
Remarks on the Isotriviality of Multiloop Algebras

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Summary. For a given finite dimensional simple Lie algebra $\mathfrak{g}$ over an algebraically closed field of characteristic 0, we establish the existence of a positive integer $d(\mathfrak{g})$ with the property that any multiloop algebra $L(\mathfrak{g}, \sigma_1, \dots, \sigma_n)$ based on $\mathfrak{g}$ is split by a finite étale cover of degree $d(\mathfrak{g})$ of the base ring $k[t_1^{\pm 1}, \dots, t_n^{\pm 1}]$.

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1 Introduction

Let $R_n = k[t_1^{\pm 1}, \dots, t_n^{\pm 1}]$ denote the Laurent polynomial ring in n variables over an algebraically closed field k of characteristic 0. Given a finite dimensional simple Lie algebra $\mathfrak{g}$ over k , consider the class of Lie algebras $\mathcal{L}$ over R_n with the property that

$$\mathcal{L} \otimes_{R_n} S \simeq_{S\text{-Lie}} \mathfrak{g} \otimes_k S \quad (1)$$

for some faithfully flat and finitely presented extension S of R_n . This interesting class of algebras has deep connections to the centreless cores of Extended Affine Lie Algebras (see [ABFP], [ABFP2] and [GP2] for details and further references). For $n = 1$ for example, the algebras in question turn out to be the derived affine Kac–Moody Lie algebras modulo their centres (see [P1] and [P2] for details).

The algebras as in (1) are the *forms* of $\mathfrak{g}$ for the *fppf*-topology on $\text{Spec}(R_n)$. They can be classified by non-abelian cohomology (see §1.1 below for references). The quintessential example of forms—and the ones that are of

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most interest to infinite dimensional Lie theory—are the multiloop algebras based on $\mathfrak{g}$ (see definition below).²

One of the major results of [GP1] is that any algebra $\mathcal{L}$ as in (1) is *isotrivial*, i.e., trivialized by a *finite* étale cover of R_n . This result has important implications for the classification of Extended Affine Lie Algebras by cohomological methods ([GP2, Theorem 5.13]). Relevant to the present work is the fact that the Isotriviality Theorem is of an abstract nature: It does not give any information on the degree of the finite étale cover needed to trivialize a given form. In this paper we show the existence of a degree which depends only on $\mathfrak{g}$ with the property that *any* multiloop algebra based on $\mathfrak{g}$ is trivialized by a finite étale cover of R_n of this degree.

1.1 Notation and conventions

Throughout, k denotes an algebraically closed field of characteristic 0. We fix once and for all a set $(\zeta_n)_{n \geq 1}$ of compatible primitive n -roots of unity of k , i.e., $\zeta_{\ell n}^\ell = \zeta_n$. The category of commutative associative unital k -algebras is denoted by $k\text{-alg}$.

For a given R in $k\text{-alg}$, by an R -group we will understand a group scheme over $\mathrm{Spec}(R)$. If $\mathbf{G}$ is an R -group, the pointed set of non-abelian Čech cohomology on the $fppf$ and étale sites of $X = \mathrm{Spec}(R)$ with coefficients in $\mathbf{G}$, will be denoted by $H_{fppf}^1(R, \mathbf{G})$ and $H_{\acute{e}t}^1(R, \mathbf{G})$, respectively. These pointed sets measure the isomorphisms of sheaf torsors (with respect to the chosen topologies) over R under $\mathbf{G}$ (see [SGA3] Exp. IV.6, [DG] III §4, and Ch. IV §1 of [M] for basic definitions and references).

If $\mathbf{G}$ is affine, flat and (locally) of finite presentation over R , then by faithfully flat descent all of our sheaf torsors are representable (by affine R -schemes). They are thus *torsors* in the usual sense. Furthermore, given that $\mathbf{G}$ is necessarily smooth³ all torsors are locally trivial for the étale topology. In particular, $H_{\acute{e}t}^1(R, \mathbf{G}) = H_{fppf}^1(R, \mathbf{G})$. These assumption on $\mathbf{G}$ are present in all of the situations that arise in our work.

