

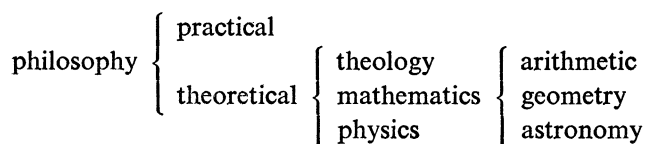
Physics and Philosophy in the *Almagest*

The *Almagest* begins with a preface [I, 1; Hei 1, 4]¹⁾ in which the whole work is dedicated to a certain Syrus, about whom we know nothing. He must have been rather closely connected with Ptolemy, who dedicated other works to him also, thus the Planetary Hypotheses, the Handy Tables, and the Tetrabiblos, i.e. the main body of his astronomical and astrological writings.

This introductory chapter is of considerable importance from a philosophical point of view. In general, the *Almagest* is a highly technical and closely reasoned account of mathematical astronomy, written in a style which but rarely reveals anything about the personality of its author and his more philosophical opinions. The subject matter is everything and the author stands back all the time to let it appear in as objective a light as possible. In fact, the preface is one of the very few sections of the work in which we get more than a glimpse of Ptolemy's ideas on such general questions as epistemology, the classification of the sciences, the human value of scientific research, and the ethical implication of the study of astronomy.

The Classification of Knowledge

Let us first consider the Ptolemaic classification of the various kinds of knowledge. It can be illustrated by the following scheme:



Here the first major division is between practical and theoretical knowledge. Ptolemy shows that this distinction is essential by means of an example: A man may have great insight in moral questions, and even extend it through his everyday experience of life, without any specialized education in ethics; but it is impossible to acquire any knowledge of the universe without theoretical studies in astronomy. How important this distinction was to Ptolemy appears from the fact that the Preface opens with a

1) This means Book I, Chapter 1, Heiberg's edition vol. 1, page 4. All references to the *Almagest* will be given in a similar way.

praise of the 'genuine philosophers' who first noted it. These philosophers are not mentioned by name, but the distinction between practical knowledge resulting in human skill in arts and crafts, and theoretical knowledge leading to intellectual understanding is a commonplace among all philosophers of the Peripatetic School both in Antiquity and the Middle Ages. Therefore, the commentator Theon of Alexandria (ed. Rome, p. 321) has no difficulty in tracing it back to Aristotle himself, who adverts to it in many of his works (e.g. *Metaph.* vi, 1, 1025 b ff.). However, it is worth noticing that Ptolemy does not include the third Aristotelian category – poetic knowledge – in his scheme, although he was deeply impressed by the universal truth, the aesthetic function, and the emotional value of astronomy.

Since Ptolemy had a marked predilection for theoretical knowledge, we must now consider his division of this branch into theology, mathematics, and physics. Here again he followed Aristotle, who deduced these categories by his theory of abstraction. At the bottom of the scale we find 'physics', or natural science, which was defined by Ptolemy as the study of the ever changing material world. For Ptolemy physics was concerned with questions like e.g. whether a material substance is hot, or cold, or sweet, etc. Most of its objects belong to the corruptible part of the universe inside the sphere of the Moon. This is, roughly speaking, in agreement with Aristotle, who defined physics as the study of 'nature', and nature as the principle of 'motion' or change (*Phys.* ii, 1, 192 b, cf. *Metaph.* vi, 1, 1025 b). This implies both that physical objects are material, and that most of them (all apart from the heavenly bodies) are subject to generation and corruption. What Ptolemy omits to mention – but certainly maintains – is that in physics such objects are studied with particular regard to their materiality.

At a more abstract degree of knowledge we find the science of mathematics, which to Ptolemy is an investigation into the nature of the forms and motions of material bodies, implying notions like figure or shape, quantity, magnitude, space, and time. Again this is in agreement with the Aristotelian doctrine (*Phys.* ii, 2, 193 b) of mathematics as an abstract science of the physical world, studying the same bodies as one does in physics, but without regard to their constituent matter and concentrating on their 'mathematical' properties.

It is one of the main tenets of Aristotelian epistemology that it is possible to carry the process of abstraction even further and to study the world under still more general points of view in terms of being, existence, cause, effect, and similar 'metaphysical' concepts. The science resulting from this most general kind of inquiry is called theology both in Aristotle (*Metaph.* vi, 1, 1026 a) and in Ptolemy. In the former it is crowned with a proof of the existence of a Supreme Being called God and conceived as the 'Prime Mover' of the whole universe. In another place it is defined more directly as the knowledge of the invisible, immaterial, and unchanging God (*Metaph.* iv, 8, 1012 b, cf. xii, 7, 1072 b). Ptolemy also infers the existence of God in this philosophical sense from a metaphysical argument: The senses are incapable of analyzing the phenomena of the material world into their constituent matter, forms, and motions. This can be accomplished only by reason, with the result that reason not

only shows us motion as something different from matter and form, but also reveals an ultimate cause of all motion, i.e. God.

On Certainty in the Various Fields of Knowledge

The three main branches of theoretical knowledge have not the same epistemological status. In his Preface Ptolemy is concerned primarily with the degree of certainty which they provide. Here he finds characteristic differences which to him are sufficient motivation for his personal preference for mathematics as the most perfect discipline. Like Plato, Ptolemy is deeply impressed by the perfect character of mathematical truth. Theology has no similar certainty, because its object is absolutely invisible and incomprehensible. Physics, too, is unable to attain absolute truth because of the corruptibility and obscurity of matter. This is why there is no hope that philosophers will ever agree, or arrive at a common opinion on these two branches of knowledge.

