
Preface

Millennia of astronomical observations were fully understood only when the seminal ideas of Galileo, Kepler, and Newton were impacted by mathematics. The subsequent theoretical elaborations of the laws of motion by d'Alembert, Lagrange, Hamilton, Liouville, and others are at the basis of all mechanical devices that affect modern life in essentially all its practical aspects. Lagrange, in the preface to his *Traité de Mécanique Analytique* [101], sets his vision of mechanics as a branch of mathematics, building on physical principles.¹ Classical mechanics stands as perhaps the most successful example of what contemporary scientists call “interdisciplinary” research. The complexity of astronomy in Newton’s day is countered today by the complexity of disciplines such as chemistry and biology. Classical mechanics is a chief example of the scientific method of organizing a “complex” collection of information into theoretically rigorous unifying principles. In this sense it represents one of the highest forms of modeling. The elegance and depth of the theoretical thinking coupled with its ubiquitous applications make it comparable, in applied sciences, to Euclid’s geometry. It also has a foundational value comparable to calculus, both as a fundamental language of applied sciences and as a catalyst of new concepts and discoveries.

This book collects my lectures in rational mechanics delivered at the School of Engineering of the University of Rome, Tor Vergata, from 1986 to 1998. The main vision was mathematical modeling, and the layout is theoretical. The required background includes a working knowledge of linear algebra (vector calculus, algebra of matrices), multivariate calculus, basic theory of ordinary

¹From the Preface of [101]: “. . . *On ne trouvera point de Figures dans cet Ouvrage. Les méthodes que j’y expose ne demandent ni constructions, ni raisonnements géométriques ou mécaniques, mais seulement des opérations algébriques, assujetties à une marche régulière et uniforme. Ceux qui aiment l’Analyse, verront avec plaisir la Mécanique en devenir une nouvelle branche, . . .*

Tel est le plan que j’avais tâché de remplir . . .

On a conservé la notation ordinaire du Calcul différentiel, parce qu’elle répond au système des infiniment petits . . .”

differential equations, and elementary physics. While the Lebesgue integral is used, a working knowledge of it is not required. Its use is mainly intended as a unifying feature, bridging from discrete systems whose material properties are described by a series of weighted Dirac masses, to continuum systems. In practice, in problems and examples only the Riemann integral is used.

The geometry of rigid motions is presented along with some of its implications to mechanical devices and the theory of Poincot precessions. The latter is remarkable, since it explains the phenomenon of equinoctial precessions only by the geometrical rolling of the Poincot cones. While Lagrange's equations are often assumed as a principle, here they are derived and put on a mathematical footing. Conversely, the cardinal equations, which are often passed over in favor of the Lagrangian and Hamiltonian formalism, are shown to be the basis of such a formalism. Classical topics, such as gyroscopes, precessions, spinning tops, effects of rotation of Earth on gravitational motions, variational principles, n -body problem, and celestial mechanics, are revisited to underscore the role of mathematics, without which they can be only perceived but not fully explained. Attention is paid to the theory of small oscillations and Lyapunov's stability, including stability and instability of Poincot precessions and celestial motions. The connection between mechanics and geometrical optics is traced to their common variational principles. The former leads to a maximum-rank Hamiltonian system, and the latter yields a degenerate Huygens canonical system. The degeneracy is overcome by Euler's theorem of homogeneous functions. The Hamilton variational formalism naturally leads to the symplectic formalism and canonical transformations. These identify, among the transformations that preserve the variational and canonical nature of the Hamilton equations, those that preserve the hidden geometry of the motion (Lie), such as Poisson and Lagrange brackets and volumes (Liouville's theorem). They also transform paths in phase space into "immobile points" in the transformed phase space, thereby providing an integration method of Hamiltonian systems (Jacobi). The basic techniques of integration of the Hamilton–Jacobi equations (complete integrals, envelopes, separability, etc), are presented in their natural symplectic formalism. Some classical facts have been given new proofs based on more modern mathematical language. These include the local minimality of the stationary points of the action, the notion of envelopes, and the Hamiltonian form of Huygens systems in geometrical optics. In the last chapter we give a very brief introduction to continuum and fluid mechanics, mainly to underscore the mathematical and modeling ideas needed in transitioning from finite degrees of freedom to nonrigid continuum systems.

Classical mechanics has evolved from these seminal principles into numerous fields, including statistical mechanics, relativity, ergodic theory, symplectic geometry, and continuum mechanics including elasticity and fluid dynamics. The methods of investigation are diverse, ranging from probability and measure theory and classical analysis to algebra, topology, and Riemannian geometry. We have refrained from going into any of them both because of the specialized and massive nature of each of them, and because

of our goal of giving essentially a “calculus-type” introduction to the applied sciences. As such, all topics are mutually interlaced, each providing the background for the indicated several directions that mechanics takes on. However, some more mathematical parts might be omitted at a first reading. They include the parametric equations of fixed and moving centrodes (§11c of Chapter 1); the theory of radial potential (§4.3c), which, while used for some atomic potentials, in this context has mostly a theoretical value; finding the principal axes of inertia of a planar rigid system (§5c of Chapter 4), and some mathematical remarks on the stationary points of a functional (§§1.1c–1.2c and §1.5c of Chapter 9).

I learned classical mechanics from Giorgio Sestini (1908–1991) at the University of Florence, Italy, back in the mid 1970s. Sestini’s teachings impressed upon me the artful balance between physics and mathematics, neither being permitted to be self-absorbing. These notes reflect those lectures, including some exercises and problems taken from my old class notes.

I would like to extend special thanks and express my deep appreciation to my close collaborators Fabrizio Daví, Alessandro Tiero, Luciano Teresi, Luigi Chierchia and Vieri Mastropietro. These fine researchers helped me, over the years and at various stages, in teaching Classical Mechanics at the Univ. of Rome Tor Vergata. They conducted recitation sessions for the students and assisted them in practice and problem solving sessions. They suggested a number of ways of improving my draft notes into a usable tool and suggested topics and problems aimed at expanding the scope and clarity of the course. I am very much indebted to them.

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made precise some thermodynamical facts in the chapter on fluid dynamics. I am very much grateful to James.

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