

Preface

This memoir is an outgrowth of earlier work of Moser and of Bangert on solutions of a family of nonlinear elliptic partial differential equations on $\mathbb{R}^n$ and of research of the authors on an Allen–Cahn PDE model of phase transitions. The simplest example of the class of equations studied by Moser and Bangert is

$$-\Delta u + F_u(x, u) = 0, \quad x \in \mathbb{R}^n, \quad (\text{PDE})$$

where F is periodic in all of its arguments. Our earlier work was for equations of the form

$$-\Delta u + G_u(x, u) = 0, \quad x \in \mathbb{R}^2, \quad (\text{AC})$$

where G is a double-well potential, e.g., $G(x, u) = a(x)u^2(1 - u)^2$ with $a(x) > 0$ and 1-periodic in the components x_1, x_2 of x . The behaviors of F and G in u are rather different. However, the study of solutions of (AC) that lie between 0 and 1 can be reduced to a similar study for (PDE). Namely, taking G restricted to $\mathbb{R}^2 \times [0, 1]$, extending it evenly and 2-periodically about $u = 0$, and rescaling the u variable yields an equation of the form of (PDE).

Moser initiated the study of a much more general family of equations than (PDE). His goal was to establish a version of the theory of Aubry and of Mather on monotone twist maps for partial differential equations. Toward that end, Moser and then Bangert studied solutions of their equations that possessed two additional properties: a certain minimality in a variational setting, and a so-called “without self intersections property” that will be explained later.

The goal of this monograph is to develop and study the rich structure of the set of solutions of the simpler model case (PDE), which both contains our earlier work on (AC) and expands the work of Moser and Bangert to include solutions that merely have local minimality properties. Minimization arguments are an important tool in our investigations. We begin in Part I by following Moser and using minimization arguments to obtain an ordered family of solutions of (PDE) that are 1-periodic in $x_1, \dots, x_n$. Suppose there is a gap, i.e., no other members of this class, between a pair of such periodics. Then an ordered family of heteroclinic solutions

in x_1 (and periodic in $x_2, \dots, x_n$) between the pair are obtained by minimizing a “renormalized functional” associated with (PDE). Such basic heteroclinic solutions were originally obtained by Bangert. His argument was based on Moser’s work and was not variational in nature. Our minimization approach is crucial for the construction of more complex solutions of (PDE) that, in the language of dynamical systems, shadow (or are near) formal concatenations of the basic heteroclinic states. These new multitransition solutions of (PDE) defined on $\mathbb{R} \times \mathbb{T}^{n-1}$ are studied in detail in Part II. They are obtained as local minima of the renormalized functional via a constrained minimization problem.

Whenever there is a gap between a pair of the basic heteroclinics in x_1 , a second renormalized functional can be introduced and used to obtain ordered families of heteroclinic solutions in x_2 between them. The existence of such solutions by nonvariational arguments was also originally carried out by Bangert. The minimization approach to this new family of basic solutions of (PDE) is given in Part I. Lastly, it is used in Part III to construct further solutions of (PDE) defined on $\mathbb{R}^2 \times \mathbb{T}^{n-2}$ that shadow formal concatenations of the heteroclinics in x_2 .

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