It is obvious that here Ptolemy is more agnostic than Aristotle. To the latter theology was the same as metaphysics, and he would surely have maintained that metaphysics is able to analyze the whole realm of existence in terms of metaphysical relations of an absolutely true character. But to Ptolemy theology is defined by means of the concept of God as a transcendent being beyond human comprehension, – with the corollary that theological statements are less certain than those of mathematics. Moreover, the reference to the disagreement between philosophers reveals that Ptolemy belongs to a later age than Aristotle. In fact, they were separated by a period of five hundred years in which one conflicting school had succeeded another and many different opinions on the nature of God had been ventured. This explains Ptolemy's reticence towards theology. He is certainly no unbeliever, as both the preface and several other places in the *Almagest* show. But he cannot ascribe to theological statements the same epistemological quality as to mathematical theorems. As we are going to see, this view entails the consequence that Ptolemy warns us against too much anthropomorphism in science. Before entering upon the very complicated theory of latitudes in Book XIII (see page 355) he tries to refute the opinion of those philosophers who maintain that astronomers ought to construct simple theories only, since the simplicity of the Supreme Being should be reflected in the description of His works. Ptolemy reminds us, first, that philosophers are not agreed upon what 'simple' means, and second, that what is simple to God may appear far from simple to man. Simplicity in God does not entail 'simplicity' in the description of nature.

In the same way physics has to be put in a more humble position for the reason that its subject matter – the material world – is both obscure and corruptible. The obscurity of matter is a commonplace in all Greek philosophy and it is no wonder that Ptolemy subscribes to this idea. It is a little more curious that he refers also to the corruptibility of matter as an obstacle making absolute truths about the physical world impossible.

Ptolemy's Conception of Mathematics

It would seem that the reason for this opinion is a conception of science slightly different from that of Aristotle to whom a true scientific statement has the character of being invariably true at all times and under all circumstances. Thus the statement that heavy bodies have a natural tendency to move towards the centre of the world is always true regardless of all the contingent vicissitudes of actual heavy bodies. So the existence of change and corruptibility in the physical world does not exclude the possibility that there may be invariable relations among its ever changing things, expressed in eternally true statements. Here Ptolemy seems to disagree when stating in the preface that *astronomy alone is concerned with the investigation of a world remaining the same throughout all eternity [...] which is a characteristic property of science* [I, 1; Hei 1, 6]. In other words, to Ptolemy, eternally true statements can only be about eternally unchanging objects.

We must now examine Ptolemy's conception of mathematics a little closer. First, mathematics is a very general science. It can be grasped through the senses or without them, just as there is a mathematical aspect of any kind of material being – corruptible as well as incorruptible. Perishable substances change with their changing forms and are thus objects of mathematical study, just as well as the unchanging forms of eternal beings of an ethereal nature, i.e. the heavenly bodies.

Next, mathematics leads to absolutely certain truths which, once established, can never be subjected to doubt. This is because mathematical truth is acquired by means of logical proofs, regardless of whether they are concerned with arithmetic or geometry. It is this logical character which conveys a certainty never to be attained through the testimony of the senses, and which is foreign to physics. Ptolemy confesses that we here have his own deepest personal motive for becoming a mathematician.

Mathematics, Astronomy, and Astrology

One could think that this might have disposed him to become a 'pure' mathematician. He did, in fact, a small amount of theoretical research in this field (see page 47) but the preface does not conceal that astronomy was his main interest, at any rate when he wrote the *Almagest*: *This is the reason which has moved us to devote ourselves – to the best of our abilities – to this pre-eminent science in general [i.e. mathematics], but particularly to that branch of it which is concerned with the knowledge of the Divine and Celestial Bodies*, after which follow the words quoted above on the unchanging nature of the heavens and the ensuing timelessness of astronomical truths.

At this point there arises a difficulty: How could Ptolemy become an astronomer if only mathematics was able to satisfy his longing for eternal truth? This question seems to be connected with an apparent ambiguity in Aristotle's opinion on where astronomy should be placed in the hierarchy of knowledge. In the *Physics* (ii, 2, 194 a) he calls astronomy *rather physical than mathematical* and compares it with optics and

harmonics (i.e. the theory of music). But in the *Metaphysics* (xii, 8, 1073 b) astronomy appears as *one of the mathematical sciences nearest to philosophy*. In Ptolemy so much is clear that astronomy is not tainted with the obscurity and uncertainty ascribed to physics, notwithstanding the fact that it is concerned with the heavenly bodies. These bodies have a material character both to Aristotle and Ptolemy, although their matter is of a particular, ethereal nature. Nevertheless, this does not prevent astronomy from being of a loftier status than the other natural sciences.

The solution of this puzzle seems to be that Ptolemy drew a different boundary between mathematics and physics from Aristotle's. It is not said in so many words, but it is as if he thinks as follows: If one studies the changing and corruptible material world the resulting science belongs to physics, even if it has a mathematical form. On the other hand, a study of the unchanging and eternal heavens of a similar mathematical form belongs to the science of mathematics.

If this is the correct interpretation, it is clear that Ptolemy's classification of science rests upon a different foundation from that of Aristotle. The latter had defined the limits of physics, mathematics, and metaphysics (or theology) by means of the formal objects only of these sciences; and since physics studies natural objects in their material form he was compelled to include any natural science in physics, even if its statements were expressed by means of mathematics. Ptolemy goes a step beyond Aristotle in using the material object of science also as a basis for classification. It is the different properties of celestial and terrestrial matter which enable him to lift astronomy out of the realm of physics into the domain of mathematics.

It should be noticed that at this particular point he had no great following among ancient philosophers of science, who in general kept to the more logical and unambiguous Aristotelian principle of regarding the formal object only. Later, in the Middle Ages, the difficulty was felt anew and gave rise to the doctrine of *scientiae mediae*. These are sciences which, like astronomy, optics, harmonics, and mechanics, are concerned with a mathematical description of the material world. Accordingly they have something in common with both physics and mathematics without being totally subsumed under either of these headings²⁾.

