V_1^\kappa \quad (2.18)$$

if

$$p_1 V_1^\kappa = p_2 V_2^\kappa \quad (2.19)$$

or

$$\frac{p_1}{p_2} = \left(\frac{V_2}{V_1} \right)^\kappa \quad (2.20)$$

or

$$\frac{V_2}{V_1} = \left(\frac{p_1}{p_2} \right)^{1/\kappa} \quad (2.21)$$

2.5.3 Temperature Rise During Adiabatic Compression

Since

$$p_1 V_1 / p_2 V_2 = T_1 / T_2 \quad (2.22)$$

eliminating V_2/V_1 gives the temperature rise during adiabatic compression:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{(\kappa-1)/\kappa} \quad (2.23)$$

Now the work done in expanding from V_1 to V_2 is

$$\int_{V_1}^{V_2} p dV = p_1 V_1 \kappa \int_{V_1}^{V_2} \frac{dV}{V^\kappa} = \frac{p_1 V_1 - p_2 V_2}{\kappa - 1} \quad (2.24)$$

from which the work done

$$W = \frac{p_1 V_1}{\kappa - 1} \left[1 - \left(\frac{p_2}{p_1} \right)^{(\kappa-1)/\kappa} \right] \quad (2.25)$$

when V_2 is unknown.

Note the following:

1. The above expressions are for the total work done in changing the volume in a **non-flow** process within a cylinder.
2. For steady flow process in reciprocating compressors or positive displacement blowers, the work involved in receiving and delivering the gas and the work required to compress the gas must be taken into account.
3. If \dot{V} is the volume dealt with per minute at p_1 and T_1 then

$$\text{work done per minute (actual)} = (p_2 - p_1) \dot{V} \quad (2.26)$$

4. The ideal compression process from p_1 to p_2 is a **reversible adiabatic** (isentropic) process [3]:

$$\text{work done per minute (ideal)} = \frac{\kappa}{\kappa - 1} p_1 \dot{V} \left[\left(\frac{p_2}{p_1} \right)^{(\kappa-1)/\kappa} - 1 \right] \quad (2.27)$$

5.

$$\text{Roots efficiency} = \frac{\text{work done isentropically}}{\text{actual work done}} \quad (2.28)$$

$$= \frac{[k/(k-1)] p_1 \dot{V} (r^{(k-1)/k} - 1)}{p_1 \dot{V} (r - 1)} \quad (2.29)$$

where

$$r = p_2/p_1 = \text{pressure ratio} \quad (2.30)$$

also

$$\kappa/(\kappa - 1) = C_p/R \quad (2.31)$$

therefore

$$\text{Roots efficiency} = \frac{C_p}{R} \left(\frac{r^{(\kappa-1)/\kappa} - 1}{r - 1} \right) \quad (2.32)$$

6. For a Roots type air blower values of pressure ratio, r , or 1.2, 1.6 and 2 give Roots efficiencies of 0.945, 0.84 and 0.765, respectively. These values show that the efficiency decreases as the pressure ratio increases.
7. In a Roots type blower the volumetric efficiency is a function of the internal leakage, known as 'slip'. Slip arises due to the reverse flow through the internal clearances. It depends upon the leakage area, the intake air density and the pressure difference across the leakage path.

Slip is expressed as:

$$S = k(\Delta p/\rho)^{1/2} \quad (2.33)$$

where k is a constant for a particular machine, Δp is the pressure difference, ρ is the intake air density, and S is the slip expressed in terms of volumetric flow rate. Slip is independent of the speed of the machine.

8. The polytropic process describes those situations where there is a flow of heat ($dQ \neq 0$). In such circumstances, the adiabatic equations are used but the exponent κ is replaced by the exponent n . For air, $n = 1.3$.

2.5.4 Power Requirements

(a) Isothermal compression

The power required for an isothermal compression is

$$P = p_1 \dot{V}_1 \ln \left(\frac{p_2}{p_1} \right) \quad (2.34)$$

where \dot{V}_1 is the volume of gas entering the compressor at suction. For fans, the isothermal power is determined by

$$P = \dot{m}_f \left[\frac{1}{\dot{m}_f} (p_2 - p_1) \dot{V}_1 + (v_2^2 - v_1^2)/2 \right] \quad (2.35)$$

Or simply

$$\text{Power}_{\text{air}} = \dot{V} p_{\text{tot}} \quad (2.36)$$

where \dot{V} is the volumetric discharge of the fan per minute and p_{tot} is the total pressure (static + dynamic).

