

Instructional Alternatives via a Virtual Setting: Rich Media Supports for Teacher Development

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Introduction

Thought Experiments in Mathematics Teaching (ThEMaT), a U.S. National Science Foundation funded project, has created two-dimensional representations of cartoon characters engaged in classroom mathematics. The classroom stories are presented as “live” animations and printed comic strips. By using these stories in meetings of study groups of teachers, the project’s initial thesis has proven itself—rich media technologies can be used to represent classroom stories that stimulate practitioners to engage in revealing conversations about practice; practitioners respond to these stories by producing alternative stories, stories that have happened or could happen in their own classrooms (see Herbst and Nachlieli 2007; Herbst and Miyakawa 2008; Miyakawa and Herbst 2007a, b; Weiss and Herbst 2007).

Based on findings in the literature and on the teaching experience of project staff, we created a model of teaching word problems, word problems of the sort typically encountered in a US Algebra 1 classroom. Our model of the *instructional situation* (Herbst 2006) of doing word problems describes the responsibilities of the students and teacher and the nature of the objects of trade between them. Our model serves as a baseline to interpret ‘usual’ instructional moves and ‘alternative’ instructional moves. We used the model’s hypotheses to invent the two alternatives for the discussion of one problem, which we then represented in the comic strips and animations.

Representing customary and non-standard teaching in a virtual space has particular affordances when it comes to supporting teachers’ reflection on instruction-

Data collection and analysis for Thought Experiments in Mathematics Teaching is supported by NSF grant ESI 0353285 to the Universities of Michigan and Maryland. Any opinions, findings and conclusions, or recommendations expressed in this material are those of the authors and do not necessarily reflect views of the National Science Foundation.

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al alternatives. Whereas sampling from a large corpus of video records may help teachers conceive of alternative moves a teacher might carry out in a given situation, any one video case narrates one story forcefully and provides few supports for alternative stories that might have happened instead or could have happened in another setting. After all, what happened, happened! And, in addition, video is limited to what has actually happened; it can be hard to find examples of teachers trying non-standard instructional moves. It can be especially hard to find a teacher who might want to teach in her classroom exactly what a teacher educator would like prospective teachers to consider, let alone to be able to videotape the exact occasion on which they were able to carry out this instruction.

Animations and comic strips of cartoon characters are a more malleable medium. In this medium, the Animated Teacher can carry out an instructional alternative that teachers would be unlikely to try. This Animated Teacher can also do and say exactly what a teacher educator might wish, though if misused the result may read more like a fable or a fairy tale than a representation of practice. In addition, in this medium, animation designers can create alternative responses, for example, to a student comment. Each of these alternatives exists on the same existential plane and represent as faithfully what might have happened. Inasmuch as they only sketch stories, these virtual representations of teaching invite the formulation of alternatives, the second-guessing of moves, and the projection of the circumstances and settings of viewers (Herbst and Chazan 2006). Such representations of teaching can support teachers by providing opportunities to ponder how different instructional alternatives might play out, as opposed merely to considering alternatives to a particular classroom video. When one of the alternatives includes a non-standard instructional move, there is an opportunity to consider the potential costs and benefits of such an instructional move.

In this chapter, we present an instructional story with variants and illustrate how groups of teachers have used two alternative enactments of a classroom task to reflect on what it means to help students learn to solve word problems.

Using Animations in Teacher Preparation

One of the key challenges in teacher education is to help future teachers imagine possibilities for instructional interaction that they themselves have not seen or experienced. In the service of providing future teachers with images of alternative practice, many teacher educators use videos or visits to classrooms of teachers using such practices. In this chapter, we explore the potential of animations of classroom practice as a new “technology” for teacher preparation. While animations are clearly “authored” texts, without the authenticity of videotaped events that have actually occurred, they have other affordances that may offset this liability. In particular, within the hypothetical space of an animated classroom, as suggested earlier, prospective teachers can observe the “same” teacher carry out the same lesson in different ways with the “same” group of students. The ways in which particular teacher moves might play out differently can then be the focus of conversations about pedagogical actions.