The status of astronomy as a part of mathematics is also, perhaps, the explanation of the very remarkable fact that the *Almagest* is completely free of astrology. This is not because Ptolemy did not believe in the possibility of making predictions from the stars. On the contrary, his *Tetrabiblos* is one of the most comprehensive manuals of judicial astrology ever written (see page 400). But astrology is concerned with the influence of the celestial world upon the terrestrial, and the influence of the stars is very closely connected with their physical nature. This means that in astrology one cannot abstract from the material qualities of the stars; for that reason astrology must be classified as a part of physics, and not treated on a par with mathematical

2) Thus St. Thomas Aquinas maintains that *some of the sciences applying mathematics to natural phenomena are placed in between, for instance music and astronomy. However, they are more related to mathematics because what interests the physicist in them is rather the material aspect, but what interests the mathematician is more the formal aspect.* – In *Libr. Boethii de Trinitate* q. 5, a 3, ad 6^m.

astronomy even if it makes use of mathematical computations. This is true without regard to the fact that in popular terminology a ‘mathematician’ often means an astrologer, as we see e.g. from St. Augustine’s fulminations against them (*De civ. Dei* v, 5, but cf. *De Trin.* ix, 6).

The Human Value of Astronomy

The special status of astronomy makes it useful to other sciences. Thus it helps theology because it draws our attention towards the celestial world and helps us to discover the First Cause of its motion – the Prime Mover being God Himself: *It opens the road to the Divine world by imparting knowledge of a force which is eternal and different from every other. It alone is able to discover the relations between eternal substances not subject to any influence, and the sensible, moving and movable world, through the phenomena, order, and disposition of their motions.*

This argument follows the Aristotelian proof of the existence of God as the Prime Mover. When theology in the Middle Ages rose to the top of the hierarchy of knowledge, the argument explained also why astronomy was given the most honourable position among the various sciences. Ptolemy himself comes very near to the same attitude when he maintains that the study of astronomy has ethical consequences and is a step on the road to human perfection: *More than any other thing it contributes to make us better, making us more aware of what is good and beautiful in the moral life. For those who study this subject find a harmony between the Divine things and the beautiful order of the propositions. This makes them love this Divine Beauty and makes them accustomed to take it as a model of their conduct.* Thus the harmony and beauty of the celestial world become a kind of second nature in those who study it. If this is a testimony of personal experience, and not only a repetition of an old Platonic idea from the *Timaios* (90 c), it certainly reveals something of the spiritual life of Ptolemy himself.

Also physics may profit from mathematics and astronomy, since the particular properties of material bodies are revealed through their characteristic motions which are described mathematically. Thus the natural motion of

the corruptible is	along a straight line
the incorruptible is	circular
the heavy and passive is	towards a centre
the liquid and active is	away from a centre

This is traditional Aristotelian physics. But Ptolemy proves to have been influenced by later schools of thought when he characterizes heaviness as a passive, and lightness as an active quality of matter. This was one of the main features of the physics of the Stoics³).

3) On the influence of Stoic philosophy upon Ptolemy see Lammert (1922). On active and passive qualities, see Galen, *Introd. s. medic.* 9, quoted by Sambursky (1959) p. 119.

This is, therefore, the final purpose of the *Almagest*: *We shall always strive to augment this Love of the Science of eternal things, not only by making us acquainted with the achievements of those who have studied these things before us, but also by contributing a little ourselves with discoveries made in the brief interval since then. Thus we shall seek to describe briefly every important contribution from our own time [...]. Finally, in order to attain the purpose of this work we shall in a suitable order comment upon anything useful to the theory of the Heavens. In order to make the exposition as brief as possible we shall only summarize that which has already been ascertained by the Ancients, concentrating upon problems not sufficiently dealt with or proved before.*

The Plan and Disposition of the *Almagest*

After the Preface follows a short chapter [I, 2; Hei 1, 8 ff.] in which the general plan of the *Almagest* is outlined. It will contain

- A. A brief general part dealing with the Earth and its position in the universe as a whole [I, 3–11].
- B. A long special part comprising 3 sections:
 - 1) Spherical-astronomical problems and their mathematical solutions [I, 12–II];
 - 2) the theory of the motion of Sun and Moon [III–VI] as a necessary preliminary to the theory of the planets;
 - 3) the theory of the stars, divided into two sub-sections
 - a) the fixed stars [VII–VIII],
 - b) the five planets [IX–XIII].

Section B 3 is said to be the core of the whole work (perhaps most of it is due to Ptolemy himself whereas much of sections 1 and 2 was known to his predecessors, in particular Hipparchus). The method of exposition must everywhere be to start with the most certain observations to be found in previous or contemporary observers; then the next step must be to connect such series of observations by means of a mathematical theory, developed in the form of geometrical models.

This programme is in perfect agreement with what Greek astronomers had been used to doing for centuries, and contrasts clearly with the purely algebraic theoretical structures applied by their Babylonian colleagues. But – as we are going to see – Ptolemy does not conceive his method as a purely inductive process in which theories are created from observations in a more or less automatic way. On the contrary, Ptolemy usually adopts a manner of exposition in which the general features of the final model are presupposed or postulated already at the beginning. Occasionally this style makes the *Almagest* difficult: a model is going to be built up upon empirical data described in terms of the model itself. Therefore, the ordinary structure of a chapter in the *Almagest* is composed of 4 parts:

- 1) a brief, qualitative sketch of the series of phenomena to be explained;
- 2) a preliminary account of the geometrical model as a postulate without proof;
- 3) a meticulous deduction of the parameters of the model from carefully selected and recorded observations, often by means of reiterative methods;
- 4) a verification that with these parameters the model really explains the phenomena in a quantitative way.

On the other hand this method may be regarded as a historical support of the view that a purely inductive procedure in science is impossible – a certain amount of theory has to be present already at the beginning as an indispensable framework for the selection and description of the ‘relevant’ empirical data.

The Nature of Astronomical Theories

Before tackling the problem which interests him most of all, viz. how to give a theoretical account of the motions of the Sun, Moon, and planets, Ptolemy has to decide upon a number of methodological and other philosophical questions of a more general nature than those touched upon in Book I (cf. page 26).

These questions are not dealt with in a really systematic way. Accordingly we have to extract them, first from a number of occasional remarks scattered all over the *Almagest*, and second, from an investigation of the methods actually employed by Ptolemy for constructing planetary theories. In other words, we must compare what Ptolemy says he is going to do with what he actually does. This will give us a much deeper insight into his philosophy of science than the very general considerations found in Book I.