(b) *Adiabatic compression*

$$P = \frac{k}{k-1} p_1 \dot{V}_1 \left[\left(\frac{p_2}{p_1} \right)^{(k-1)/k} - 1 \right] \quad (2.37)$$

Since for air $\kappa = 1.4$

$$P = 3.5 p_1 \dot{V}_1 \left[\left(\frac{p_2}{p_1} \right)^{0.286} - 1 \right] \quad (2.38)$$

For multistage air compressors with the same inlet temperature at each stage

$$P = 3.5 p_1 \dot{V}_1 \left[\left(\frac{p_2}{p_1} \right)^{0.286/n} - 1 \right] \quad (2.39)$$

where n is the number of stages.

In Table 2.4, the power required to compress air is shown as a function of the gauge pressure for single stage, two stage and three stage compressors.

Compressor volumetric efficiency is dependent upon altitude. The exact location of a compressor must be taken into account when selecting a suitable machine. In Table 2.5 the effect of altitude on compressor volumetric efficiency is shown for 400 kPa and 700 kPa applications.

Table 2.4 Power required to compress air [4]

Gauge pressure (bar)	Theoretical adiabatic power	kW/100 dm ³ /s	
	Single stage	Two stage	Free air Three stage
0.5	4.0	—	—
1.0	7.5	—	—
2.5	15.0	14	—
5.0	23.0	20	19
7.0	28.0	24	22
10.0	34.0	28	27
14.0	40.0	32	32

Note: 1 bar = 100 kPa

Table 2.5 Effect of altitude on compressor volumetric efficiency [5]

Altitude above sea level (m)	Barometric pressure (mbar)	Percentage relative volumetric efficiency compared with sea level	
		400 kPa	700 kPa
	1,013	100.0	100.0
500	945	98.7	97.7
1,000	894	97.0	95.2
1,500	840	95.5	92.7
2,000	789	93.9	90.0
2,500	737	92.1	87.0

2.6 Gas Flow Through Pipes

The flow of gas through pipes forms an essential element of any pneumatic conveying system. In most systems, plant layouts dictate that the compressor is located some distance from the feeder. A piping system is required to connect the compressor to the feeder. Further, in order to design the actual pneumatic conveyor, an understanding of the behaviour of the gas flow through the total system is essential.

2.6.1 Types of Flow

There are two basic types of flow in a pipe: (a) laminar; (b) turbulent.

(a) Laminar flow

This is characterized by streamlined concentric cylindrical layers of fluid flowing past one another in an orderly fashion. The velocity is greatest at the centre of the pipe and decreases sharply at the wall or boundary layer.

(b) Turbulent flow

This is characterized by an irregular random movement of fluid particles across the main stream without an observable frequency or pattern.

The exact limit of laminar and turbulent flow is defined by the Reynolds number (*Re*);

$$Re = \frac{\rho v D}{\eta} \tag{2.40}$$

where ρ is the density of fluid, v the fluid velocity, D the pipe diameter, and η the dynamic viscosity of the fluid ($N\,s/m^2$). (Viscosity in gases rises with temperature within moderate ranges of pressure (3,500 kPa) viscosity is not influenced by more than 10%).

In Fig. 2.1, the variation of dynamic viscosity of air with temperature is shown for various pressures. Useful data for air at various temperatures are given in Table 2.6.

Note that a useful method of calculating the dynamic viscosity of air is by using Rayleigh's criterion

$$\eta_1/\eta_0 = (T_0/T_1)^{0.77}$$

where at sea level

$$\eta_0 = 1.783 \times 10^{-5} \text{ N s/m}^2 \text{ (Pa s)}$$

$$T_0 = 288.16 \text{ K}$$

2.6.2 Pipe Roughness

The absolute roughness (k) of many types of pipe materials has been approximated. A factor known as relative roughness is used to relate the internal surface conditions of a pipe to its diameter, using the ratio (k/D). Charts relating the relative roughness to the pipe diameter are available. For turbulent flow $Re > 2,300$; for laminar flow $Re < 2,300$. In most cases, for pressure piping the flow is turbulent.

