
Preface

Optimal control of partial differential equations (PDEs) is by now, after more than 50 years of ever-increasing scientific interest, a well-established discipline in mathematics with many interfaces to science and engineering. During its development, the complexity of the systems to be controlled has also increased significantly, so that today, for example, fluid-structure interactions, magneto-hydronechanical, and electromagnetical as well as chemical and civil engineering problems can be dealt with. However, the numerical realization of optimal controls based on, say, optimality conditions together with the simulation of states has also become an issue in scientific computing, as the number of variables involved may easily exceed a couple of millions and, hence, structure-exploiting discretization and corresponding adaptive algorithms have to be developed.

There are several trends to be observed in this discipline. One is to increase the complexity of the system description in terms of genuinely nonlinear partial differential equations and hybrid couplings to ordinary or even event-driven dynamics together with constraints not only on the controls but also on the states. This kind of investigation typically focuses on a more accurate modeling with respect to the physical description and typically subsumes simple domains for the components. On the other hand, we observe a development, in particular in multiscale modeling, where even classical equations are considered on very complicated domains such that the focus is on the increasing complexity of the geometry of the underlying domain. It obviously is desirable, but also mostly prohibitive, to ask for everything: a very detailed modeling of the process, the handling of very complex geometries, and a timely or even real-time capable numerical realization. However, in the context of modern industrial or science applications, this often turns out to be impossible if one insists on very high accuracy.

Model reduction or effective modeling for optimal control problems involving systems of PDEs on complicated domains, therefore, has been the focus of many research initiatives in the last decade. The dynamical system describing the behavior of states may be replaced by a low-dimensional one using, for in-

stance, a technique that has come to be known as the reduced basis approach or, possibly greedy, proper orthogonal decomposition. Here, the geometry is built into the simulation tools that provide the empirical bases using “snapshots” which, in turn, makes this approach difficult for hierarchically ordered domains. On the other hand, one may use “surrogate models” on simple domains, models that are simplifying approximations of both the state equation and the domain.

A third avenue, the one along which we will proceed in this monograph, is based on asymptotic analysis. The method we describe and further develop aims at combining techniques of homogenization and approximation in order to cover optimal control problems defined on reticulated domains, such as lattice structures, honeycomb structures, hierarchical structures, or networked domains in general. Here, error estimates are introduced in order to control the quality of the controls obtained on the approximation level.

Our interest is mainly in problems where the control is exerted at, for example, highly oscillating boundaries or interfaces associated with such structures, but we also ask for “controls in the coefficients”, that is, for controlling material parameters on the microscopic ε -level. Our research is strongly motivated by recent developments in multiscale modeling and simulation in a variety of applications. However, from a mathematical point of view, only the aspect ratio – that is, the relation of, say, thickness versus length – is relevant (as long as one does not enter molecular dynamics). In that respect, we can also relate our research to networked structure mechanics in civil engineering, such as flexible structures, masts of all kinds, and gas, water, and traffic networks.

As for material sciences, metallic, ceramic, or polymeric foams are particularly interesting because of their mechanical properties, such as being extremely lightweight and at the same time adequately stiff. Similarly, complex conductors on the micro-level exhibit graphlike structures, and carbon nanotubes are used in many applications. They themselves serve as gridlike domains supporting processes like electromagnetic wave propagation. However, properly assembled in thousands or millions on a waferlike substrate, they can be used as reactors for catalytic processes.

Mathematically speaking, elliptic, parabolic, and hyperbolic systems on reticulated domains are used to model these applications which, in turn, exhibit a genuine multiscale character. Engineers define cost or merit functions that are to be optimized with respect to various control actions along the boundary of the structure and, even more challenging, with respect to material properties. Certainly, both states and controls have to be taken into account as being constrained.