The most obvious feature of this philosophy is Ptolemy’s deep conviction that things are not what they seem. The real structure of the Universe is hidden from our bodily senses and can be revealed only through the eyes of our mind. As deeply as any Platonist, Ptolemy must accordingly distinguish the observable phenomena from their hidden causes. This is only a particular consequence of the common Greek idea that what really exists must be without change, so that all changing phenomena must be mere appearances. On the other hand, science is founded upon the belief that even the most disorderly events in nature are caused by immutable laws to be uncovered by the scientist.

This general attitude has immediate implications for astronomy. Already in the preface to Book I Ptolemy calls attention to the perfect order and unchanging beauty of the heavens which had drawn his inclination towards the study of the stars in their courses. An obvious example of this order is the perfect regularity with which the heavens perform their diurnal rotation. But this is a somewhat particular case, and in general such beautiful regularities do not belong to our immediate experience.

This is most easily seen in the five planets whose normal motions towards the East among the fixed stars are sometimes interrupted by retrograde movements towards the West. Even the most perfect heavenly body, the Sun itself, is not untainted by

disorder; for even if it has no retrograde motion, the unequal lengths of the four seasons show that it performs its course along the ecliptic with a changing velocity. Thus most celestial phenomena exhibit an intriguing mixture of order and disorder, forcing us to admit that what we perceive by our senses are not the real motions, but only appearances.

Therefore, the task of the theoretical astronomer is clear: by means of a suitable intellectual effort he must try to prove beyond doubt that even the most confused and disorderly celestial phenomena can be explained in terms of invariable, orderly laws⁴). Accordingly his aim must be first to discover and to formulate such laws, and second, to demonstrate that the phenomena can be deduced from them. We know already that in order to cope with this problem we must proceed by way of mathematics. In Greek astronomy – as opposed to Babylonian calculations – this is the same as to say that astronomical theories must be expressed in geometrical terms.

From the very infancy of Greek science one particular geometrical concept had been proposed as the principal conceptual tool of the theoretical astronomer. If we are to believe Geminus (*Elem. Astr.* i, 19) it was the Pythagoreans who first *assumed that the motions of the Sun, Moon, and planets are circular and uniform*. What this means is clearly stated in the *Almagest* [III, 3; Hei 1, 216] where Ptolemy considers a point moving upon the circumference of a circle in such a way that a line from the centre to the moving point describes equal angles in equal times. This is equivalent to saying that the point moves with a constant angular velocity, a term of more recent origin and unknown to Ancient or Mediaeval astronomers.

This notion of uniform, circular motion is fundamental. In fact, the history of Greek theoretical astronomy is, to a very large extent, the history of a long series of efforts at explaining away the observed irregularities in the heavens by resolving even the most complex celestial motions into a set of uniform, circular components. Here Aristotle had advocated a very strict position, admitting only component circular motions which were

- 1) uniform as seen from their own centres, and
- 2) concentric with the universe as a whole.

As is well known, a theory fulfilling these requirements was proposed by Eudoxos (*Metaph.* xii, 8, 1073 b), but it failed because it was unable to account for the varying distances of the planets (cf. Herz, 1887, and Neugebauer, 1953). Consequently both Apollonius and Hipparchus were forced to abandon the second condition, admitting into their theories circles with centres other than that of the universe. On the other hand they retained the first condition, using only motions with constant angular velocities relative to their own centres.

Ptolemy's position is rather complex. We notice first that every time he begins his investigation of the theory of a particular planet, he explicitly acknowledges the validity of the principle of uniform, circular motion. Thus in the introduction to the

4) This is the famous program of *saving the appearances*, cf. Mittelstrass (1962) and Wasserstein (1962).

Solar theory [III, 3; Hei 1, 216] he says that in order to *explain the apparent irregularity of the Sun it is necessary to assume in general that the motion of the planets is uniform and circular*. The reason given for this dogmatic statement is metaphysical, the principle being said to be the only one *consistent with the nature of Divine Beings* (i.e. the planets) [IX, 2; Hei 2, 208]. Consequently, the construction of a theory along these lines is *a great thing, the accomplishment of which is the goal of a philosophically founded mathematical science* [ibid.]. However, it is also *an enterprise which for many reasons is connected with great difficulties*.

In IX, 2 Ptolemy enumerates some of these difficulties. However, already at this place it should be noted that one of the most intriguing features of his planetary theories is passed over in silence. Ptolemy always claims that he adheres to the principle of uniform circular motion without any formal modifications. But, as we shall see, this is mere lip service to a venerated traditional dogma from which Ptolemy does not hesitate to depart when he finds it opportune to do so. In fact, he takes an important step beyond Hipparchus and all earlier astronomers by admitting into planetary theory also circular motions which are uniform only as seen from a point other than the centre of the circle (see later page 277)⁵. Consequently they are non-uniform with respect to the centre. For this departure from a well-established tradition Ptolemy was often blamed by later astronomers⁶). This is, of course, unjustified from a purely astronomical point of view, since a theoretical science cannot be confined by assertions of a non-scientific character. But it is, nevertheless, a little disappointing that Ptolemy everywhere pretends to stick to tradition without commenting on the points where he discards it.

The General Part of the Almagest

The Preface to the Almagest considered on the preceding pages contained, as it were, Ptolemy's general philosophy of science. We shall now examine what he calls the 'General Part' of the Almagest, i.e. the 6 chapters [I, 3–8; Hei 1, 10–31] in which he describes the heavenly sphere, the various motions observed in the heavens, and the shape, position, size, and immobility of the Earth. In general, the doctrines here described characterize what has become known as the Ptolemaic Universe, although they stem, all of them, from a much earlier period in Greek Natural Philosophy, and were first described in a coherent manner by Aristotle. This is the reason why Ptolemy is able to deal very briefly with them. But it should be remembered that in the *Planetary Hypotheses* he gave a personal contribution to this cosmology, attempting, among other things, to compute the distances of the planets from the Earth, and the size of the whole universe. These latter problems will be investigated in a later chapter

5) This means that it is a mistake to consider Ptolemaic methods as a kind of rudimentary Fourier analysis, cf. Aaboe (1960 b).