2.6.3 General Pressure Drop Formula

The following general formula for pressure in piping was developed by Darcy:

$$\Delta p = \lambda_L \rho L v^2 / 2D \quad (2.41)$$

where λ_L is the friction factor and L the pipe length (metres).

(a) *Friction factor laminar flow*

In the range $0 < Re < 2,300$ the friction factor:

$$\lambda_L = 64/Re \quad (2.42)$$

(b) *Friction factor turbulent flow*

For turbulent flow, the friction factor can be found from Fig. 2.2 relating the friction factor to the Reynolds number.

For compressed air pipework, the following equation for the pressure drop (Δp) in a straight pipe can be used with good approximation [8]:

$$\Delta p = 1.6 \times 10^3 \dot{V}^{1.85} L / (D^5 p_1) \quad (2.43)$$

where Δp is the pressure drop (Pa), \dot{V} is volumetric flow rate (m^3/s), L is pipe length (m), D is pipe diameter (m), p_1 is initial pressure (Pa). In Eq. 2.43 the

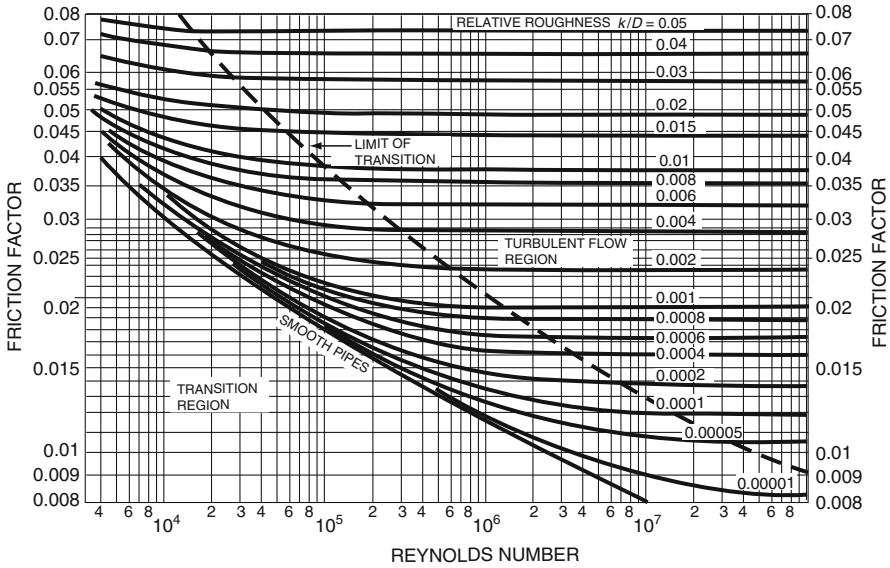


Fig. 2.2 Friction factor versus Reynolds number

temperature in the pipe is assumed to be the same as the ambient temperature. For practical situations, it can be assumed that $\lambda_L \cong 0.02$ within a wide range, provided that $Re > 10,000$.

There are several methods of obtaining the gas friction factor. The reader should be cautious in noting that there is a difference in definition resulting in a factor of 4 when using the Fanning friction (f_L) as distinct from the friction factor (λ_L) used in the text.

For simplicity, it is recommended that the Blasius equation [7] or the Koo equation [9] be used. For $Re < 10^5$ the Blasius equation [7] states:

$$\lambda_L = 0.316/(Re)^{0.25} \quad (2.44)$$

The Koo equation [9] for f is given by

$$f_L = 0.0014 + 0.125/(Re)^{0.32} \quad (2.45)$$

Note that $\lambda_L = 4f_L$.

A set of curves (Fig. 2.3) produced using the Blasius equation (2.44) provides a quick method of obtaining the pressure loss for various air flow rates. For easy reference, Table 2.7 provides information pertaining to the volume of air which can be carried by various sizes of pipes at given velocities. (It is common practice to size compressed air mains in a velocity range $v = 6 - 9$ m/s.)