Given an R -group $\mathbf{G}$ and a morphisms $R \rightarrow S$ in $k\text{-alg}$, we let $\mathbf{G}_S$ denote the S -group $\mathbf{G} \times_{\mathrm{Spec}(R)} \mathrm{Spec}(S)$ obtained by base change. For convenience, we will under these circumstances denote $H_{\acute{e}t}^1(S, \mathbf{G}_S)$ simply by $H_{\acute{e}t}^1(S, \mathbf{G})$.

To a given finite (abstract) group F corresponds a finite constant R -group that we denote by $\mathbf{F}_R$, or simply by $\mathbf{F}$ if R is understood from the context.

² In nullity 1 all forms are loop algebras [P2]. An example of B. Margaux shows that already in nullity 2 there exists forms which are not multiloop algebras. See [GP2] §3.6.

³ Since R is of characteristic zero, the geometric fibers of $\mathbf{G}$ are smooth by Cartier's theorem [DG], II.6.1.1. The smoothness criterion on fibers ([EGA IV] §17.8.1) shows that $\mathbf{G}$ is smooth over R .

The following notation will be extensively used throughout this work.

$$R_n = k[t_1^{\pm 1}, \dots, t_n^{\pm 1}], \quad R_{n,d} = k[t_1^{\pm \frac{1}{d}}, t_2^{\pm \frac{1}{d}}, \dots, t_n^{\pm \frac{1}{d}}], \quad R_{n,\infty} = \varinjlim_d R_{n,d},$$

and

$$F_n = k((t_1))((t_2)) \dots ((t_n)), \quad F_{n,d} = k((t_1^{\pm \frac{1}{d}}))((t_2^{\pm \frac{1}{d}})), \dots, ((t_n^{\pm \frac{1}{d}})).$$

1.2 R_n -torsors under finite constant groups

The extension $R_{n,d}/R_n$ is finite Galois. Its Galois group $\Gamma_{n,d}$ is henceforth identified with $(\mathbb{Z}/d\mathbb{Z})^n$ acting naturally on $R_{n,d}$ via our fixed choice of compatible roots of unity; namely $\bar{\mathbf{e}}(t_i^{1/d}) = \zeta_d^{e_i} t_i^{1/d}$ for all $\mathbf{e} = (e_1, \dots, e_n) \in \mathbb{Z}^n$, where as usual $\bar{\cdot} : \mathbb{Z}^n \rightarrow (\mathbb{Z}/d\mathbb{Z})^n$ is the canonical map.

We fix once and for all an algebraic closure $\overline{F_n}$ of F_n containing all of the fields $F_{n,d}$. This defines a geometric point a of $X = \text{Spec}(R_n)$, and we consider the corresponding algebraic fundamental group $\pi_1(R_n, a)$ (see [SGA1] V.7). Since X is connected, normal and noetherian, we have an explicit realization, henceforth denoted by X^{sc} , for the universal covering of X at a (ibid. V.8.2). In particular, we have a canonical homomorphism $\text{Gal}(F_n) \rightarrow \pi_1(R_n, a)$. We recall for convenience the following fundamental fact (see [GP3] corollary . 2.10 for details).

Proposition 1.1. *1. $\text{Spec}(R_{n,\infty})$ is simply connected (i.e., it does not have any nontrivial finite étale covers), and it is naturally isomorphic to the simply connected covering X^{sc} of $\text{Spec}(R_n)$ at the geometric point a .
2. $\pi_1(R_n, a)$ is the Galois group of the pro-covering $R_{n,\infty}/R_n$, namely*

$$\pi_1(R_n, a) = \varprojlim_d \text{Gal}(R_{n,d}/R_n) = \varprojlim_d (\mathbb{Z}/d\mathbb{Z})^n = \widehat{\mathbb{Z}^n}.$$

3. The canonical map $\text{Gal}(F_n) \rightarrow \pi_1(R_n, a)$ is an isomorphism. □

The next basic lemma—especially its proof—gives a complete description of R_n -torsors under a finite constant group.