6) See for instance Ibn al-Haitham's critique, in Pines (1962), and also Copernicus, *De rev. Praef.* iii b.

(page 391 ff.). At present we shall consider only those cosmological opinions which Ptolemy found it necessary to include in the *Almagest* as prerequisites to the understanding of planetary theory.

The Diurnal Rotation of the Heavens

The third chapter of Book I [I, 3; Hei 1, 10] begins with some speculations on how the Ancients might have come to the view of the heavens as a rotating sphere. All the time they saw the Sun, Moon, and stars rising above the Eastern horizon, culminating in the middle of the heavens, and setting beneath the Western horizon, moving continually upon parallel circles with a common centre at the Northern pole of the heavens. The fact that the nearer a star is to the pole, the smaller its circle will be, leads to the idea of a rotating sphere upon which they are placed.

In fact, any other idea will be unable to save the phenomena. Thus Ptolemy turns down the conception that all stars perform rectilinear motions through an infinite space, a view perhaps advocated by Xenophanes of Colophon (about 570–475 B.C. (Kirk and Raven, p. 173)) but ascribed by Theon (ed. Rome, p. 338 f.) to Epicurus (about 342–271 B.C.), which is wrong, since Epicurus speaks clearly of the rotation of the heavens (Diog. Laert, X, 92). This hypothesis would explain the phenomena of rising and setting, but not the fact that e.g. the Sun and the Moon retain their size and even look a little larger at the horizon than higher up. If they moved along an infinite straight line they would rise and set as vanishing points. Furthermore, it seems impossible to explain the impression that it is the same stars which return at the same places night after night.

Another critique is directed against those who like Xenophanes maintain that the stars arise from the Earth and are kindled only to extinguish when they fall to the Earth again. This idea is ridiculed by Ptolemy from the fact that the Earth is spherical; this implies that when a star is rising for one observer it will be setting for another, which makes this theory impossible.

Ptolemy continues with the assertion that any other figure of the heavens than that of a sphere would change the relative distances of the stars during their daily revolution, thus making the constellations change contrary to what experience shows.

A further argument is that if the heavens were not a sphere sundials would not show the correct time – presumably a reference to the hemispherical sundial, or *polos* (Herodotus, ii, 109; cf. Dicks, 1954 p. 77) used in various forms by the Babylonians, Greeks, and Romans.

To these astronomical reasons for the spherical shape of the heavens are added two others, one mathematical and the other physical. First, the sphere is that among all geometrical forms which has both the smoothest and easiest motion, and the greatest volume relative to its size. The latter is the isoperimetric property of the sphere, already known to Zenodorus, who is difficult to date, but certainly lived before Ptolemy's time (Theon, ed. Rome, p. 360 ff.; cf. v. d. Waerden, 1954, p. 306).

The physical argument rests upon the assumption that the celestial matter, or ether, is composed of small particles or molecules. These are finer and more regular than those of which other substances are composed, and must therefore be spherical. A further assumption is not stated explicitly in the *Almagest*, but is formulated by Theon: *any physical body consisting of similar parts will have the same figure as these parts*. Therefore, the whole ethereal body, i.e. the heavens, must be spherical (Theon, ed. Rome, p. 49). A further conclusion drawn from this principle is that the individual heavenly bodies (the stars) must also be spherical. The latter is confirmed by the more acceptable assertion that otherwise they would not appear in the same shape to observers at different places of the Earth. This would be a good argument if Ptolemy did not, at other places, deny any observable parallax of the heavenly bodies apart from the Moon (and, perhaps, the Sun).

The Spherical Shape of the Earth

That not only the heavens, but also the Earth is spherical is proved by a series of arguments based on astronomical facts [I, 4; Hei 14]. Here Ptolemy begins by considering the times at which the same Lunar eclipse is observed by different observers. The important thing is that a Lunar eclipse is an objective phenomenon in the sense that the entrance of the Moon into the shadow of the Earth happens at a definite instant, independently of where the observer is situated. Now it is a fact that the local time of, say, the beginning of an eclipse is not the same to all observers. The event happens later the further the observer is situated towards the East, the time difference being proportional to the distance. This shows that the Earth has a curvature relative to the East–West-direction.

Furthermore, this curvature makes the Earth a convex body. Were it concave, i.e. hollow, the stars would rise earlier for a Western observer than for an Eastern. Were it flat, they would rise simultaneously for all observers (Figure 2.1).

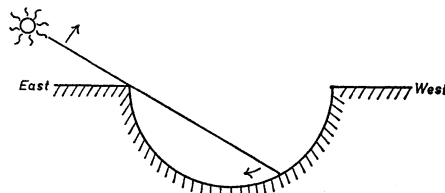


Fig. 2.1

Now one could imagine that the Earth was a circular cylinder with its axis in the North–South direction; this would be consistent with the curvature proved above, but not with the fact that the height of a star above the horizon will change as the observer travels North; this implies that his horizon (the tangential plane to the Earth) has no fixed direction relative to the axis as it would have on a cylinder (Figure 2.2).

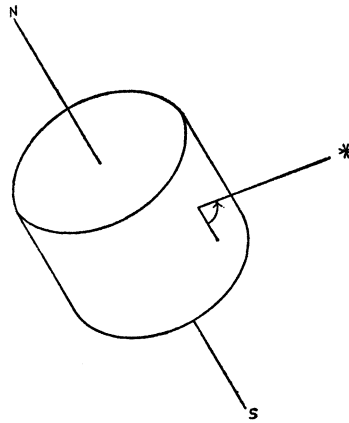


Fig. 2.2

Furthermore, travelling North we observe how stars gradually disappear forever from the Southern sky, at the same time as more and more stars become circumpolar in the Northern. On a cylinder the same stars would be either visible or invisible without regard to the position of the observer (Figure 2.3).