We explore the potential of this new technology for teacher preparation by examining an animation involving a word problem. The students in this animated class generate four potential answers to the word problem. The teacher is faced with the challenge of this diversity in student responses. In one alternative, the teacher asks the students to use their knowledge of the problem to determine which of these answers is a reasonable response to the question and which of these answers cannot possibly be correct. In the second alternative, the teacher asks one student to show how they obtained the correct answer and uses this response to convince students that the other answers are not correct.

This pairing of animations was used with a group of prospective teachers and their mentors (supervising teachers) in after-school meetings during the student teaching semester of the program. The prospective teachers were in their final semester; they had already completed the coursework for their B.A. in mathematics and had taken two “methods” classes in the College of Education. During this semester, they had teaching responsibilities in their mentors’ classrooms. In bringing together prospective teachers and mentors, the research project sought to test a model for understanding student and teacher responsibilities in the solving of word problems. However, for participants, this was not the focus of the sessions; with the participants, we sought to have conversations around specific incidents in teaching where prospective teachers might ask mentors (explicitly or tacitly) why teaching works the way that it does, and where mentors might explain to prospective teachers why, in their view, it does work this way. We call the knowledge at play in these questions and responses practical rationality (Herbst and Chazan 2003), what Schön (1983) calls ‘knowledge-in-action’ and ‘reflection-in-action.’ This is the knowledge that enables practitioners to do what they do; such rationality is common to people who perform the same job. Thus, we hoped that the rationality of the practice of teaching of mathematics would come to the fore around examples of teaching that were not the teaching of any particular mentor or prospective teacher, in a hypothetical classroom onto which the experiences of prospective teachers and mentors might be projected. In one of the sections, we analyze the conversation the prospective teachers and mentors had around these two alternatives to illustrate the kinds of conversations about teaching that can be stimulated by the use of animations in the service of teacher education (an analysis of teacher and student responsibilities when doing word problems can be found in Chazan, Sela, & Herbst, in review).

The “Stories” in the Animated Alternatives

The two animated alternatives we focus on involve a class doing an algebra word problem. The problem is a standard motion problem (Yerushalmy and Gilead 1999). The interaction in the classroom is in many ways consistent with Gerofsky’s (2004) notion of word problems as a genre of problem and the interaction around them as a genre of classroom interaction (which we call “instructional situation”, Herbst 2006). The following development is common to the two alternatives:

Narrator: the class was working on the following problem:

A and B are 280 miles apart. A truck and a cab started traveling at the same time towards each other on the same road. The cab traveled from A towards B at an average speed of 80 miles per hour. The truck traveled from B towards A at an average speed of 60 miles per hour. How long after they set off did the two vehicles meet?

Both of the animated alternatives around this problem begin with students sharing the answers they had gotten for this problem. While the correct answer for the problem is 2 hours, the answers that were offered by the students are: 14, negative 14, 2, 2 hours and 20 minutes. We present with a comic strip the beginning of the story, which is common to the two alternatives (Fig. 1).

The teacher at this point in the animation is faced with a question of how to proceed. How does a teacher deal with multiple student answers in the context of a word problem in an algebra class? Most concretely, on whom should the teacher call? Should the teacher call on a student who has the correct answer and focus on the correct answer and how it was obtained? Will this help the students who did not get that answer? Should students with the incorrect answers be called upon? Should their answers be addressed in the public space, if so, as answers, or as outcomes of solution methods whose flaws can be brought to the attention of the class and thus learned from? How does the teacher help students with incorrect answers understand both that their answers are incorrect and why their answers are incorrect?

From this point in the animated story, there are two diverging alternatives. In one alternative, the teacher makes what, in our model for the teaching of word problems, is a non-standard move. In this alternative, the teacher focuses on the answers as answers, not on the solution methods that led to them. Rather than have students show how they have solved the problem, the teacher focuses on the answers that students obtained and asks students to decide whether or not these answers are reasonable given the circumstances described in the problem. The students resist this move by repeatedly asking for the teacher to tell them which answer is correct and asking the teacher to show how to solve the problem. The other alternative, one that is more standard according to our model, focuses first on the correct answer and moves swiftly to understanding how students got their answers; the question of judging the reasonableness of the answers is set aside, the focus is on how the students solved the problems, whether the solution method is correct or not, and, if not, how it might be corrected.