Lemma 1.2. *Let $\mathbf{F}$ be a finite group, and let $d(\mathbf{F})$ be the maximum of all orders of abelian subgroups of $\mathbf{F}$. If $\mathbf{X}$ is an R_n -torsor under $\mathbf{F}$, then there exists a positive integer $d(\mathbf{X})$ with the following properties:*

- (a) $d(\mathbf{X}) \leq d(\mathbf{F})$
- (b) $d(\mathbf{X})$ divides $|\mathbf{F}|$
- (c) $\mathbf{X}$ is trivialized by the base change $R_n \rightarrow R_{n,d(\mathbf{X})}$.

In particular, the base change $R_n \rightarrow R_{n,|\mathbf{F}|}$ also trivializes $\mathbf{X}$.

Proof. The classes of such torsors are parametrized by $H_{\text{ét}}^1(R_n, \mathbf{F})$. This same pointed set classifies the Galois covers of R_n with a given action of the finite group F . We have natural correspondences (see [SGA1] XI.5)

$$\begin{aligned} H_{\text{ét}}^1(R_n, \mathbf{F}) &\simeq H_{\text{ct}}^1(\pi_1(R, a), F) \simeq H_{\text{ct}}^1(\widehat{\mathbb{Z}}^n, F) \\ &\simeq \text{Hom}_{\text{ct}}(\widehat{\mathbb{Z}}^n, F) / \text{inner automorphisms of } F. \end{aligned}$$

Consider a cocycle (i.e., a continuous homomorphism) $u : \widehat{\mathbb{Z}}^n \rightarrow F$ representing an R_n -torsor $\mathbf{X}$ under $\mathbf{F}$. Set $K = \ker(u)$ and $F_0 = u(\widehat{\mathbb{Z}}^n)$. Then the induced cocycle $u_0 : \widehat{\mathbb{Z}}^n \rightarrow F_0$ corresponds to a connected Galois extension $\mathbf{X}_0$ of R_n with Galois group $F_0 \simeq \widehat{\mathbb{Z}}^n/K$. By definition $|F_0| \leq d(F)$.

All connected components of $\mathbf{X}$ are isomorphic to $\mathbf{X}_0$. There are $(F : F_0)$ similar connected components and F acts transitively on them according to the way that F permutes the right cosets of F_0 . We summarize this by writing $\mathbf{X} = \coprod_{(F:F_0)} \mathbf{X}_0$. The torsor $\mathbf{X}_0$ is trivialized by the extension $S_0 = R_{n,\infty}^K$; more precisely $\mathbf{X}_0 \simeq \text{Spec}(S_0)$ and $S_0 \otimes_{R_n} S_0 \simeq S_0 \times \cdots \times S_0$ under the map $x \otimes y \mapsto (\sigma(x)y)_{\sigma \in F_0}$.

Clearly $|F_0|\widehat{\mathbb{Z}}^n \subset K$. Since the subgroup $|F_0|\widehat{\mathbb{Z}}^n$ of $\widehat{\mathbb{Z}}^n$ corresponds to the Galois extension $R_{n,|F_0|}$ we see that $\mathbf{X}_0$ is trivialized by the extension $R_{n,|F_0|}/R_n$. We summarize this by writing $\mathbf{X}_0 \times_{R_n} R_{n,|F_0|} = \coprod_{F_0} R_{n,|F_0|}$.

The isomorphisms

$$\mathbf{X} \times_{R_n} R_{n,|F_0|} \simeq \left(\coprod_{(F:F_0)} \mathbf{X}_0 \right) \times_{R_n} R_{n,|F_0|} \simeq \coprod_{(F:F_0)} \left(\coprod_{F_0} R_{n,|F_0|} \right) \simeq F_{R_{n,|F_0|}}$$

are compatible with the action of F . This shows that the R_n -torsor $\mathbf{X}$ is trivialized by $R_{n,|F_0|}$, hence also by $R_{n,|F|}$ (since $|F_0|$ divides $|F|$ and therefore $R_{n,|F|}$ is an extension of $R_{n,|F_0|}$).