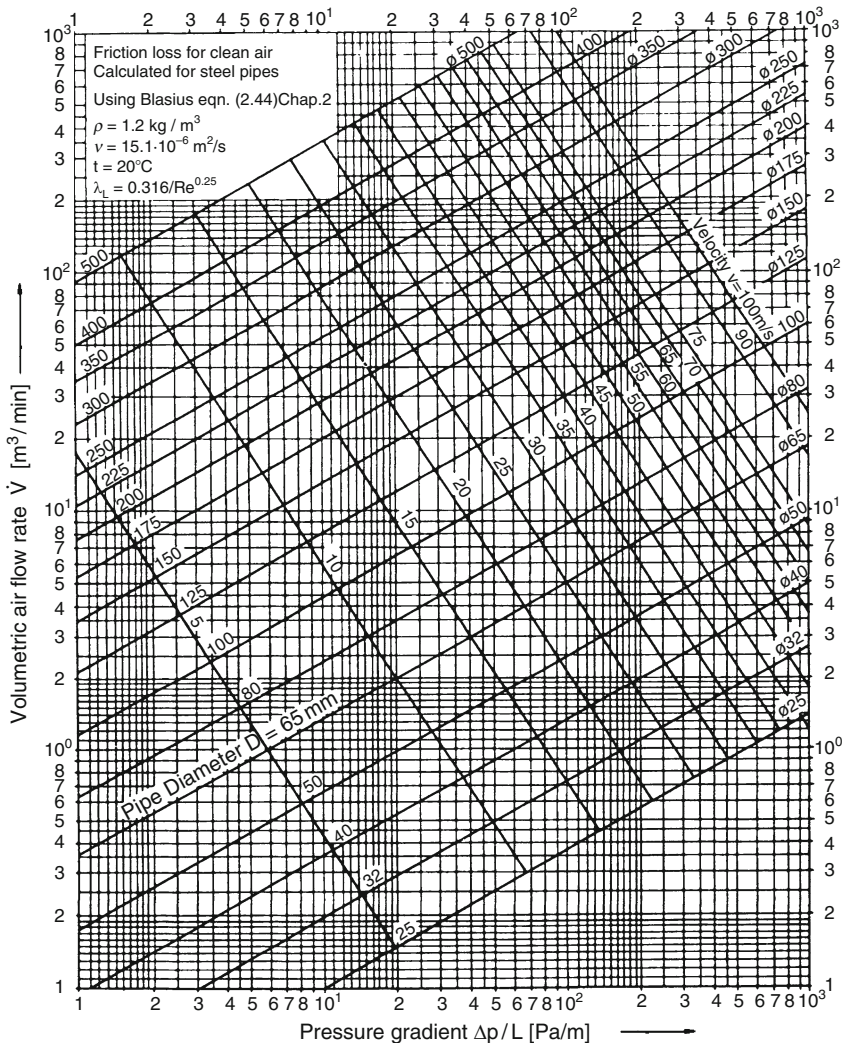


Fig. 2.3 A set of curves produced using the Blasius equation

2.6.4 Resistance Due To Pipe Fittings

In a system layout where a complex pipe layout is required, it is essential that the pressure losses generated by pipe fittings are taken into account. This practice is especially pertinent to low-pressure air supply lines. Allowances for losses due to fittings can be accounted for by finding an equivalent length in metres for typical fittings. Table 2.8 gives the equivalent length for fittings in relation to pipe diameter.

Table 2.7 Volume of compressed air carried by medium grade steel pipes, of minimum bore, to BS 1387, at given velocities [9]

Velocity, $v \text{ (m s}^{-1}\text{)}$	Volume of air ($10^{-3} \text{ m}^3/\text{s}$) through medium grade steel pipe, to BS, minimum bore, D (mm)											
	15	20	25	32	40	50	65	80	100	125	150	200
3.0	0.6	1.1	1.7	3.0	4.1	6.5	10.9	15.1	25.7	39.2	56.2	98.5
3.5	0.7	1.3	2.0	3.5	4.7	7.6	12.7	17.6	30.0	45.7	65.5	115.0
4.0	0.8	1.4	2.3	4.0	5.4	8.7	14.6	20.1	34.2	52.2	74.9	131.0
4.5	0.9	1.6	2.6	4.5	6.1	9.8	16.4	22.6	38.5	58.8	84.2	147.0
5.0	1.0	1.8	2.8	5.0	6.8	10.8	18.2	25.1	42.8	65.4	93.6	164.0
5.5	1.1	2.0	3.1	5.5	7.4	11.9	20.0	27.6	47.1	71.9	103.0	181.0
6.0	1.2	2.1	3.4	6.0	8.1	13.0	21.8	30.1	51.3	78.5	112.0	197.0
6.5	1.3	2.3	3.7	6.5	8.8	14.1	23.7	32.6	55.6	85.0	122.0	213.0
7.0	1.4	2.5	4.0	7.0	9.5	15.1	25.5	35.1	59.9	91.5	131.0	230.0
7.5	1.5	2.7	4.3	7.5	10.1	16.2	27.3	37.6	64.2	98.0	140.0	246.0
8.0	1.6	2.8	4.5	8.0	10.8	17.3	29.1	40.1	68.5	105.0	150.0	263.0
8.5	1.7	3.0	4.8	8.5	11.5	18.4	31.0	42.6	72.8	111.0	159.0	278.0
9.0	1.8	3.2	5.1	9.0	12.2	19.5	32.8	45.1	77.1	118.0	169.0	296.0

