

Contents

Part I Proofs

1	Predicate Logic	3
1.1	Inductive Definitions	3
1.1.1	The Fixed Point Theorem	3
1.1.2	Inductive Definitions	6
1.1.3	Structural Induction	8
1.1.4	Derivations	8
1.1.5	The Reflexive-Transitive Closure of a Relation	10
1.2	Languages	10
1.2.1	Languages Without Variables	10
1.2.2	Variables	11
1.2.3	Many-Sorted Languages	12
1.2.4	Substitution	13
1.2.5	Articulation	15
1.3	The Languages of Predicate Logic	16
1.4	Proofs	18
1.5	Examples of Theories	23
1.6	Variations on the Principle of the Excluded Middle	30
1.6.1	Double Negation	30
1.6.2	Multi-conclusion Sequents	30
2	Models	35
2.1	The Notion of a Model	35
2.2	The Soundness Theorem	38
2.3	The Completeness Theorem	39
2.3.1	Three Formulations of the Completeness Theorem	40
2.3.2	Proving the Completeness Theorem	40
2.3.3	Models of Equality—Normal Models	43
2.3.4	Proofs of Relative Consistency	44
2.3.5	Conservativity	46

2.4	Other Applications of the Notion of Model	49
2.4.1	Algebraic Structures	49
2.4.2	Definability	51
Part II Algorithms		
3	Computable Functions	55
3.1	Computable Functions	55
3.2	Computability over Lists and Trees	58
3.2.1	Computability over Lists	58
3.2.2	Computability over Trees	60
3.2.3	Derivations	61
3.3	Eliminating Recursion	62
3.4	Programs	65
3.4.1	Undecidability of the Halting Problem	66
3.4.2	The Interpreter	66
4	Computation as a Sequence of Small Steps	71
4.1	Rewriting	72
4.2	The Lambda-Calculus	81
4.3	Turing Machines	92
Part III Proofs and Algorithms		
5	Church's Theorem	101
5.1	The Notion of Reduction	101
5.2	Representing Programs	102
5.3	Church's Theorem	108
5.4	Semi-decidability	111
5.5	Gödel's First Incompleteness Theorem	112
6	Automated Theorem Proving	117
6.1	Sequent Calculus	117
6.1.1	Proof Search in Natural Deduction	117
6.1.2	Sequent Calculus Rules	118
6.1.3	Equivalence with Natural Deduction	120
6.1.4	Cut Elimination	126
6.2	Proof Search in the Sequent Calculus Without Cuts	130
6.2.1	Choices	130
6.2.2	Don't Care Choices and Don't Know Choices	130
6.2.3	Restricting the Choices	131
7	Decidable Theories	139
8	Constructivity	143

9	Epilogue	149
	References	151
	Index	153



<http://www.springer.com/978-0-85729-120-2>

Proofs and Algorithms

An Introduction to Logic and Computability

Dowek, G.

2011, XII, 156 p., Softcover

ISBN: 978-0-85729-120-2