2 Bounds on the isotriviality of multiloop algebras

Let $R \in k\text{-alg}$. Recall that by an R -form of $\mathfrak{g}$ we understand an algebra $\mathcal{L}$ over R for which there exists a faithfully flat and finitely presented extension S/R in $k\text{-alg}$ such that

$$\mathcal{L} \otimes_R S \simeq_{S\text{-Lie}} \mathfrak{g} \otimes_k S$$

(isomorphism of S -Lie algebras). Thus, by definition, forms are trivialized by some *fppf* cover S of R . In the case of Laurent polynomials, we have a very precise control over the trivializing base change.

Theorem 2.1 (Isotriviality, [GP1] and [GP3]). *Every R_n -form $\mathcal{L}$ of $\mathfrak{g}$ is isotrivial (i.e., trivialized by a finite étale cover of R_n). More precisely, there exist a positive integer $d = d(\mathcal{L})$, such that*

$$\mathcal{L} \otimes_{R_n} R_{n,d} \simeq_{R_{n,d}} \mathfrak{g} \otimes_k R_{n,d}.$$

□

At first, it does not seem reasonable to expect the existence of a “universal” d in the Isotriviality Theorem that splits all forms of $\mathfrak{g}$. The purpose of this note is to show that such a d thus indeed exists for multiloop algebras. More precisely, we have

Proposition 2.2. *Let $d(\mathfrak{g})$ be the maximum of all orders of abelian subgroups of the (finite) group of automorphisms of the root system corresponding to $\mathfrak{g}$. Any multiloop algebra $\mathcal{L}(\mathfrak{g}, \sigma_1, \dots, \sigma_n)$ is trivialized by the Galois extension $R_{n,d(\mathfrak{g})}$ of R_n . In particular, every multiloop algebra based on $\mathfrak{g}$ is trivialized by a finite étale cover of degree $d(\mathfrak{g})^n$.*

We begin by recalling the definition of multiloop algebras. Let $\sigma = (\sigma_1, \dots, \sigma_n)$ be a commuting family of finite order automorphisms of the k -algebra $\mathfrak{g}$. Let d be a common period of the σ_i , i.e., $\sigma_i^d = 1$.

For each $(i_1, \dots, i_n) \in \mathbb{Z}^n$, consider the simultaneous eigenspaces

$$\mathfrak{g}_{i_1 \dots i_n} := \{x \in \mathfrak{g} : \sigma_j(x) = \zeta_d^{i_j} x \text{ for all } 1 \leq j \leq n\}$$

(which of course depend only on the i_j modulo d). The multiloop algebra associated to this data is the k -Lie subalgebra $\mathcal{L}$ of $\mathfrak{g} \otimes_k R_{n,\infty}$ defined as follows:

$$\mathcal{L} = \mathcal{L}(\mathfrak{g}, \sigma) = \bigoplus_{(i_1, \dots, i_n) \in \mathbb{Z}^n} \mathfrak{g}_{i_1 \dots i_n} \otimes t_1^{i_1/d} \dots t_n^{i_n/d} \subset \mathfrak{g} \otimes_k R_{n,d} \subset \mathfrak{g} \otimes_k R_{n,\infty}. \quad (2)$$

Observe that $\mathcal{L}$ does not depend on the choice of period d . Observe also that $\mathcal{L}$ is in fact an R_n -submodule of $\mathfrak{g} \otimes_k R_{n,\infty}$. In particular, $\mathcal{L}$ has a natural R_n -Lie algebra structure. One easily verifies ([ABP2.5] Lemma 4.7) that

$$\mathcal{L} \otimes_{R_n} R_{n,d} \simeq_{R_{n,d}} \mathfrak{g} \otimes_k R_{n,d}. \quad (3)$$

Since $R_{n,d}/R_n$ is free of finite rank (hence *fppf*), $\mathcal{L}$ is an R_n -form of $\mathfrak{g}$ which is trivialized by the extension $R_{n,d}/R_n$. Note that the integer d can be arbitrarily large (there exist automorphisms of any given finite order). Proposition 2.2 implies the existence of a fixed d that splits all multiloop algebras based on a given $\mathfrak{g}$.