As teachers of algebra, we tend to focus on the solution method by which a solution was derived. In the context of doing word problems, teachers of an algebra class typically would like to see their students solve the problem by writing an equation and then solving the equation. After all, the function of the word problem in the curriculum is often two-fold, to provide a rationale for the importance of equation solving by illustrating its potential to answer questions embedded in a non-mathematical context and to provide a setting for the practice of equation solving. Such a

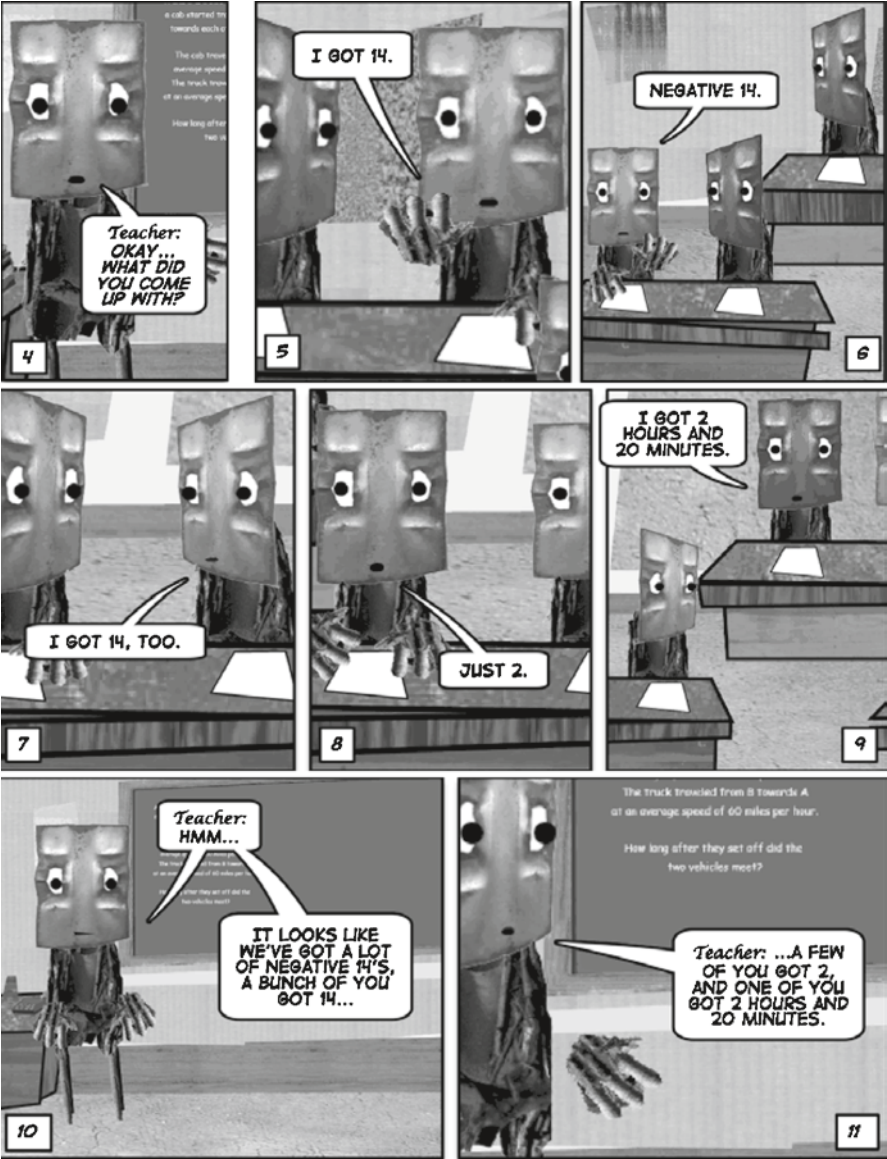


Fig. 1 Common introduction to the two alternatives. (The character set ThExpians M and the comic strip are © University of Maryland and University of Michigan)

focus on solution method however can obscure other important mathematical lessons, like learning to check the reasonableness of one's solutions. Particularly when it comes to solving applied problems, it is important to verify that the solution one has come to mathematically indeed is a reasonable solution to the problem. There are algebraic solution techniques that sometimes can lead a person to a solution to an equation that is not the solution to the problem as articulated in a context. Thus,

on occasion, it seems valuable to consider asking students to argue for the reasonableness of the solutions they have found. An important task for teacher education is to help prospective teachers consider how and when to engage students in discussion about the reasonableness of answers. In the context of these word problem animations, this general lesson is bound up with the question of whom to call when there are multiple answers, and why?

