

Preface

The present book is an introduction to a new field in applied group analysis. The book deals with symmetries of integro-differential, stochastic and delay equations that form the basis of a large variety of mathematical models, used to describe various phenomena in fluid mechanics and plasma physics and other fields of nonlinear science.

Because of its baffling complexity the mathematical study of nonlocal equations is far from completion, although the equations have been intensively studied in numerous applications over more than fifty last years using both numerical and analytical methods. The principal aim of analytical approaches is to obtain exact solutions, admitted symmetries, conservation laws and other mathematical properties, which allow one to make sound decisions in more detailed applied investigations.

Classical Lie group theory provides a universal tool for calculating symmetry groups for systems of differential equations. Consequently, group theoretical methods appear efficient in analyzing different phenomena using mathematical models that employ differential equations. However Lie's methods cannot be directly applied to integro-differential equations, infinite systems of differential equations, delay equations, etc. Hence it is natural to extend the ideas of modern group analysis to these mathematical objects that up to recently were not in mainstream of classical group theoretical approaches.

The book is designed for specialists in nonlinear physics interested in methods of applied group analysis for investigating nonlinear problems in physical, engineering and natural sciences. It is based on our research results and various courses and lectures given to undergraduate and graduate students as well as professional audiences over the past thirty years. The book can also serve as a textbook on symmetries of integro-differential, stochastic and delay equations for graduate students in applied mathematics, physics and engineering.

In the preparation of this monograph the roles were distributed in the following way. The first chapter was written by N.H. Ibragimov. The second and third chapters are the result of collaboration between Y.N. Grigoriev and S.V. Meleshko. The fourth chapter was prepared by V.F. Kovalev. Chapters five and six are the work of S.V. Meleshko.

Organization of the Book

The contents of this book have been assembled from results scattered across many different articles and books published over the last thirty years.

The monograph includes six chapters. The first chapter contains an introduction to the methods of Lie group analysis of ordinary and partial differential equations. The basic notions of this mathematical area: continuous transformation groups, algebras of their generators, determining equations and methods of finding invariant solutions of differential equations are presented and illustrated by numerous examples. New trends in modern group analysis are also reflected. The intention of the chapter is to give the basic ideas of classical and modern group analysis to beginner readers and provide useful materials for advanced specialists.

The second chapter presents a survey of different methods for constructing symmetries and finding invariant solutions of integro-differential equations. An introduction to these methods is carried out using simple model equations, allowing the reader to follow the calculations in detail. The chapter includes substantial generalization of the original scheme of the group analysis method to equations with nonlocal operators. In the concluding sections of the chapter this regular method of obtaining admitted Lie groups is illustrated by applications to different integro-differential equations.

The results of group analysis of the Boltzmann kinetic equation and some similar equations with squarely nonlinear integral operators are described in the third chapter. These equations form the foundation of the kinetic theory of rarefied gas and coagulation. The main point of interest here is the isomorphism of the Lie group of point transformations admitted by the full Boltzmann equation and the Euler inviscid gas dynamic system. This remarkable fact allows us to obtain representations of all invariant solutions with one and two independent variables of the Boltzmann equation. For equations with few number of independent variables the proposed method allows us to derive constructive proofs of the completeness of admitted Lie groups. The representations of all invariant solutions are also presented.

The fourth chapter is entirely devoted to a group analysis of the Vlasov–Maxwell and related type equations. The equations form the basis of the collisionless plasma kinetic theory, and are also applied in gravitational astrophysics, in shallow-water theory, in the theory of pulverulent suspensions, etc. Nonlocal operators in these equations appear in the form of the functionals defined by integrals of the distribution functions over momenta of particles. Much of the importance of the approach used in this chapter for calculating symmetries stems from the procedure of solving determining equations using variational differentiation. The set of symmetries obtained comprises symmetries for the Vlasov–Maxwell equations of the non-relativistic and relativistic electron and electron–ion plasmas in both one- and three-dimensional cases, and symmetries for Benney equations. In the concluding sections of this chapter the procedure for symmetry calculation and the renormalization group algorithm go hand in hand to present illustrations from plasma kinetic theory, plasma dynamics, and nonlinear optics, which demonstrate the potentialities of the method in construction of analytic solutions to nonlocal problems of nonlinear physics.

The fifth and sixth chapters present new fields of application of group analysis to stochastic and delay differential equations. In the fifth chapter a definition of determining equations for calculation of the Lie algebras admitted by stochastic dynamical systems is formulated. This gives an opportunity to derive determining equations for symmetries of Itô and Stratonovich dynamical systems.

The sixth chapter deals with symmetries of delay differential equations. In recent years these equations have been intensively studied in biology, in population dynamics and bioscience problems, in control problems, etc. The equations have a nonlocal character because their solutions demand a knowledge of not only current conditions, but also of conditions at certain previous moments. The concept of determining equations is also introduced here, and is then used for classification of invariant solutions of the second-order ordinary delay differential equations.

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