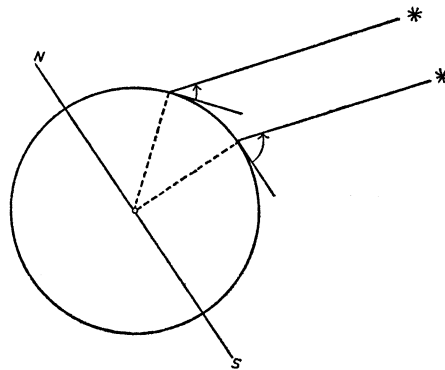


Fig. 2.3

The result is that the Earth must have a double curvature. Finally, Ptolemy draws attention to the well known experience that an observer on board a ship approaching the coast will see first the top of the mountains, then gradually more and more.

It is interesting to compare this chapter with the classical text on the spherical shape of the Earth in Aristotle's work *On the Heavens* (ii, 14, 297 a). Here Aristotle begins with a series of physical arguments concerning gravitation, having previously proved the Earth to be at the centre of the universe. This latter opinion is first established by Ptolemy in the following chapter. The proofs based on theories of gravitation have

to wait, and he is left with purely astronomical arguments. This gives his treatment more unity and coherence than the corresponding chapter in Aristotle. On the other hand one cannot help wondering why Ptolemy ignores one of the most striking proofs found in Aristotle, viz. that during a Lunar eclipse the Earth casts a circular shadow on the surface of the Moon (*De caelo* ii, 14, 297 b).

The Central Position of the Earth

So far Ptolemy has established that the Earth is spherical and placed somewhere inside a spherical universe. Now follows a proof that its only possible position is at the centre of the heavenly sphere [I, 5; Hei 1, 17]. The proof is indirect since Ptolemy shows that any other position is impossible for astronomical reasons, leaving out any 'physical' argument derived from the theory of gravitation.

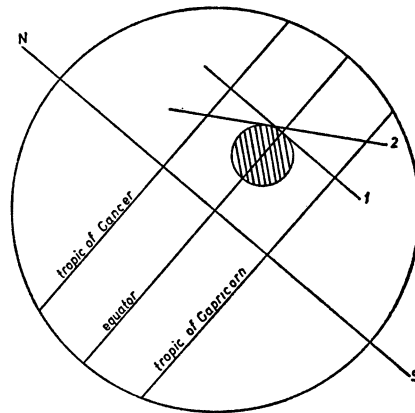


Fig. 2.4

1) If the Earth were placed outside the axis but at the same distance from each pole (i.e. in the plane of the celestial equator, see Figure 2.4) there would be trouble with the equinoxes. On the terrestrial equator (position 1 of the horizontal plane) day and night could never be equal everywhere simultaneously, since the horizontal plane would divide both the celestial equator and any other diurnal arc of the Sun into unequal parts. At other places (position 2) either the same would be true, or else the equinoxes would not fall in the middle between Summer and Winter solstice. This is because the circle halved by the horizon would no longer be the celestial equator, but some other circle lying asymmetrically with respect to the two tropics.

Moreover, a traveller moving East would see the relative distance of the stars change, and the constellations with them. He would also find the time between the rising and culmination of a star at one place different from the time between culmination and setting at another place on the same parallel.

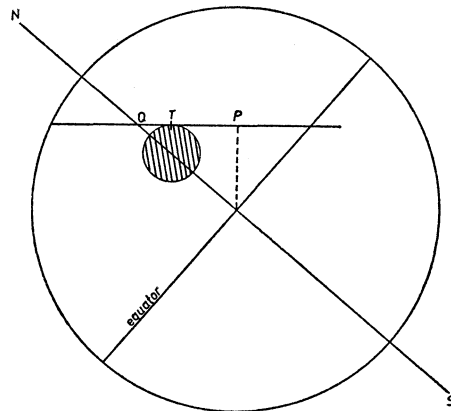


Fig. 2.5

2) Next we suppose the Earth to be placed on the axis but nearer to one pole than to the other (Figure 2.5). In this case an observer at a point T outside the terrestrial equator will find the heavenly sphere divided into two unequal parts by his horizontal plane. In particular the ecliptic would be divided into unequal parts too, and we would not always see 6 zodiacal signs above and 6 below the horizon, which is contrary to experience.

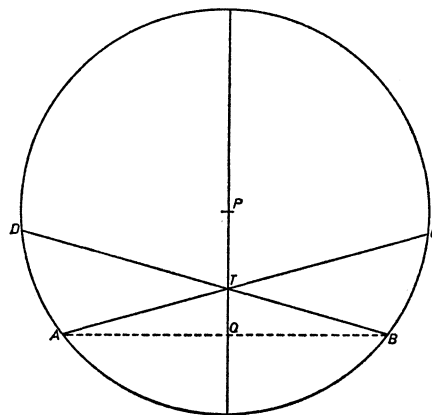


Fig. 2.6

Another consequence would be that at the equinoxes (if such dates exist for the given position of the Earth), the Sun would neither rise in the East nor set in the West. This argument is not made explicit. Figure 2.6 shows the horizontal plane of Figure 2.5. The dotted line AQB is the trace of the plane of a circle parallel to the equator and halved by the horizontal plane. If this circle is so situated that the Sun is able to reach it, then there will be a date when day and night are equal. Looking at the horizon

from above, the trace of this circle will be the line AQB. It follows that a gnomon at T will cast a morning shadow along TC and an evening shadow along TD; these two shadows are not diametrically opposed, contrary to what is observed everywhere on the Earth.

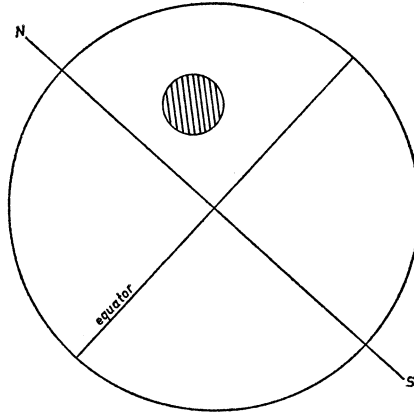


Fig. 2.7

3) Finally the Earth is supposed to lie neither on the axis, nor with the same distance from the poles (Figure 2.7). This assumption leads to all the objections raised against the two former positions, and is clearly impossible.

