A Study of the Influence of the Reynolds Number on Jet Self-Similarity Using Large-Eddy Simulation

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Abstract Three round jets at Reynolds numbers 1,800, 3,600 and 11,000 are computed using Large-Eddy Simulations based on low-dissipation numerical schemes and explicit selective filtering, in order to study the influence of the Reynolds number on jet self-similarity. At lower Reynolds number, the jet flow achieves self-similarity more rapidly, and then develops at a higher rate. The effects of the Reynolds number on the velocity moments as well as on the budget for the turbulent kinetic energy across the self-similar jet flow are however found to be weak.

1 Introduction

The self-similarity region of jet flows has been investigated extensively over the last fifty years. Reference solutions for round jets have for instance been provided by the experimental works of Wygnanski and Fiedler [1], Panchapakesan and Lumley [2] and Hussein et al. [3]. Discrepancies between the solutions are however observed. They are expected to result from differences in the measurement methods and in the jet initial conditions, the diameter-based Reynolds numbers ranging in particular from 1.1×10^4 to 10^5.

The effects of the Reynolds number on jet flow development have indeed been shown to be significant up to Reynolds numbers around $\mathrm{Re}_D = u_j D/\nu \simeq 10^4$, where u_j and D are the jet inlet velocity and diameter, and ν is the kinematic molecular viscosity. They have been examined experimentally specially by Lemieux and Oostuizen [4], Namer and Ötügen [5], Kwon and Seo [6] and Deo et al. [7], as well as numerically by Bogey and Bailly [8]. In experiments, variations of the Reynolds number might however result in modifications of other flow initial conditions, whereas the latter computational study only dealt with the transitional jet region.

V. Armenio et al. (eds.), *Direct and Large-Eddy Simulation VII*,
ERCOFTAC Series 13, DOI 10.1007/978-90-481-3652-0_2,

It appears therefore now worthwhile to study the influence of the Reynolds number on jet self-similarity using simulations. Computations of self-similar jets have been performed previously by Boersma et al. [9], Freund [11] and Uddin and Pollard [12] for instance. In Bogey and Bailly [10], a round jet at a Reynolds number 11,000 has also been calculated by a Large Eddy Simulation (LES) using an approach combining low-dissipation schemes [13] and relaxation filtering [8]. Jet self-similarity was reached on the computational domain, and described in detail. A good agreement with the data obtained by Panchapakesan and Lumley [2] at the same Reynolds number was found.

Given these results, two jets with the same initial conditions as the jet at Reynolds number 11,000 except for the diameter, providing Reynolds numbers of 1,800 and 3,600, have been simulated by LES with the aim of characterizing the influence of the Reynolds number on self-similar round jets. Preliminary comparisons are reported here.

2 Simulation parameters

Three round jets at Mach number 0.9 and at Reynolds numbers 1,800, 3,600 and 11,000 are computed by LES using the same numerical set-up, on grids containing from 18 to 44 millions of points, extending, respectively up to 90, 120 and 150 jet radii r_0 in the downstream direction, see in Table 1 for additional parameters. The jet inflow conditions such as the mean flow profiles, the shear-layer thickness or the forcing used to seed the turbulent transition are identical except for the diameter. They are described in detail in a previous paper [10].

The LES are performed using low-dispersion low-dissipation finite-difference and Runge–Kutta schemes [13], in combination with the application of an explicit filtering to the flow variables in order to remove the smaller scales discretized without appreciably affecting the larger scales. This LES methodology has been used successfully for transitional jets [8,14]. It does not lead in particular to an artificial decrease of the effective Reynolds number of the flow, as it might be the case with eddy-viscosity-based LES modellings. In addition the budgets for the turbulent kinetic energy are also computed directly from the flow-governing equations. All the energy terms are estimated explicitly [10].

Table 1. Number of grid points and simulation time, as well as location of the end of the potential core x_c, decay constant B, spreading rate A and centerline filtering activity s_{filt} in the self-similar jets.