e.g. for $v = 3 \text{ m/s}$ and $D = 100 \text{ mm}$, $\dot{V} = 25.7 \times 10^{-3} \text{ m}^3/\text{s}$

Table 2.8 Resistance of pipe fittings (equivalent length in m) [10]

Type of fitting	Nominal pipe size (mm)									
	15	20	25	32	40	50	65	80	100	125
Elbow	0.26	0.37	0.49	0.67	0.76	1.07	1.37	1.83	2.44	3.2
90° bend (long)	0.15	0.18	0.24	0.38	0.46	0.61	0.76	0.91	1.2	1.52
Return bend	0.46	0.61	0.76	1.07	1.2	1.68	1.98	2.6	3.66	4.88
Globe valve	0.76	1.07	1.37	1.98	2.44	3.36	3.96	5.18	7.32	9.45
Gate valve	0.107	0.14	0.18	0.27	0.32	0.40	0.49	0.64	0.91	1.20
Run of standard tee	0.12	0.18	0.24	0.38	0.40	0.52	0.67	0.85	1.2	1.52
Through side outlet of tee	0.52	0.70	0.91	1.37	1.58	2.14	2.74	3.66	4.88	6.40

2.7 Illustrative Examples

Example 2.1. Standard reference conditions (SRC)

A flow rate of $100 \text{ m}^3/\text{min}$ is rated at SRC. Determine the equivalent free air flow rate at a site with atmospheric pressure $p_{\text{atm}} = 84 \text{ kPa(abs)}$ and temperature $t = 25^\circ\text{C}$.

From equation (2.4):

$$\frac{p_1 \dot{V}_1}{T_1} = \frac{p_2 \dot{V}_2}{T_2}$$

In this example:

$$\frac{101.325 (100)}{273 + 15} = \frac{84(\dot{V}_2)}{273 + 25}$$

Therefore

$$\dot{V}_2 = 124.8 \text{ m}^3/\text{min (FAD)}$$

The above conversion could also have been made using the density ratio. From Eq. 2.5:

$$pV = MRT$$

(for air $R = 0.287 \text{ kJ}/(\text{kg}/\text{K})$). By definition density, $\rho = p/RT$

$$\rho = \begin{cases} 1.293 \text{ kg}/\text{m}^3 & \text{at STP} \\ 1.225 \text{ kg}/\text{m}^3 & \text{at SRC} \end{cases}$$

The 'site' density in this example:

$$\rho = \frac{84}{0.287 \times 298} = 0.98 \text{ kg}/\text{m}^3$$

$$\dot{V}_2 = \frac{100 \times 1.225}{0.98} = 125 \text{ m}^3/\text{min (FAD)}$$

Example 2.2. Effect of altitude on blower performance

A Roots type blower is selected at sea level conditions with an intake volume of $50 \text{ m}^3/\text{min}$ and a pressure rise 90 kPa . Temperature at site is 25°C . Determine the blower's power requirements at both sea level and at an altitude of $1,500 \text{ m}$.

(a) *At sea level*

The intake density $\rho = 1.18 \text{ kg}/\text{m}^3$. The slip for such a blower size $\dot{V} = 11.7 \text{ m}^3/\text{min}$. The displacement required would be

$$\dot{V} = 50 + 11.7 = 61.7 \text{ m}^3/\text{min}$$

A typical machine would have a speed $n = 1,270 \text{ rev}/\text{min}$.