But Ptolemy has still another reason for the central position of the Earth, taken from eclipses of the Moon. It is a matter of experience that such eclipses occur only at oppositions (or Full Moons) when the Sun, Earth and Moon are on a straight line connecting diametrically opposite points of the heavens. But if the Earth were not central it could happen that it came between the Sun and the Moon at positions which were not diametrically opposite but had a difference of longitude smaller than

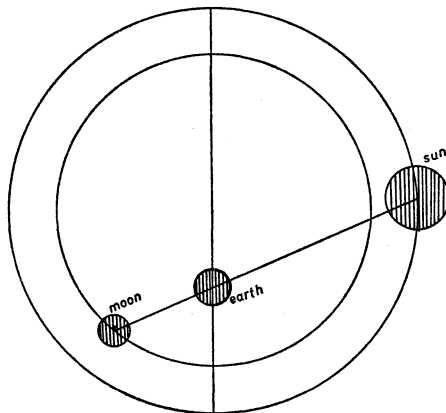


Fig. 2.8

180°. This argument is only hinted at, but Theon has developed it more fully (ed. Rome, p. 78ff.). It rests on the assumption that the Earth is outside the centre of the universe, but that both Sun and Moon are moving upon circles around this centre (Figure 2.8). In that case it is obvious that in the eclipse shown in the figure the line through Sun, Earth, and Moon will not divide the heavens into equal parts.

The Size of the Earth

Long before Ptolemy's time the absolute size of the Earth had been determined to a fair degree of accuracy by Eratosthenes (about 275–194 B.C.) and Posidonius (about 135–50 B.C.). Their results are not mentioned in the *Almagest*. Perhaps Ptolemy was unaware of them, however difficult to believe that may be. But the account of Eratosthenes' method and result is known from the *De motu circulari corporum caelestium* by the astronomer Cleomedes (2nd century A.D.), and it seems that this book was unknown to Ptolemy when he wrote the *Almagest*. The reason is that he never mentions Cleomedes as the discoverer of atmospheric refraction. This astronomically very important phenomenon is mentioned in the *Almagest* [I, 3; Hei 1, 13], but only as explanation of the changing apparent size of the heavenly bodies near the horizon. In his *Optics* (V, 23–30) Ptolemy examines astronomical refraction in more detail, but this work was completed at a later date. In any case, only relative measures of Earth, Sun, and Moon are given in the *Almagest* [V, 16; Hei 1, 426] with the radius of the Earth as the preferred astronomical unit (cf. below p. 213).

In the present chapter [I, 6; Hei 1, 20] Ptolemy is concerned only with the size of the Earth compared with the distance of the fixed stars. Since the latter are supposed to belong to a sphere (the firmament) concentric with the Earth, the problem is to find the ratio between the radius of the Earth and that of the starry sphere. Here Ptolemy argues that the Earth must be likened to a mere point compared with the firmament. The main reason is that the fixed stars keep their mutual distances and constellations unchanged no matter where the observer places himself on the Earth. In other words, the fixed stars are too far away to show any daily parallax. This word is not used here, but introduced later in the *Almagest* [V, 11; Hei 1, 401].

The insignificant size of the Earth appears also from the fact that any horizontal plane (i.e. any tangential plane to the surface of the Earth) is seen to divide the heavenly sphere into two exactly equal parts. If the Earth had an appreciable size only a plane through its centre could do that.

Particularly impressive is the fact that the point of a gnomon, or the centre of an armillary sphere, represents the centre of the Earth equally well wherever they are placed on its surface. (This means that stellar coordinates measured with such instruments may be regarded as geocentric, whereas the Moon is so near that it is necessary to reduce observed apparent positions to true geocentric positions by means of a particular theory of parallaxes).

The absence of daily parallax in the fixed stars would have set Ptolemy free to adopt

the hypothesis of the rotating Earth as an explanation of the diurnal motion of the heavens. Although this hypothesis was known to Greek astronomers, for instance, Hicetas, Ecphantus, Philolaus, Heracleides, and Aristarchus, Ptolemy refers to it in the *Almagest* only to refute it. This refutation is the purpose of the following chapter.

The Immobility of the Earth

That the Earth remains immovable at the centre of the universe is first proved very briefly by an astronomical argument [I, 7; Hei 1, 21]. If the Earth moved away from the centre it would necessarily come into one or another of the impossible positions already refuted [I, 5]. If Ptolemy was aware that this argument is questionable he does not tell us so. In fact, having just proved that the Earth is like a point compared with the firmament, Ptolemy would have to admit that a displacement of the Earth equal to, say, its own diameter would not influence the observed phenomena. Perhaps he would maintain that it would affect the relative position of Earth, Moon, and Sun and thus be impossible for the reason based on the phenomena of eclipses. In any case the proof needs further elucidation which is omitted, Ptolemy hastening on to a series of proofs based on physical considerations and comparable to those given by Aristotle who, by the way, dismisses the astronomical proof as quickly as Ptolemy (*De caelo* ii, 14, 296 a).

The first of the physical proofs is based on the Aristotelian theory of gravity. Ptolemy seems to regard gravitation very much like Aristotle as a natural tendency of heavy bodies to move towards the centre of the world as their 'natural place' of rest. Since we know that the Earth is placed in the middle of the universe, it follows that all heavy bodies moving freely under gravity must strike the surface of the Earth. This happens always in a vertical direction perpendicular to the tangential plane of the Earth at the point of impact. Since the vertical is conceived here as the direction towards the centre of the world, and since a sphere concentric with the universe is the only surface with a tangential plane everywhere perpendicular to the vertical, we have here an argument for the Earth being spherical and concentric with the universe. But this is no more than what we know already, and no proof of the immobility of the Earth, which might still have a diurnal motion around the centre. It would have been necessary to underline the principle that heavy bodies move towards the centre in order to be at rest there, but this is not stressed by Ptolemy.