Re_D	$n_x \times n_y \times n_z$	Tu_j/D	x_c	B	A	s_{filt}
1,800	$411 \times 211 \times 211$	0.77×10^5	$24.1r_0$	5.8	0.096	0.082
3,600	$531 \times 261 \times 261$	0.77×10^5	$17.1r_0$	6.1	0.091	0.153
11,000	$651 \times 261 \times 261$	1.35×10^5	$13.5r_0$	6.4	0.087	0.334

3 Results

In the jets, mean flow and turbulence properties, including the second-order and third-order velocity moments, and the energy budgets, have been evaluated from the LES fields over a long time period to ensure statistical convergence. Some comparisons are provided here to give some insight into the Reynolds number effects on jet self-similarity.

A first illustration is provided by the vorticity fields presented in Fig. 1. The transitions from laminar shear layers toward fully-developed turbulence can be seen. As the Reynolds number increases, the presence of fine scales is more visible. The initial development of the jet occurs also more rapidly, leading to a decrease of the length of the potential core, in agreement with experimental findings. The core lengths x_c, defined by $u_c(x_c) = 0.95u_j$ where u_c is the centerline mean axial velocity, are consequently $24.1r_0$, $17.1r_0$ and $13.5r_0$ at Reynolds numbers 1,800, 3,600 and 11,000, refer to Table 1.

To determine the axial locations at which the simulated jet flows achieve self-similarity, profiles of turbulence intensities along the centerline are presented in Fig. 2. The establishment of self-similarity, obtained when constant values are observed on the jet axis, is shown to take place more rapidly at lower Reynolds number, i.e., closer to the end of the potential core, in agreement with experimental and numerical results [8,15]. More quantitatively, self-similarity seems to be reached around $x = 60r_0$, $70r_0$ and $120r_0$ at Reynolds numbers 1,800, 3,600 and 11,000.

The effects of the Reynolds number of the mean flow development are considered from the variations of the centerline mean axial velocity u_c. The

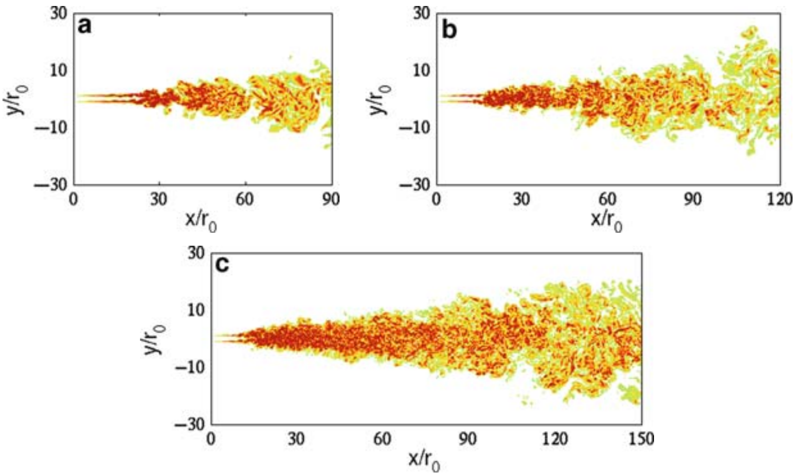

Fig. 1. Snapshots of vorticity norm $|\boldsymbol{\omega}| \times x/u_j$, in the plane $z = 0$, for the jets at Reynolds numbers: (**a**) 1,800, (**b**) 3,600 and (**c**) 1,1000. The color scale ranges for levels from 4 to 20.

