

Preface

This book has its roots in a course I taught for many years at the University of Paris. It is intended for students who have a good background in real analysis (as expounded, for instance, in the textbooks of G. B. Folland [2], A. W. Knap [1], and H. L. Royden [1]). I conceived a program mixing elements from two distinct “worlds”: functional analysis (FA) and partial differential equations (PDEs). The first part deals with abstract results in FA and operator theory. The second part concerns the study of spaces of functions (of one or more real variables) having specific differentiability properties: the celebrated Sobolev spaces, which lie at the heart of the modern theory of PDEs. I show how the abstract results from FA can be applied to solve PDEs. The Sobolev spaces occur in a wide range of questions, in both pure and applied mathematics. They appear in linear and nonlinear PDEs that arise, for example, in differential geometry, harmonic analysis, engineering, mechanics, and physics. They belong to the toolbox of any graduate student in analysis.

Unfortunately, FA and PDEs are often taught in separate courses, even though they are intimately connected. Many questions tackled in FA originated in PDEs (for a historical perspective, see, e.g., J. Dieudonné [1] and H. Brezis–F. Browder [1]). There is an abundance of books (even voluminous treatises) devoted to FA. There are also numerous textbooks dealing with PDEs. However, a synthetic presentation intended for graduate students is rare, and I have tried to fill this gap. Students who are often fascinated by the most abstract constructions in mathematics are usually attracted by the elegance of FA. On the other hand, they are repelled by the never-ending PDE formulas with their countless subscripts. I have attempted to present a “smooth” transition from FA to PDEs by analyzing first the simple case of one-dimensional PDEs (i.e., ODEs—ordinary differential equations), which looks much more manageable to the beginner. In this approach, I expound techniques that are possibly too sophisticated for ODEs, but which later become the cornerstones of the PDE theory. This layout makes it much easier for students to tackle elaborate higher-dimensional PDEs afterward.

A previous version of this book, originally published in 1983 in French and followed by numerous translations, became very popular worldwide, and was adopted as a textbook in many European universities. A deficiency of the French text was the

lack of exercises. The present book contains a wealth of problems. I plan to add even more in future editions. I have also outlined some recent developments, especially in the direction of nonlinear PDEs.

Brief user's guide

1. Statements or paragraphs preceded by the bullet symbol **•** are **extremely important**, and it is essential to grasp them well in order to understand what comes afterward.
2. Results marked by the star symbol **★** **can be skipped** by the beginner; they are of interest only to advanced readers.
3. In each chapter I have labeled propositions, theorems, and corollaries in a continuous manner (e.g., Proposition 3.6 is followed by Theorem 3.7, Corollary 3.8, etc.). Only the remarks and the lemmas are numbered separately.
4. In order to simplify the presentation I assume that all vector spaces are over \mathbb{R} . Most of the results remain valid for vector spaces over \mathbb{C} . I have added in Chapter 11 a short section describing similarities and differences.
5. Many chapters are followed by numerous exercises. Partial solutions are presented at the end of the book. More elaborate problems are proposed in a separate section called “Problems” followed by “Partial Solutions of the Problems.” The problems usually require knowledge of material coming from various chapters. I have indicated at the beginning of each problem which chapters are involved. Some exercises and problems expound results stated without details or without proofs in the body of the chapter.

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Haim Brezis
Rutgers University
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