Ptolemy proceeds with a refutation of those who cannot imagine so great and heavy a body as the Earth to be at rest without its inclining to one side or another. Aristotle had refuted this idea by means of his doctrine of natural motion: no substance has more than one natural motion (*De caelo* i, 2, 269 a), and since that of earth and other heavy particles is towards the centre of the world, it follows that the Earth as a whole cannot have a natural tendency to move sideways. It is interesting that here Ptolemy has a quite different argument based on the physical notion of pressure. As already mentioned, all matter is supposed to consist of molecules

(page 37). Now the Earth is extremely small compared with the celestial spheres. This means that it is subjected to an immense pressure by the ether molecules. Since these are all alike and surround the Earth evenly on all sides, the pressure will act uniformly everywhere on the Earth, and keep it in place at the centre, the pressure at one point of the surface being equal and opposite to that at the opposite point.

The argument is not quite clear, since Ptolemy on the one hand operates with the notion of ethereal molecules exerting a pressure towards the centre, while on the other hand he acknowledges that incorruptible matter has a natural, circular motion around the centre. Furthermore, he tries also to take into account the natural down- and upward motions of the heavy and light molecules in the neighbourhood of the Earth, which is a further complication. But in any case, it is obvious that Ptolemy's belief in the immobility of the Earth rests upon a line of thought of a more physical character (in the modern sense of the word) than the Aristotelian doctrine of natural places and natural motions, although he has not succeeded in liberating himself from the latter⁷).

Having established his own position, Ptolemy devotes the remaining part of this chapter to an attack on 'certain people' who assumed a diurnal rotation of the Earth around its own axis. At the beginning Ptolemy admits that from a purely astronomical point of view nothing speaks against the hypothesis of a rotating Earth, which is even simpler than that of a celestial universe rotating about an immobile Earth. Theon (ed. Rome, p. 89 ff.) develops this point, showing in great detail that all the diurnal phenomena are the same in either case. This means that Aristarchus's hypothesis can only be refuted by 'physical' arguments.

These arguments are based on the assumption that the rotation of the Earth would create atmospheric phenomena contrary to experience; for instance, that clouds would be overtaken by the Earth and that we should always have a strong wind towards the East. Ptolemy is aware that the supporters of the hypothesis tried to avoid this difficulty by the assumption that the atmosphere takes part in the rotation with the same velocity; but that is refuted for reasons taken from the doctrine of the free fall of bodies. If a heavy body is thrown upwards along the vertical it is supposed to return to the Earth along the same geometrical line, thus striking the surface at a point different from that from which it started. This does not agree with everyday experience. Ptolemy is aware, too, that thrown or flying bodies could be, as it were, united with the air. In that case they would not be overtaken by the Earth but would remain at the same place forever, contrary to what we observe in projectiles and birds. It has to be admitted that this is a very weak argument, indicating that Ptolemy has not gone deeply into the Aristotelian doctrine of violent motion.

7) That Ptolemy is on the move away from a purely Aristotelian doctrine is seen also from his (now lost) treatise *On the balance*. According to Simplicius he maintained here that neither water nor air has any weight at its natural place, since e.g. divers never feel the weight of the water above them, even at great depth. – Simplicius: *Comm. in de Caelo* IV, 4, cf. Thomas: *Greek Mathematical Works*, II, 411. It would have been interesting to see how such an anti-Aristotelian doctrine was dealt with in Ptolemy's other lost work *On the Elements*.

The Two Principles of the Motion of the Heavens

Until now we have only discussed phenomena connected with the daily motion of the heavens around the axis through the poles. This motion causes all the stars to move on circles parallel to the equator. Ptolemy explains the etymology of this name by the fact that the equator always and everywhere is divided into two equal parts by the horizon just as day and night are equal at the dates when the Sun is on the equator [I, 8; Hei 1, 26].

In the present chapter he explains the reasons why this first motion of the heavens is not the only one. In fact while the majority of the stars always rise and set at the same points of the horizon, there are a few wandering stars which behave otherwise. This is the case with the Sun, the Moon, and the five planets. Further observations show that these seven bodies perform individual motions towards the East relative to the fixed stars and at the same time they take part in the diurnal rotation. It looks as if they are moving on a circle inclined to the equator. This is, in fact, exactly the case for the Sun, whose apparent path among the fixed stars is a great circle, called the ecliptic, and intersecting the equator at two diametrically opposed points called the vernal and the autumnal equinox. At the former the Sun crosses the equator in the direction from South to North and at the latter in the opposite way.

The six remaining planets perform an even more complicated motion. Roughly speaking, they follow the ecliptic, completing their revolutions in slightly variable periods. But at the same time they have a motion in latitude, i.e. they depart a little from the ecliptic. In the first approximation it would seem that they revolve on great circles forming a small angle with the ecliptic and cutting it at two opposite points called the ascending and the descending node.

Conclusion

What has been said until now comprises everything that Ptolemy deemed necessary for the understanding of the picture of the universe upon which the rest of the *Almagest* is based. To the history of astronomy these first chapters are interesting not only because of what Ptolemy says, but because of the many points he is passing over in silence. Only one important question must be mentioned here. It is often said that the *Almagest* deals with theoretical astronomy in a purely mathematical and formalistic way, as if Ptolemy were uninterested in the question of the physical relevance of his geometrical models. But, as we have seen, there is a picture of the physical structure of the universe behind the *Almagest*. Thus the fixed stars are supposed to exist in a particular sphere concentric with the Earth. As for the planets, Ptolemy usually describes their motions by means of the purely mathematical concept of a circle, referring but rarely to material spheres, e.g. in the theory of the Moon [IV, 6; Hei 1, 301]. But this should not make us think that he belongs to what has been called the mathematical school of astronomers. We know from his *Planetary Hypo-*

theses that the doctrine of spheres was, in fact, an essential part of his astronomy and that Ptolemy is one of the sources of the many Mediaeval speculations on how the geometrical models could be transformed into a machinery of spheres and how the theories of the individual planets could possibly be compatible from this physical point of view (see p. 393 ff.). On the other hand, one is grateful that he did leave this physical point of view out of consideration in the *Almagest*. Had he chosen a different course, the work would have lost in clarity, and a lot of 'physical niceties' would have obscured the magnificent mathematical structure which is Ptolemy's lasting claim to fame as one of the greatest theoretical astronomers of all time.

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Jones

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