Preface

This book consists of two parts. Part I (Chapters 1–8) gives a basic introduction to wavelets and the related mathematics. It is designed for a one-semester course on wavelet analysis aimed at graduate students or advanced undergraduates in science, engineering, and mathematics. This part is an outgrowth of lecture notes to just such a course that I have now taught, with variations, for several years at the University of Massachusetts at Lowell. It can also be used as a self-study or reference book by practicing researchers in signal analysis and related areas or by anyone else interested in wavelet theory. Since the expected audience is not presumed to have a high level of mathematical background, much of the needed machinery is developed from the beginning, with emphasis on motivation and explanation rather than mathematical rigor. The underlying mathematical ideas are explained in a conversational, common-sense way rather than the standard definition-theorem-proof fashion. A notation is introduced that makes it possible to present signal analysis in a clean, general, modern mathematical language having its roots in linear algebra. Each chapter ends with a set of straightforward problems designed to drive home the concepts just covered. The only prerequisites for the first part are elementary matrix theory, Fourier series, and Fourier integral transforms.

Part II (Chapters 9–11) consists of original research and is written in a more advanced style. It can be used for a higher-level second-semester course or, when combined with Chapters 1 and 3, as a reference for a research seminar. Its theme may be described as an attempt to unify signal analysis and physics. This quest is based on the observation that the electromagnetic and acoustic waves used as carriers of information lend themselves naturally to a particular space-time form of wavelet analysis. The existence, and virtual uniqueness, of this analysis is guaranteed by the very structure of the differential equations governing these physical waves (Maxwell's equations for electromagnetic waves and the wave equation for acoustic waves). Namely, those equations are invariant under the group of *conformal transformations of space-time*, which includes dilations and translations – the basic operations of wavelet analysis – as a subgroup.

Although wavelet analysis is a young field, several excellent books have already appeared on the subject. *Ten Lectures on Wavelets* by Ingrid Daubechies (SIAM, 1992) is a comprehensive account containing (but not restricted to) the wonderful lectures she gave at the NSF–CBMS wavelet conference held at the University of Lowell in June 1990. I have relied heavily on Ingrid Daubechies' monograph while writing Chapters 5–8 of this book. *An Introduction to Wavelets* by Charles Chui (Academic Press, 1992) is readable and concise, emphasizing especially the important connection with splines. *Wavelets and Operators* by Yves Meyer (Cambridge University Press, 1993) is a translated and updated version of the earlier *Ondelettes et Opérateurs*, by a master of classical analysis who is also one of the founders of wavelet theory. In addition, there exist by now several collections of essays and research papers on wavelets, including *Wavelets and Their Applications*, edited by Mary-Beth Ruskai et al. (Jones and Bartlett, 1992); *Wavelets: A Tutorial in Theory and Applications*, edited by Charles Chui (Academic Press, 1992); *Wavelets: Mathematics and Applications*, edited by John Benedetto and Michael Frazier (CRC Press, 1993); and *Progress in Wavelet Analysis and Applications*, edited by Yves Meyer and Sylvie Roques (Editions Frontières, Gif-sur-Yvette, France, 1993).

Why write yet another book? Having taught several courses in wavelet analysis to a variety of audiences with backgrounds in science, engineering and signal analysis, I discovered that most of my students found the existing books quite difficult – mainly because the presupposed level of mathematical sophistication seemed to be quite high. I believe that some of this difficulty is a language problem. Most scientists, engineers, and applied researchers are unfamiliar with modern mathematical notation and concepts. Thus, a casual reference to "measurable functions" or "$L^2(\mathbf{R})$" can be enough to make an aspiring pilgrim weary. Also, none of the monographs with which I am familiar contain exercises that can be assigned as homework problems, and this makes them less suitable as textbooks. I hope that this volume will fill the need for a book truly aimed at the level of a competent and motivated student or researcher, even if he or she may not be highly trained in modern mathematics.

Numerous graphics illustrate the concepts as they are introduced. The notation and basic concepts needed for a clean and fairly general treatment of wavelet analysis are developed from the beginning without, however, going into such technical detail that the book becomes a mathematics course in itself and loses its stated purpose. There is no attempt to be comprehensive in the coverage; one of the goals of the book is to enable readers to extract more specialized information from the literature for themselves. Nor is there an attempt to be comprehensive in the literature citations. The number of research papers on wavelets and related topics has grown astronomically in the past few years, and it is very difficult for anyone to keep up with all the aspects of this literature. An extensive survey of wavelet literature, including abstracts, was prepared by Stefan Pittner, Josef Scheid, and Christoph Ueberhuber. It is available from the Institute for Applied and Numerical Mathematics, Technical University of Vienna. Another large bibliography on wavelets and digital signal processing was compiled by Reiner Creutzburg and appears in *Progress in Wavelet Analysis and Applications*, edited by Yves Meyer and Sylvie Roques (cited earlier). A survey of the literature on time-frequency methods and related topics is available upon request from Hans Feichtinger at the electronic mail address fei@tyche.mat.univie.at.

Although I have done my very best to make this book accessible, I cannot claim that it is easy reading for everyone. The level of presentation becomes more advanced as the language is established and the concepts internalized. Certain aspects of signal analysis that are usually glossed over are explained carefully here, such as the fundamental algebraic distinction between the space-time domain $\mathbf{R}^n$ and the wave-number-frequency domain $\mathbf{R}_n$ (the latter is the *dual space* of the former; see Section 1.4). What *is* claimed is that, apart from the stated prerequisites of matrix theory, Fourier series, and Fourier integrals, the book is largely self-contained, enabling a diligent reader without much previous mathematical background to understand the concepts sufficiently well to apply them, or to tackle some of the more mathematically oriented books or delve into the research literature.