Below, we present the two alternatives in detail, reviewing key elements of the alternative stories and providing commentary. For each alternative we suggest specific sets of questions that might be addressed to both prospective and practicing teachers about these stories.

The Alternative “A Correct Solution Method” and Questions for Teacher Discussion

In this alternative, the Animated Teacher focuses on the right answer by choosing to call on Orange, a student who had the right answer, to come up to the board and show what he did.

We discuss this alternative in three parts. In the first part of this alternative, Orange uses a common ‘method’, one that the teacher has presumably taught: a “Distance equals Rate times Time” chart¹ to generate an equation to solve. Orange writes on a board a chart and an equation, and does not explain what he has written (Fig. 2):

The teacher encourages him “That looks good. Well done, Orange”. Questions that might be addressed to teachers for this part of the animation include:

- Why might a teacher want a class to examine Orange’s work? What are the pros and cons of choosing Orange’s work for examination?
- Is Orange’s work sufficient? Orange did not speak, do we as teachers, or do the other students in the class, need further explanation?

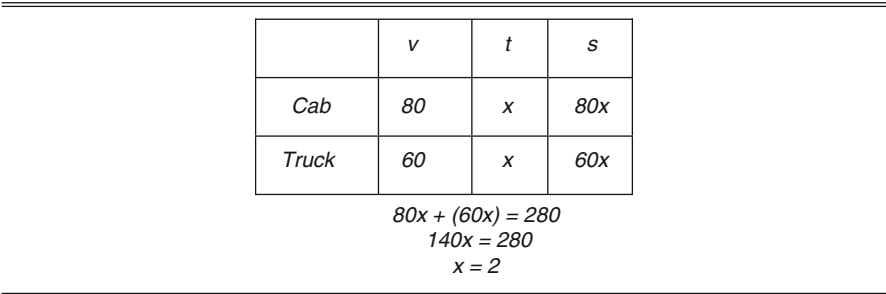


Fig. 2 Orange’s solution

¹ See Hall et al. (1989) though rather than writing $d=r * t$, they write $v * t=s$.

- Is the chart an important component of the solution process? How is the equation generated from the chart? Do you think this is a useful tool for teaching students how to solve word problems, why or why not?
- Should the teacher affirm Orange's work (at this stage) as correct, why or why not?

In the second part of this alternative, Purple takes the initiative and has the courage to ask what is wrong with his solution:² "I got 2 hours and 20 minutes. I don't know what I did wrong. Can I show what I did?" The teacher does not take up the request to spend classroom time on understanding Purple's solution. The teacher says: "Why don't you just be sure to copy the correct answer from the board and try to see what went wrong in yours. If you're still having trouble, we can talk after class."

Questions that might be addressed to teachers at this part of the animation include:

- Is Purple's question a typical question, why or why not? Is it a good question for a student to ask? Would you like your students to ask this sort of question, why or why not?
- Do you like how the teacher responded to Purple or not? If not, how should the teacher react to Purple's question? What effect might the teacher's response have on Purple? On other students in class?
- What might have been some reasons for the teacher's response?

In the third part of this alternative, after setting aside Purple's question, the teacher decides to question the students who though the answer was 14: "I'm curious how so many of you got 14 as an answer. Can somebody show us how you got that answer?" Red comes to the board and uses Orange's table to show the class that thinking about the speeds as directed quantities (the velocity of the truck is negative) results in another equation (one that does not adequately capture the situation) and a solution (Fig. 3).

