Preface

The purpose of this book is to introduce the reader to some of the main abstract tools of nonlinear functional analysis and their applications to semilinear elliptic Dirichlet boundary value problems.

In the first chapter we outline some general results on Fréchet differentiability, Nemitski operators, weak and strong solutions of the linear Laplace equation, linear compact operators and their eigenvalues and Sobolev spaces. This last topic is discussed in greater generality in Appendix A.

Chapter 2 deals with the Banach contraction principle and with a fixed point theorem for increasing operators. In Chap. 3 we study the local inversion theorem, the Hadamard–Caccioppoli global inversion theorem and the case in which the map to be inverted has fold singularities. Chapter 4 is concerned with the Leray–Schauder topological degree. Variational methods are discussed in Chap. 5. Minima, the mountain pass theorem and the linking theorem are stated and proved. Chapter 6 deals with local and global bifurcation theory.

The abstract results collected in first part of the book are applied in the second part to prove existence and multiplicity results for semilinear elliptic Dirichlet boundary value problems on bounded domains in $\mathbb{R}^N$. We emphasize that the choice of the appropriate abstract tool depends on the behavior of the nonlinearity f as well as on the kind of results one expects.

First, in Chap. 7, we outline how a semilinear elliptic boundary value problem can be transformed into an operator equation in an appropriate Banach or Hilbert function space. In Chap. 8 we consider the case in which, roughly, f is sublinear at infinity and one can prove a priori estimates for possible solutions. In this case one can use degree theory or variational methods or the global inversion theorem. Chapter 9 deals with asymptotically linear problems, for which one can also use several different approaches such as global bifurcation or variational methods. In Chap. 10 we study problems with asymmetric nonlinearities, when the behaviors at $+\infty$ and $-\infty$ are different. If one aims to find the precise number of multiple solutions, the most appropriate approach turns out to be the global inversion theorem in the presence of fold singularities. But one can also use sub- and super-solutions jointly with degree theoretical arguments. Nonlinearities that are superlinear at infinity are considered in Chap. 11 by means of the mountain pass or linking theorems.

In all of the preceding chapters we do not consider more general sophisticated versions of problems, but prefer to study model cases containing the main features of the arguments without unnecessary technical details.

The last two chapters of the book are concerned with slightly more advanced topics of current research. In Chap. 12 a class of quasilinear elliptic problems is discussed using critical point theory. Here the corresponding Euler functional is not C^1, and hence a new form of the mountain pass theorem has to be proved. Chapter 13 deals with nonlinear Schrödinger equations on R^N. We prove the existence of ground and bound states as well as semiclassical states.

The book is addressed to senior undergraduate and graduate students of mathematics as well as to students of applied sciences, who wish to utilize a modern approach to the fascinating topic of nonlinear elliptic partial differential equations.

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