# Preface

The formulation and evaluation—sometimes approximate to be sure—of functional integrals has become increasingly important in a variety of disciplines. The material presented in this text offers the interested reader a modern treatment of a number of topics in this important field, and several exercises at the end of most chapters may help clarify certain aspects. Our emphasis is principally on quantum theory, but similar techniques may find application elsewhere as well. A well-known example observes that the position and momentum in quantum mechanics have their natural analogs with time and frequency in signal analysis.

A few general thoughts may be helpful. Integrals involving $N$ integration variables, when $N < \infty$, are familiar and well understood, even if they are often difficult to evaluate. The present textbook deals with integrations when $N = \infty$, i.e., for infinitely many variables. Such integrals have the usual problems of finite-dimensional integrals—such as conditional convergence, finitely additive measures, etc.—and a number of their own special problems. Integrals with an infinite number of variables are often called *functional integrals* or *path integrals*, and they occur in many areas of science, especially quantum mechanics, quantum field theory, and stochastic processes. We will include discussions on all these topics, and of course, the needed background will also be presented. Traditional discussions of these subjects by physicists, regrettably, are all too often rather cavalier and occasionally misleading—we shall try to point out some of the usual weak points in standard presentations, and even more importantly, we shall try to do better in our own discussions.

While there already exist a variety of other sources that offer good treatments of applications of functional integrals, our presentation instead favors conceptual and methodological developments, and as such there are some novel aspects to our discussion. Besides the standard material, it is noteworthy that we also present: (i) a different and more widely applicable approach for the analysis of quantum constraints, and (ii) a potentially new way to overcome conventional—but unacceptable—results for scalar nonrenormalizable relativistic quantum field theories.

A descriptive summary of the contents may help the reader gauge the overall scope and tenor of the material. After some general remarks (Chapter 1), an analysis of the probability of a single random variable and its three pure forms of realization (Chapter 2) form the background for a discussion of the probability aspects of random functions (Chapters 3 and 4), such as the most important and well-known Wiener process and the equally important but less well-known Poisson processes. These processes may be described via functional integrals or by suitable differential equations, each way offering different vantage points. Quantum phenomena are described by similar expressions with the appearance of $i = \sqrt{-1}$ in critical places. This makes for a formal similarity between probabilistic and quantum phenomena, but often masks a substantial mathematical gap that needs to be bridged to relate the two stories (Chapters 5 and 6). Special quantum states, the so-called coherent states, are especially helpful in bridging this mathematical gap (Chapter 7), and in a final version (Chapter 8) of physically motivated formulations, the probabilistic aspects of phase space paths are united with the quantum aspects of phase space paths to offer a natural formulation of the quantum theory that is both rigorous and truly invariant under classical canonical coordinate transformations leading thereby, in the author's view, for the first and only time, to a rigorously defined version of quantum mechanics that is strictly geometric in nature! This happy result has arisen by seeking further developments that are both physical and natural, and constitutes one of the main points of the book. The presence of constraints, which limit the classical phase space and often introduce unphysical (gauge dependent) variables, can be a challenge when any theory is quantized. Several commonplace techniques may lead to incorrect answers; instead we focus on a formulation, the so-called projection operator method (Chapter 9), that although possibly more difficult to apply, leads to acceptable results and does so with just the usual classical variables and nothing else. Infinitely many variables, as are appropriate to discuss field theories, adds several complications of a new kind, such as divergences that are typically handled by renormalized perturbation theory (Chapter 10). Yet some theories, the so-called nonrenromalizable theories, cannot be successfully studied by such conventional techniques. Finally (Chapter 11), we offer a novel way to deal with these insoluble models on the basis of insight gained by studying some soluble nonrenormalizable models within functional integral formulations. Here we see how such integrals help expose the source of unwanted divergences and support procedures that eliminate those divergences.

This book is intended for physicists, mathematical physicists, applied mathematicians, chemists, engineers, and others who seek alternatives to the study of partial differential equations as a means to examine the properties of certain systems. Part of the material in this book has been used for a one-semester graduate course on Functional Integration (including elements of all chapters except Chapter 11). Exercises at the end of most chapters help enforce the principal topics.

**Acknowledgements**

Over the years the author has profited from various forms of interaction with a number of people, and it is fitting to thank some of them on this occasion. They include: R. Alicki, E. Aslaksen, K. Baumann, B.G. Bodmann, W.R. Bomstad, I. Daubechies, G.G. Emch, H. Ezawa, H.-Y. Fan, H. Gausterer, J.-P. Gazeau, J. Govaerts, H. Grimmer, L. Hartmann, G.C. Hegerfeldt, C.J. Isham, A. Kempf, J.S. Lee, H. Leutwyler, J. Lewandowski, E. Lieb, J.S. Little, P. Maraner, J. McKenna, K. Muttalib, H. Narnhofer, E. Onofri, T. Paul, K.A. Penson, W.P. Petersen, A. Rubin, S.V. Shabanov, L. Shepp, B.-S. Skagerstam, J.J. Stankowicz, R.F. Streater, L. Streit, J.Y. Su, E.C.G. Sudarshan, W. Tomé, T.T. Truong, G. Watson, and C.-J. Zhu.

Thanks are also expressed to Jennifer Klauder for producing several of the figures for this book, and to Glenn Watson for numerous suggestions for text improvement.

Gainesville, Florida *John R. Klauder*
May 2010

Springer

http://www.springer.com/978-0-8176-4790-2

A Modern Approach to Functional Integration
Klauder, J.R.
2011, XVI, 282 p. 9 illus., Hardcover
ISBN: 978-0-8176-4790-2
A product of Birkhäuser Basel