Orange asks why Red wrote negative 60. Blue joins Red: "Yeah, that's right. I did the same thing, but I had 80 as negative...so I got $x = -14$." Red replies that it's because they traveled in opposite directions. The teacher praises the students that thought about negative speed, but says it isn't appropriate to use a negative number for speed here.

$$\begin{aligned} 80x + (-60x) &= 280 \\ 20x &= 280 \\ x &= 14 \end{aligned}$$

Fig. 3 Red's solution

² Note that this is often something that students might ask when they have an incorrect answer and when they do not understand why what they did was incorrect, see Office of Educational Research and Improvement (1998).

Questions that might be addressed to teachers at this part of the animation include:

- Given that the correct answer had been given, why might a teacher continue the discussion of the problem?
- If a teacher wanted the class to discuss 14 as a solution, should this discussion occur before Orange presented a correct solution, why or why not?
- Why might a teacher be curious about the 14 and not about the 2 hour and 20 minutes?

The Alternative “Reasonableness of Answers” and Questions for Teacher Discussion

In this alternative, instead of having students share how they solved the problem, the Animated Teacher focuses on the reasonableness of the answers obtained. We discuss this alternative in two parts.

In the first part of this alternative, the students are asked to work in groups in order to figure out which one of the solutions: 14, negative 14, 2 or 2 and 20 minutes, is correct, and also what they need to do to show that the correct answer is indeed correct. A few minutes later, Green presents one group’s findings: “In our group, we couldn’t decide which one was correct; all of them seemed okay for different reasons. Can we solve it together on the board?” The Animated Teacher returns the question to the class: “Let’s go back and try to see how you decided which one of the solutions is correct.” One student wants to rule out the negative 14 answer by saying: “Time can’t be negative.” The teacher does not acknowledge this bid initially and directs the discussion to the meaning of 14 in this context by asking questions like: “What does the number 14 represent?”

Questions that might be addressed for this part of the animation are:

- Is this a good context/problem for group work?
- Should the teacher have said that one (and only one) of these answers is correct?
- How could Green’s group decide that each solution is reasonable?
- How might a teacher react now?

In the second part of this alternative, the teacher continues to ask students to decide whether or not a given answer is reasonable and fits with the given situation. The conversation becomes heated when there is a return to the answer negative 14. The teacher asks: “What does it mean when we get negative 14 as an answer? Can you explain with the story what negative 14 might mean?” Students are not able to answer these questions to the teacher’s satisfaction and the teacher continues to probe. The students begin to lose patience, saying things like: “Just tell us what the answer is! Show us how to get it! Let’s just solve it together on the board.” The teacher settles the class down, saying it is important to understand how the numbers they get are related to the story, and that it allows them to decide if a solution is reasonable, “You have to use your commonsense to see if your answer is correct. Without doing that, you could write down an answer that couldn’t possibly be true”. The teacher

makes one more pass at negative 14 and then relents and indicates that negative 14 is ruled out, but still wants to continue to discuss the other answers. When one of the students says he is sure that 2 is the right answer, the teacher asks if it is the only correct answer, and asks to explain by using the story, and not by writing an equation and solving it. Eventually, one student articulates that 14 couldn't possibly work because the cab and truck would travel too far, then another student illustrates how with 2 hours the cab and truck would both be between A and B and the sum of the distances traveled would be 280. Even after this progress, some students seem perturbed by the fact that they haven't solved the problem. Blue comments: "I don't understand why we are spending so much time on this one problem without even solving it".

Questions that might be addressed here include:

- Why might students start to lose their patience?
- What are students' expectations about solving word problems? About time allocation?
- How is the questioning of the students influencing their participation and their understanding?
- Is it a problem if classroom conversation gets heated?

Comparing the Two Alternatives

After having seen both alternatives and having discussed them both, in discussions with teachers, one might consider the following questions:

- What is the difference between the approaches taken in the two alternatives?
- What are the advantages of each of them? Disadvantages? Constraints?
- Could a teacher enact both of these strategies in one classroom session? If so, would there need to be a particular order, or could either order work?