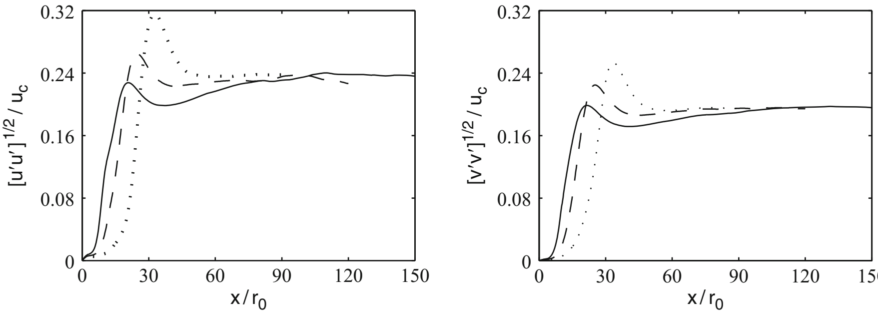

Fig. 2. Variations along the jet centerline of turbulence intensities $[u'u']^{1/2}/u_c$ (left) and $[v'v']^{1/2}/u_c$ (right), for Reynolds numbers 1,800 (dotted lines), 3,600 (dashed lines) and 11,000 (solid lines).

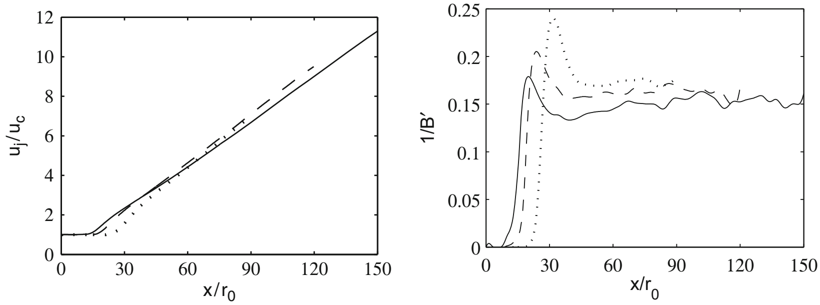

Fig. 3. Axial variations of the inverse of centerline mean velocity u_j/u_c (left) and of the inverse of local decay constant $1/B'$ (right), for Reynolds numbers 1,800 (dotted lines), 3,600 (dashed lines) and 11,000 (solid lines).

inverse of u_c is represented in Fig. 3 for the three jets. After the transitional region, it seems to vary linearly as expected. In self-preserving round jets, the mean velocity evolves indeed as $u_c/u_j = B \times D/(x - x_0)$ where B is the decay constant and x_0 denotes a virtual origin. The establishment of mean flow self-similarity is then investigated by plotting the inverse of the local decay constant defined as $1/B' = \mathrm{d}(u_j/u_c)/\mathrm{d}(x/D)$ in Fig. 3. This mean flow parameter tends to asymptotic values in the downstream direction, indicating self-similarity and providing decay constants $B = 5.8$, 6.1 and 6.4 at $\mathrm{Re}_D = 1{,}800$, 3,600 and 11,000, respectively. In the same way, values of 0.096, 0.091 and 0.087 are reported in Table 1 for the spreading rates A governing the variations of the jet half-width $\delta_{0.5}$. The self-similar mean flow therefore develops at a higher rate at lower Reynolds number, in agreement with the recent experimental data obtained for plane jets by Deo et al. [7].

To characterize the turbulent fields in the self-similar jets, the profiles of second-order and third-order velocity moments are calculated across the jets, and plotted as functions of $y/\delta_{0.5}$. Illustrations are given in Fig. 4 with the