From (3) it follows that to a multiloop algebra $\mathcal{L}$ corresponds an R_n -torsor $\mathbf{X}_{\mathcal{L}}$ under the group $\mathbf{Aut}(\mathfrak{g})$.⁴ By descent theory, $\mathbf{X}_{\mathcal{L}}$ is representable by an affine R_n -scheme. The isomorphism class of the R_n -torsor $\mathbf{X}_{\mathcal{L}}$ will be denoted by $[\mathbf{X}_{\mathcal{L}}]$. Thus,

$$[\mathbf{X}_{\mathcal{L}}] \in H_{\text{ét}}^1(R_{n,d}/R_n, \mathbf{Aut}(\mathfrak{g})) \subset H_{\text{ét}}^1(R_n, \mathbf{Aut}(\mathfrak{g})). \quad (4)$$

As mentioned above the extension $R_{n,d}/R_n$ is finite Galois, and its Galois group $\Gamma_{n,d}$ can be identified with $(\mathbb{Z}/d\mathbb{Z})^n$ acting naturally on $R_{n,d}$ via our fixed choice of compatible roots of unity; namely

⁴ Strictly speaking, the structure group is the affine R_n -group $\mathbf{Aut}(A \otimes_k R_n)$. As already mentioned, this harmless slight abuse of notation is used throughout.

$\bar{\epsilon}(t_i^{1/d}) = \zeta_d^{e_i} t_i^{1/d}$ for all $\mathbf{e} = (e_1, \dots, e_n) \in \mathbb{Z}^n$. If we now let $\Gamma_{n,d}$ act on $\mathbf{Aut}(\mathfrak{g})(R_{n,d}) = \text{Aut}_{R_{n,d}\text{-alg}}(\mathfrak{g} \otimes R_{n,d})$ by conjugation, i.e., $\gamma\sigma = (1 \otimes \gamma)\sigma(1 \otimes (-\gamma))$, we have a natural correspondence $H^1(\Gamma_{n,d}, \mathbf{Aut}(\mathfrak{g})(R_{n,d})) \simeq H_{\text{ét}}^1(R_{n,d}/R_n, \mathbf{Aut}(\mathfrak{g}))$, where the H^1 on the left-hand side is the usual non-abelian cohomology. In terms of $H^1(\Gamma_{n,d}, \mathbf{Aut}(\mathfrak{g})(R_{n,d}))$, the loop torsor $\mathbf{X}_{\mathcal{L}}$ corresponds to the cocycle $\alpha_{\mathcal{L}} \in Z^1(\Gamma_{n,d}, \mathbf{Aut}(\mathfrak{g})(R_{n,d}))$ given by $(\alpha_{\mathcal{L}})_{\bar{\epsilon}} = \sigma_1^{-e_1} \dots \sigma_n^{-e_n} \otimes \text{id} \in \text{Aut}_{R_{n,d}\text{-alg}}(\mathfrak{g} \otimes R_{n,d})$.

Recall that we have an exact sequence of R_n -groups

$$1 \rightarrow \mathbf{G}^{ad} \rightarrow \mathbf{Aut}(\mathfrak{g}) \rightarrow \mathbf{Out}(\mathfrak{g}) \rightarrow 1, \quad (5)$$

where $\mathbf{G}^{ad}$ comes by base change from the Chevalley $\mathbb{Z}$ -group of $\mathfrak{g}$ of adjoint type, and $\mathbf{Out}(\mathfrak{g})$ is the finite constant group corresponding to the (abstract) finite group $\text{Out}(\mathfrak{g})$ of automorphisms of the Dynkin diagram of $\mathfrak{g}$ ([SGA3] Exp. XXIV Théorème 1.3 and Proposition 7.3.1). The sequence (5) is split. The choice of section $\mathbf{Out}(\mathfrak{g}) \rightarrow \mathbf{Aut}(\mathfrak{g})$ identifies $\mathbf{Out}(\mathfrak{g})$ with a subgroup of $\mathbf{Aut}(\mathfrak{g})$ so that

$$\mathbf{Aut}(\mathfrak{g}) = \mathbf{G}^{ad} \rtimes \mathbf{Out}(\mathfrak{g}). \quad (6)$$

Proof of Proposition 2.2: By the main theorem of [BM] there exists a maximal torus $\mathbf{T}$ of $\mathbf{G}^{ad}$ that is stable under the action of all the σ_i . Let $\Phi = \Phi(\mathbf{G}, \mathbf{T})$ be the corresponding root system, and $W = W(\Phi)$ its Weyl group.