In particular, state constraints genuinely give rise to Lagrange-multipliers that are measures. As a result, in a general setting, such PDEs on reticulated domains should be dealt with in the context of descriptions allowing for measures on the “right-hand side” as well as in the system equations and the geometries. This means that providing an abstract approach to optimal

control problems for PDEs in such domains is not the result of an intrinsically motivated mathematical desire to achieve the highest degree of generality—it follows the needs dictated by the applications!

Moreover, the type of reticulated structures to be considered varies drastically with the application context or with the degree of approximation needed in a given context. Namely, in a gas network or a sewer system, civil engineers trust in one-dimensional models for the gas-flow, and consequently in network-flow problems on graphs. In a river system a fully three-dimensional treatment may be necessary, but, still, the problem would be one on a networked domain—a “fat graph”. The same rationale applies to elastic multi-structures, foams, and all the way down the scales to nanotubes. Percolation networks carrying microflows can be modeled as perforated domains, but, in a further approximation, also as fat graphs.

Obviously, the aspect ratio and the cell size for the periodic structure serve as scaling parameters. An optimal control problem defined on such structures, therefore, genuinely inherits a number of scales that may be separated or even not separable at all. The fundamental problem now is: How do such optimal control problems “behave” upon changing these parameters? More precisely, what happens if we go up the scales to a “continuum” description? However, why should we do that to begin with? This question brings us back to the question of model reduction and its interplay with approximation properties. For a small scale, for example, a fine gridstructure, the numerical effort related to a discretization in order to reveal a resolution according to the scale is typically prohibitive. On the other hand, a coarse graining may miss the effects looked for. Asymptotic analysis resolves this antagonism elegantly, in that the limiting problem and approximations thereof can be taken as a surrogate optimal control problem on a simple domain such that error estimates show how far the true fine-scale solution is away from the “homogenized” solution which, in turn, can be termed suboptimal.

In order for all of this to become a mathematically sound theory, it is not sufficient to apply asymptotic expansions to all components of such an optimal control problem—namely the cost function, the state equation, the domains, and the control and observation instruments individually—and replace it by its low-order parts. An asymptotic analysis of optimal control problems as such is in order. Meanwhile, an ill-posed PDE problem—in the sense of Hadamard—due to the freedom of choosing proper control inputs may turn out to be well-posed as an optimization problem, while a well-posed PDE problem easily can exhibit ill-posedness, once integrated into an optimal control problem. Hence, well-posedness of optimal control problems is a new and interesting issue.

The book focuses on all of these aspects from two perspectives. First, a rigorous and mostly self-contained mathematical theory of PDEs on reticulated domains together with well-posedness for the governing optimal control problems is described. The concept of optimal control problems for PDEs in varying such domains, and hence in varying Banach spaces, is developed, fol-

lowed by appropriate concepts for convergence of optimal control problems in variable spaces. This comprises Part I of the book. Even though there are by now a number of textbooks for PDE-constrained optimal control available, this monograph contains a unique collection of results that are necessary to treat the optimal control problem on varying structures which are not available in a textbook otherwise. In Part II, particular examples and applications are investigated with the tools established in Part I of the book. These examples are strongly motivated by applications in mechanics and material sciences as explained above, but they can be understood without any knowledge from those fields. In order to accomplish this, the models are somewhat simplified such that, for example, only quasi-static flow in cylindrically perforated domains is considered instead of the fully time-dependent Navier–Stokes flow. Additionally, the elliptic second order problems on thin or fat graphs are scalar instead of being vectorial, which would be necessary in order to be directly applicable to problems of elasticity.

Overall, the book's first part can be seen as an advanced textbook for abstract optimal control problems, in particular on reticulated domains, which can be used in graduate courses, while its second part serves as a research monograph, using somewhat stratified applications in an exemplary manner. Part II can be of use also in seminars, building on the knowledge from a graduate course taught from Part I. For the reader's convenience, in Part II, we sometimes reintroduce the basic concepts that are dealt with in Part I on an abstract level; however, they are explicitly geared towards the particular application. Admitting some potential redundancy, we thereby keep the chapters in the second part of the book self-contained for researchers in the field.

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