Power required (from Eq. 2.38) is

$$P = 3.5 \times 100 \times \frac{61.7}{60} \left[\left(\frac{190}{100} \right)^{0.286} - 1 \right]$$

$$= 72.5 \text{ kW}$$

Temperature rise (from Eq. 2.23) is

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{(\kappa-1)/\kappa}$$

$$= 298 \left(\frac{190}{100} \right)^{0.286} = 358 \text{ K}$$

Therefore $t_2 = 85^\circ\text{C}$.

(b) At 1,500m above sea level

$p_{\text{atm}} = 84.5 \text{ kPa (abs)}$, $t = 25^\circ\text{C}$, intake density $\rho = 0.99 \text{ kg/m}^3$. For this machine slip $\dot{V} = 12.8 \text{ m}^3/\text{min}$. The displacement required to maintain $\dot{V}_f = 50 \text{ m}^3/\text{min}$

$$\dot{V}_f = 50 + 12.8 = 62.8 \text{ m}^3/\text{min}$$

This blower would be speeded up to 1,280 rev/min. The power (from Eq. 2.38) is

$$\begin{aligned} P &= 3.5 \times 84.5 \times \frac{62.8}{60} \left[\left(\frac{174.5}{84.5} \right)^{0.286} - 1 \right] \\ &= 71.3 \text{ kW.} \end{aligned}$$

Example 2.3. At processes, Roots type blower

1. Determine the work input for a Roots type blower having an induced volume of $0.03 \text{ m}^3/\text{rev}$. The inlet pressure is 1.013 bar and the pressure ratio 1.5:1.

$$p_1 = 1.013 \text{ bar}$$

$$p_2 = 1.013 \times 1.5 = 1.52 \text{ bar}$$

From Eq. 2.26

Actual work done per revolution

$$\begin{aligned} W &= (p_2 - p_1) \dot{V}_1 \\ &= (1.52 - 1.013) \times 0.03 \times 10^5 = 1,521 \text{ J} \\ &= 1.52 \text{ kJ (where 1 bar} = 10^5 \text{ N/m}^2) \end{aligned}$$

2. In (1), determine the Roots efficiency.

From Eq. 2.27

Work done isentropically per revolution

$$\begin{aligned} &= \frac{\kappa}{\kappa - 1} p_1 \dot{V}_1 \left[\left(\frac{p_2}{p_1} \right)^{(\kappa-1)/\kappa} - 1 \right] \\ &= \frac{1.4}{0.4} \times 1.013 \times 10^5 \times 0.03 \times [(1.5)^{0.4/1.4} - 1] \\ &= 1.3063 \times 10^3 \text{ J} \end{aligned}$$

From Eq. 2.28

$$\text{Roots efficiency} = \frac{1.31}{1.52} \times 100 = 86.2\%$$

3. If the inlet temperature to the blower is 15°C , determine the final temperature after compression in degrees Celsius.

From Eq. 2.23

$$\begin{aligned} T_2 &= T_1 \left[\frac{p_2}{p_1} \right]^{(\kappa-1)/\kappa} \\ &= 288(1.5)^{0.4/1.4} = 323 \text{ K} \end{aligned}$$

Therefore

$$t_2 = 50^\circ\text{C}$$

Example 2.4. Power required for compression

1. If a compressor is rated at $14 \text{ m}^3/\text{min}$ and compresses air from 100 kPa to 800 kPa , determine the power required.

For isothermal compression, from Eq. 2.34

$$\begin{aligned} P &= 100 \times 10^3 \times \frac{14}{60} \ln \left(\frac{800}{100} \right) \\ &= 48.5 \text{ kW} \end{aligned}$$

2. If a positive displacement blower is rated to compress $14 \text{ m}^3/\text{min}$ of free air to 100 kPa , determine the power required.

For adiabatic compression, from Eq. 2.38

$$\begin{aligned} P &= 3.5 \times 100 \times 10^3 \times \frac{14}{60} \left[\left(\frac{800}{100} \right)^{0.286} - 1 \right] \\ &= 66.4 \text{ kW} \end{aligned}$$

Example 2.5. Pressure loss in a pipe system

Determine the pressure loss in a piping system consisting of a length $L = 200 \text{ m}$ and a diameter $D = 70 \text{ mm}$. The air flow rate $\dot{V} = 170 \text{ dm}^3/\text{s}$ (free air), and initial pressure $p_1 = 800 \text{ kPa}$ at an ambient temperature $t = 20^\circ\text{C}$.