Let $\mathbf{B}$ be a Borel subgroup of $\mathbf{G}$ containing $\mathbf{T}$. Then $\mathbf{Aut}(\mathbf{G}^{ad}, \mathbf{B}, \mathbf{T})/\mathbf{T} \simeq \mathbf{Out}(\mathfrak{g})$ ([SGA3] Exp. XXIV Prop. 2.1), and therefore

$$\mathbf{N}_{\mathbf{Aut}(\mathfrak{g})}(\mathbf{T}) = \mathbf{N}_{\mathbf{G}^{ad}}(\mathbf{T}) \rtimes \mathbf{Out}(\mathfrak{g}).$$

Thus

$$\mathbf{N}_{\mathbf{Aut}(\mathfrak{g})}(\mathbf{T})/\mathbf{T} \simeq \mathbf{N}_{\mathbf{G}}(\mathbf{T})/\mathbf{T} \rtimes \mathbf{Out}(\mathfrak{g}) \simeq \mathbf{W} \rtimes \mathbf{Out}(\mathfrak{g}) \simeq \mathbf{Aut}(\Phi),$$

where $\mathbf{Aut}(\Phi)$ is the finite group of automorphisms of the root system Φ .

The foregoing discussion shows that the torsor $\mathbf{X}_{\mathcal{L}}$ corresponding to $\mathcal{L}$ belongs to the image of the canonical map

$$H_{\text{ét}}^1(R_n, \mathbf{N}_{\mathbf{Aut}(\mathfrak{g})}(\mathbf{T})) \rightarrow H_{\text{ét}}^1(R_n, \mathbf{Aut}(\mathfrak{g})).$$

Consider the exact sequence of pointed sets

$$H_{\text{ét}}^1(R_n, \mathbf{T}) \rightarrow H_{\text{ét}}^1(R_n, \mathbf{N}_{\mathbf{Aut}(\mathfrak{g})}(\mathbf{T})) \xrightarrow{\pi} H_{\text{ét}}^1(R_n, \mathbf{Aut}(\Phi)).$$

By the previous Lemma after the base change $R_n \rightarrow R_{n,d(\mathfrak{g})}$ the torsor $\mathbf{X}_{\mathcal{L}}$ is in the kernel of π . Since $H_{\text{ét}}^1(R_{n,d(\mathfrak{g})}, \mathbf{T}) = \text{Pic}(R_{n,d(\mathfrak{g})})^{rk(\mathfrak{g})} = 1$ the $R_{n,d(\mathfrak{g})}$ -torsor $\mathbf{X}_{\mathcal{L}} \otimes_{R_n} R_{n,d(\mathfrak{g})}$ under $\mathbf{N}_{\mathbf{Aut}(\mathfrak{g})}(\mathbf{T}) \subset \mathbf{Aut}(\mathfrak{g})$ is trivial. $\square$

Remark 2.3. If all $\sigma_i \in \mathbf{T}$, then

$$\mathbf{X}_{\mathcal{L}} \in \operatorname{im}(H^1(R_n, \mathbf{T}) \rightarrow H^1(R_n, \mathbf{Aut}(\mathfrak{g}))) = 1.$$

Thus $\mathcal{L}(\mathfrak{g}, \sigma)$ is isomorphic as an R_n -Lie algebra, and a fortiori also as a k -Lie algebra, to $\mathfrak{g} \otimes_k R_n$. The “interesting” multiloop algebras are thus to be found by considering n -tuples of commuting automorphisms of $\mathfrak{g}$ that do not belong to a torus.

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