Every chapter begins with a brief summary of its contents and a list of prerequisites. In all but the first chapter, the prerequisites are an understanding of some of the previous chapters. This should make the book more useful as a reference for working researchers, since they will be able to study their topics of interest without reading the entire volume.

Part I, called *Basic Wavelet Analysis*, consists of Chapters 1–8. A brief description of the contents is as follows: Chapter 1 contains a review of linear algebra, especially the relation between matrices and linear operators. A streamlined formalism, called *star notation*, is developed, which makes the construction of dual bases and resolutions of unity appealing and intuitive. By reinterpreting finite-dimensional vectors as functions on finite sets, the formalism is seen to generalize seamlessly to an infinite number of dimensions. When the functions depend on a discrete variable, it usually suffices to restrict the analysis to spaces of square-summable sequences, giving the so-called ℓ^2 spaces. In order to accommodate functions of a continuous variable such as time signals, the concepts of measure and integration are explained, leading to L^2 spaces. Next, the analysis of periodic functions by Fourier series and the associated resolutions of unity for ℓ^2 spaces are developed. As the periods become infinite, the Fourier series go over to the Fourier integral transforms on L^2 spaces. In Chapters 2 and 3, continuous time-frequency and time-scale (wavelet) analyses are motivated and developed, along with the associated continuous resolutions of unity. In Chapter 4 we introduce the concept of *generalized frames*, which combines the idea of resolutions of unity (both continuous and discrete) with the usual (discrete) notion of *frames*. The continuous resolutions of unity constructed in Chapters 2 and 3 are special cases. In Chapters 5 and 6 we discretize the resolutions of unity of Chapters 2 and 3 to obtain various time-frequency and time-scale *sampling theorems*. In the context of Chapter 4, this amounts to constructing *discrete subframes* of the continuous frames found in Chapters 2 and 3. Chapters 7 and 8 give an algebraic presentation of multiresolution analysis and orthonormal wavelet bases. Section 8.4 introduces a new algorithm for the construction of scaling functions and wavelets from a given filter sequence. This method is similar to diadic interpolation but is somewhat more efficient since no eigen-equation needs to be solved to obtain the initial values. It also inspires a new approach to the construction of multiresolution analyses, in particular orthonormal filter sequences, based on the statistical concept of *cumulants*.

Part II, called *Physical Wavelets*, consists of Chapters 9–11. It attempts to bridge the gap between signal analysis and physics, motivated by the observation that signals are often communicated by electromagnetic or acoustic waves. These waves satisfy Maxwell's equations and the wave equation, respectively, and we show that the structure of those equations implies the existence of *electromagnetic and acoustic wavelets* that can be used as building blocks to compose arbitrary electromagnetic and acoustic waves. The construction of these "physical wavelets" is implemented in a simple and elegant way by means of the *analytic-signal transform*, which extends the physical waves to a complex space-time domain $\mathcal{T}$, the *causal tube*. These extensions, which generalize Gabor's idea of "analytic signals," tend to *unfold* the physical waves, displaying their informational contents. All the physical wavelets in any representation can be obtained from a single "reference wavelet" by conformal space-time transformations, just as one-dimensional wavelets can all be obtained as dilations and translations of a single "mother wavelet." In Chapter 9 this construction and some of its consequences are explored in detail for electromagnetic waves. In Chapter 10 the electromagnetic wavelets are applied to radar and other electromagnetic imaging. My interest in radar was sparked by an invitation to teach a short course on wavelet electrodynamics at the tenth annual ACES (Applied Computational Electromagnetics Society) conference at Monterey, California in March 1994. The naive model I presented there has since undergone considerable evolution. The analytic signal of a given electromagnetic wave plays the role of an extended space-time *cross ambiguity function*. A novel geometric model is proposed for electromagnetic scattering, based on the simple transformation properties of the wavelets under conformal transformations. In Chapter 11 we construct the acoustic wavelets. A major difference between electromagnetic and acoustic waves is that whereas the former can travel in vacuum, the latter need a medium in which to propagate. Since all reference frames in uniform relative motion are equivalent in vacuum, relativity theory requires that Lorentz transformations be represented by unitary operators on the Hilbert space of solutions of Maxwell's equations. No such requirement applies to acoustic waves, since for them the medium determines a unique reference frame. We construct a one-parameter family of nonunitary wavelet representations of the conformal group on spaces of solutions of the wave equation.

I believe the single most exciting result in Part II is the discovery that the physical wavelets Ψ_z, which are solutions of the *homogeneous* Maxwell and wave equations, naturally split into two parts: one is an incoming wavelet Ψ_z^- that gets *absorbed* (or detected) and the other is an outgoing wavelet Ψ_z^+ that gets *emitted* just when the incoming wavelet is absorbed. This splitting is far from trivial, because of the analyticity constraints. Neither Ψ_z^- nor Ψ_z^+ are global solutions of the homogeneous equation. Rather, they each solve the corresponding *inhomogeneous* equation, and the source terms are "currents" given by one-dimensional wavelets, as in Equation (11.49). That establishes a deep connection between the physical wavelets and a certain special class of one-dimensional wavelets, and therefore between the wavelet analysis of physical waves and that of communication signals.

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