Take atmospheric pressure $p_{\text{atm}} = 100 \text{ kPa}$. From continuity $\dot{V}_f = Av$, where A is the cross-sectional area of pipe (m^2), v is velocity (m/s) and \dot{V}_f is volumetric flow rate (m^3/s). The velocity at inlet to the pipe (given $\dot{V} = 170 \text{ dm}^3/\text{s}$ (free air), is

$$\begin{aligned} v &= (100/800) \times 0.17 \times 4/(\pi \times 0.07^2) \\ &= 5.525 \text{ m/s} \end{aligned}$$

From tables the dynamic viscosity of the air is $\eta = 18.08 \times 10^{-6} \text{ Ns/m}^2$ at 293 K . The density of the air at inlet to the pipe:

$$\begin{aligned} \rho_1 &= p_1/RT_1 \\ &= 800 \times 10^3/(287 \times 293) \\ &= 9.51 \text{ kg/m}^3 \end{aligned}$$

from which the Reynolds number (Eq. 2.40) is

$$\begin{aligned}\text{Re} &= 9.51 \times 5.53 \times 0.07 \times 10^6 / 18.08 \\ &= 203,600\end{aligned}$$

Using Eq. 2.43 the pressure loss is

$$\begin{aligned}\Delta p &= 1.6 \times 10^3 \times 0.17^{1.85} \times 200 / (0.07^5 \times (800 \times 10^3)) \\ &= 8,972 \text{ Pa}\end{aligned}$$

Assuming $\lambda_L = 0.02$ and using Eq. 2.41 (since $\text{Re} > 10,000$)

$$\begin{aligned}\Delta p &= 0.02 \times 9.5135 \times 200 \times 5.525^2 / (2 \times 0.07) \\ &= 8,285 \text{ Pa}\end{aligned}$$

from which it can be seen that in spite of some simplifying assumptions, Eq. 2.43 can be used with confidence. The pressure loss calculated using Eq. 2.43 is higher than that calculated using Eq. 2.41.

Example 2.6. Calculations of Reynolds number Re and gas friction factor λ_L

Given pipes of $D = 50$ and 100 mm and gas velocities $v = 1, 10, 20$ and 40 m/s, calculate the Reynolds number and the corresponding gas friction factors λ_L and f_L , i.e. using the Blasius and Koo equations.

Reynolds number $\text{Re} = vD\rho/\eta$. At $t = 20^\circ\text{C}$, $\eta/\rho = 15.1 \times 10^{-6} \text{ m}^2/\text{s}$. At $v = 1 \text{ m/s}$ and $D = 50 \text{ mm}$

$$\begin{aligned}\text{Re} &= \frac{1 \times 0.05}{15.1} \times 10^6 \\ &= 3.31 \times 10^3\end{aligned}$$

Friction factors:

(a) Blasius

$$\begin{aligned}\lambda_L &= 0.316/\text{Re}^{0.25} \\ &= 0.04166\end{aligned}$$

(b) Koo

$$\begin{aligned}f_L &= 0.0014 + 0.125 \text{Re}^{-0.32} \\ &= 0.0107 \\ \lambda_L &= 4f_L = 4 \times 0.0107 = 0.0429\end{aligned}$$

Note that $\lambda_L = 4f_L$. Table 2.9 gives results calculated using the Blasius and Koo equations.

Table 2.9 Calculation of friction factors λ_L and f_L using the Blasius and Koo equations

D (mm)	50	50	50	50	100	100	100	100
v (m/s)	1	10	20	40	1	10	20	40
$Re \times 10^{-3}$	3.31	33.1	66.2	132.45	6.62	66.2	132.45	264.90
λ_L (Blasius)	0.0417	0.0234	0.0197	0.0166	0.035	0.0197	0.0166	0.0139
f_L (Koo)	0.0107	0.0059	0.0049	0.0043	0.0089	0.0049	0.0043	0.0037
$4f_L$	0.0429	0.0235	0.0199	0.0171	0.0355	0.0199	0.0171	0.0148

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