Mentor/Prospective Teacher Discussion of the Animations

While the previous section focused on how the animation might be used with teachers, this section illustrates our use of this animation with a particular group that consisted both of prospective teachers and their mentors. The meeting took place during the student teaching semester when prospective teachers have mentors in whose class they are teaching. Convening a group of pairs, each pair consists of an experienced teacher and his or her student teacher, to discuss the animation and its two alternatives, had some additional affordances beyond those described earlier. The study group described here consisted of thirteen teachers: six mentors and seven prospective teachers during senior's year in a four-year undergraduate secondary teacher preparation program. They watched and discussed the alternative "reasonableness of answers" first, and then they dealt with the alternative "a cor-

rect solution method”. The alternatives could have been used in a different order. Our choice was to start with a less-typical teacher response figuring that then there would be more vigorous response around the more typical teacher response when it was shown.

The study group was not a class. No grades were given; participants were given financial incentives for attending. The broad goal of the discussions was, in the context of “soft” professional development, creating conversation between prospective teachers and mentors where the prospective teachers could ask mentors how teaching is supposed to go and where mentors could share their wisdom of practice. The facilitator did not have an explicit agenda to convince participants of particular understandings of teaching.

In accordance with the project’s goals, we did not use the specific questions suggested in the previous section to probe the participants’ thinking. Specifically, the conversation about this animation and its two alternatives targeted the following key questions:

- How do teachers talk with students about how to decide whether answers to word problems are correct or incorrect?
- To what degree should teachers teach students to use the context of the word problem as a check on solutions, or is the check tied up with methods for generating and solving an equation?

Two people facilitated the group discussions. One person took the lead in directing the conversation. The other asked for points of clarification around the specific teaching aspects in each animation.

Typically, after watching an animation, the teachers in the study group were invited to share their thoughts with the group. The overall tenor of the discussions was serious and positive. The teachers treated the animations as instances of teaching practice and shared their thoughts with the group.

Discussion of the Alternative “Reasonableness of Answers”

The discussion of this alternative encouraged the prospective teachers to ask their mentors about the importance of checking the reasonableness of answers in the context of solving word problems. The mentors used this occasion to share their experience by responding to the comments that the prospective teachers raised.

The initial part of the discussion between prospective teachers and mentors was about the importance of determining if an answer is reasonable. It began when one of the mentors (Ralph³) was enthusiastic about the Animated Teacher’s spending time teaching students to judge the reasonableness of answers. Ralph seems to appreciate what the Animated Teacher did, even though he considered the teaching unusual.

³ This name and other teacher names are pseudonyms assigned to protect confidentiality.

Ralph: You know what I like about this video [the animation they just watched]? Because, I'm always telling the kids to "check the reasonableness of your answer", but I liked it that the teacher actually spent time **teaching**⁴ checking the reasonableness of the answer. Because that -14, I said [to myself], "Oh please, that's something my kids would do" [laughter]. And it would make perfect sense to them that that [time cannot be a negative number] was the correct answer.

After eight turns, a prospective teacher (Pat) makes what Sfard (2001) calls "pro-active utterance", an utterance that calls for a group reaction. She expresses her concern about teaching students to test the reasonableness of their answers: "I didn't really like the way she [the Animated Teacher] rejected the equation". The contrast between Pat's remark, as a prospective teacher, and Ralph's, the mentor, initial remark, implicitly asks: "Why did you like this teaching? And, why do you think reasonableness is so important?" Pat continues her thought by wondering why the Animated Teacher discouraged the using of equations:

Pat: I think she [the Animated Teacher] should have encouraged the fact that they [the students] used an equation and said, "If we have time after our reasoning process, maybe we'll go back to that [equation] at the end of class". Or, like, because they [the students] are going to be asked to set up equations, so discouraging that [the use of equations], I don't think is beneficial, necessarily, especially if that [writing equations] helps them organize their thoughts.

Other mentors reacted to Pat's concern by arguing for the importance of teaching to check the reasonableness of answers. They did it by describing alternative ways, which are, to their mind, more effective in accomplishing this goal. For example, Floyd offered a different activity structure, that of preparing students for exams with multiple-choice questions. He suggested that it is better to deal with reasonableness within a context where students do not have solution methods, only answers. This way they can reason about the answers without thinking about the method they have used and without being biased by the solution that they have produced:

Floyd: I wasn't against the reasonableness of the solution, but I probably wouldn't have used it [reasonableness] in that particular problem. It [the problem I would use] would've been more like a multiple choice. I always tell my students there are 4 choices, 3 of them wrong. Now explain to me why... Some days I do, instead of saying, 'What is the right answer?', I say, 'Why are the other 3 answers wrong?' [Ralph nods with agreement].