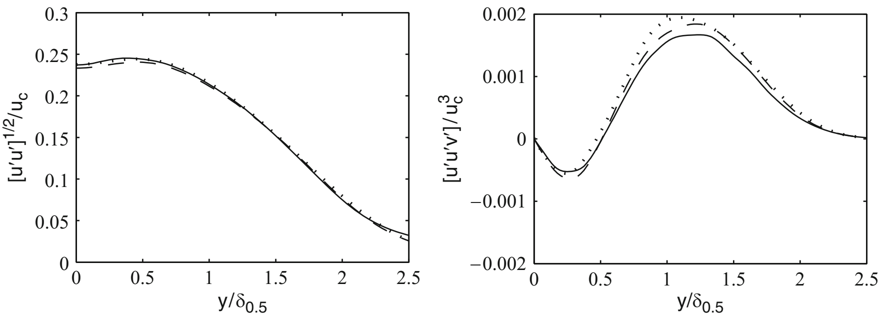

Fig. 4. Profiles of second and third-order velocity moments $[u'u']^{1/2}/u_c$ (left) and $[u'u'v']/u_c^3$ (right) across the self-similar jets, for Reynolds numbers 1,800 (dotted lines), 3,600 (dashed lines) and 1,1000 (solid lines).

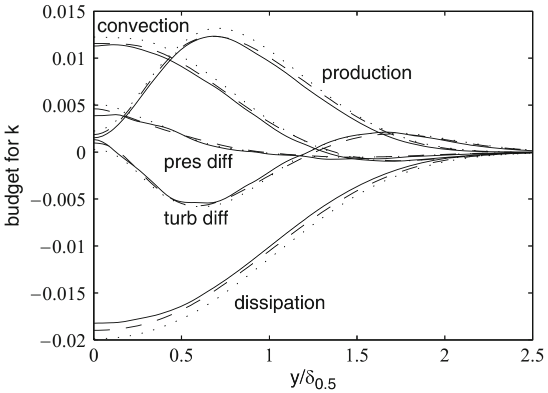

Fig. 5. Budget for the turbulent kinetic energy k across the self-similar jets at Reynolds numbers 1,800 (dotted lines), 3,600 (dashed lines) and 11,000 (solid lines): convection, production, dissipation, turbulence diffusion and pressure diffusion.

profiles obtained for $[u'u']^{1/2}/u_c$ and $[u'u'v']/u_c^3$. For the former quantity, the curves fairly collapse, whereas for the latter, the magnitude of the velocity moment only slightly increases when the Reynolds number decreases. The influence of the Reynolds number therefore appears weak on the flow features considered, over the given Reynolds number range.

Finally, the budgets calculated for the turbulent kinetic energy in the self-similarity regions of the three jets are represented in Fig. 5. Dissipation is here the sum of the viscous and filtering dissipations, whose relative contributions vary with the Reynolds number. On the centerline for example, the filtering dissipation is 33% of the total energy dissipation at $\mathrm{Re}_D = 11{,}000$ but only 8% at $\mathrm{Re}_D = 1{,}800$, as indicated in Table 1 by the centerline filtering activity s_{filt}. The shapes obtained for the different energy terms, namely for mean flow convection, production, dissipation, turbulence diffusion and pressure diffusion, are very similar. The magnitudes of the energy terms only appear to be higher at lower Reynolds number. This is particularly the case for production, dissipation and mean flow convection, whereas turbulence diffusion is surprisingly nearly unchanged. The present results suggest that the balance between the different turbulent mechanisms in the self-similar jets does not depend significantly on the Reynolds number.

4 Conclusion

The present LES show that in round jets the distance required to achieve self-similarity as well as the development of the self-similar jet mean flow depend on the Reynolds number over the range $1{,}800 \leq \mathrm{Re}_D \leq 11{,}000$, but that the effects are quite weak on the turbulence features. Further analyses will be done to support this contention.

Acknowledgments

The authors gratefully acknowledge the *Institut du Développement et des Ressources en Informatique Scientifique* of the CNRS and the *Centre de Calcul Recherche et Technologie* of the CEA for providing CPU time on Nec computers and for technical assistance.

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http://www.springer.com/978-90-481-3651-3

Direct and Large-Eddy Simulation VII
Proceedings of the Seventh International ERCOFTAC Workshop on Direct and Large-Eddy Simulation, held at the University of Trieste, September 8-10, 2008
Armenio, V.; Geurts, B.; Fröhlich, J. (Eds.)
2010, IV, 646 p., Hardcover
ISBN: 978-90-481-3651-3