From there, everyone seemed to accept this line of reasoning. In contrast to his first remark that suggests he rarely teaches students to judge the reasonableness of their answers, the first mentor (Ralph) then says that in the context of multiple choice questions, he does that kind of work all the time; his (prospective teacher) mentee even confirms he had done something of that sort on that very day. So, the sense of the group seemed to be, this is a valuable focus, but, in response to Pat, not in this sort of problem context. Mary, a prospective teacher, reinforced Floyd's point. She hypothesizes that when students first work on a problem themselves, they are too focused on their own solutions, so they cannot think of other answers:

⁴ Oral stress in vocal track.

Mary: I thought maybe the kids [in the animation] didn't really want to [spend time on reasonableness], because everyone was very focused on their answer. Like, the one kid, I think it was Blue, kept saying "No, my answer's right, my answer's right." I don't think he got anything there talking about reasonableness, because he just wanted to sit and say "My answer's right, I know my answer's right, I don't have to explain it." Some of the kids who, like, had answers were more focused on their answer than checking what the other answers were.

Discussion of the Alternative "A Correct Solution Method"

While in the alternative "Reasonableness of Answers" the Animated Teacher focuses on the reasonableness of each answer without paying attention to the 'method' used to produce it, in this alternative the teacher focuses on the right answer. The Animated Teacher calls on a student who shows the 'method' (chart/table and equation), without an explanation (i.e. how one produces the equation) and without checking the reasonableness of the answer.

We expected the study group teachers to react to this alternative in light of the previous one, saying that reasonableness is important. Indeed, while watching the animation, the teachers were very surprised by the Animated Teacher's turn, "Copy the correct answer from the board". They reacted with an uproar, loud laughter and sharp movements: two mentors clapped their hands, one mentor held his head in his hands, and one mentor got up from her chair; one prospective teacher also got up from his chair.

Similar to the initial responses to the previous alternative, prospective teachers wanted the Animated Teacher to ask Orange to explain his solution. Also similar to the previous alternative, mentors focused on explaining why the teacher's act represents the reality of classrooms. Lea (a prospective teacher) was the first to say that the Animated Teacher was wrong to not have Orange explain his answer:

Lea: If you let the first guy [Orange] explain [how he constructed the equation], the guy [Orange] just, like, put it [the chart and the equation] up the board and he's [the Teacher] like [said to Orange] "Okay, good, sit down".

Lea thinks that explaining the answer could help other students understand why they were wrong:

Lea: Maybe if he [Orange] explained what he was doing... then the guy who got 2 hours and 20 minutes would have been like, "Oh, I see what he did [and therefore I understand what I should have done]". And then he [the student who got 2 hour and 20 minutes] could have come back to his [solution] and been [thinking] like, "Oh, I see why I'm wrong".

Pat (a prospective teacher) suggests that the exact place where an explanation is needed is the equation because this is the place that she herself had difficulty with: "I think... I wouldn't have thought to do $280 = 80t + 60t$."

Mentors' reactions then came in with Ralph making a sharp shift in the conversation back to the Animated Teacher's response to the student who wanted an explanation about 2 hours and 20 minutes. His turn seemed to shift the overall tenor of the

discussion. He looked very decisive, striking the table with his hands and looking at his colleagues around the table and addressing the prospective teachers:

Ralph: But you [the prospective teachers] will [act as the Animated Teacher did by asking students to copy the right answer], and I know you will. Next year you're going to be crankin' through your lesson, and some kid's going to have an off-the-wall answer [like 2 hours and 20 minutes] and you're going to say [like the Animated Teacher did], "I'm sorry, I don't have time to explain it", because you [as a teacher] have to cover all this stuff by the end of the year.

Similar to his role in the discussion around the alternative "reasonableness of answers", Ralph talks about the distinction between theory and practice. His teaching experience allows him to contend that the prospective teachers are paying too little attention to actual classroom constraints. According to Ralph, explaining answers is great, but having to cover content does not allow teachers to spend class time on explaining each answer. It is interesting to note here that while the prospective teachers were upset that Orange did not explain the correct answer, Ralph's response focused on not explaining other wrong answers like Purple's. It seems that Ralph agreed with the prospective teachers that Orange's answer should have been explained. But he wants the prospective teachers to realize that sometimes as a teacher you cannot deal with every answer a student puts forth.

Craig (another mentor) was concerned with "how" the Animated Teacher acted, more than "what" that teacher did. He felt that the Animated Teacher acted poorly and should have said that there was no time to explain.

Craig: Seriously, if the teacher had just said that [not having the time to explain it], I would have said OK [It is fine not to spend time on Purple's wrong answer].

Craig's response suggests that there are circumstances under which not explaining the answers is appropriate. But the prospective teachers still remain troubled. Mary and Lea remind the group of the Animated Teacher's decision to ignore Purple's answer (2 hours and 20 minutes), but to address another answer (14) without any explicit reason. Mary: "And then they [the Animated Teacher] went and put another wrong answer on the board." Lea: "One wrong answer was better than another wrong answer".

Darcy, a mentor, agrees: "[the Animated Teacher said] I don't really want to see why you got it (2 hours and 20 minutes), but I want to see the 14". The study group members reply to this concern by relating to circumstances in which it makes sense to address the 14 but not to address the 2 hours and 20 minutes. Pat (prospective teacher) suggests that the Animated Teacher might have anticipated the 14, but not the 2 hours and 20 minutes. Two mentors and another prospective teacher propose that many students in the classroom got 14, which led to the decision to focus on the common answer. Ralph (mentor) suggests another circumstance, which causes teachers not to deal with an answer:

Ralph: That kid [who asks the teacher to look at his wrong answer] might have been Brandon [a student in his class], who purposely goes on the board. He knows he has the wrong answers, but he wants the attention from the class for about 5 minutes, so he'll volunteer to go do the problem on the board.

Concluding Remarks

The discussions between mentors and prospective teachers around the two alternatives of the story brought forth some ideas that both mentors and prospective teachers seem to agree upon:

- Teaching students to judge the reasonableness of their answers in the way that the Animated Teacher did is a non-standard act. What makes it non-standard, and not viable in the reality of classrooms, is that when students have solution methods as well as answers, and when it is those methods that are what must be learned, focusing simply on the answers is a counter-cultural act, because the answers are not what is important. Reasonableness of answers is better focused on in other contexts.
- Choosing to deal with one wrong answer and not with others, emphasizes the difference between the desirability of responding to all students and the reality of what actually can happen: during the lesson, the teacher has to be flexible and make in-the-moment decisions according to circumstances, and cannot act only according to generalities about what is desirable.

In terms of the mentor/prospective teacher interaction during this conversation, it is interesting to note that a prospective teacher raised concerns about the Animated Teacher's action (checking the reasonableness of answers), while the mentors appreciated an aspect of that same action. As mentors work to identify what they value in the teacher action, they use their knowledge of the practice and are reminded of a teaching context in which examining the reasonableness of answers is easier to accomplish. This difference in point of view between prospective teachers and mentors about a non-standard teaching move supported a meaningful conversation where they could examine teaching practice as a group of experienced and novice teachers. In general, across all of our study group sessions, the prospective teachers and mentors use the animations to think about their own teaching and talk about themselves and their own teaching, and to say what teachers should and should not do. Indeed, use of the animations created a venue for mentors sharing the wisdom of the practice and for prospective teachers to ask questions about teaching that are on their mind.

In terms of the affordances of this animation for teacher preparation, we think this particular animation can be used in the context of "methods" courses to have prospective teachers explore issues of teaching like which student to call when students have multiple answers. The questions we proposed earlier are designed to support the use of this animation for such a purpose.

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Zaslavsky, O.; Sullivan, P. (Eds.)

2011, X, 330 p., Hardcover

ISBN: 978-0-387-09811-1