Preface

Every time I complete a manuscript my attention turns quickly to the title of my next book. And after completing the latest version of *Mathematics for Computer Graphics*, I began to think of what should follow. It didn't take too long to identify the subject of this book: rotation transforms, which have always interested me throughout my career in computer graphics.

I knew that I was not alone in finding some of the ideas difficult, as every time I searched the Internet using search keys such as '*quaternions*', '*Euler angles*', '*rotors*', etc., I would come across websites where groups were discussing the meaning of gimbal lock, the matrix representation of a quaternion, eigenvectors, etc., and I knew straight away that I had to do my bit to clarify the subject.

One of the main problems why there is so much confusion arises through the different forms of vector and matrix notation. Some authors work with matrices that involve row vectors, rather than column vectors, which leads to a transposed matrix. In some cases, the direction of rotation is clockwise, rather than the normal positive, anti-clockwise direction. Quaternions are treated as a four-dimensional object where one has to visualise a hyper-sphere before they can be mastered. Some of the algorithms for extracting eigenvectors and their associated eigenvalue can be very sensitive to the type of matrix in use. This is all rather disconcerting.

The experienced mathematician will take all of this in his or her stride, but to a cg programmer trying to implement the best rotation algorithm and design some stable code, this is not good news. So about a year ago, I started to collect my thoughts on how to approach this subject. After a few false starts and chapter rewrites, I decided to write an introductory book that would take the reader through the foundations of rotation transforms from complex numbers to Clifford algebra rotors, touching on vectors, matrices and quaternions on the journey.

Illustrations are vital to understanding rotation transforms, especially the difference between rotated points and rotated frames. I came across many websites, technical literature and books where the illustrations confused rather than clarified what was going on, and I explored various approaches before settling for a unit cube with numbered vertices. This book contains over a hundred illustrations, which, I hope will help the reader understand the underlying mathematics.

In order to create some sort of structure, I have separated transforms for rotating points in a fixed frame, from transforms that rotate frames with fixed points. I have also separated transforms in the plane from transforms in 3D space. In all, there are thirteen chapters, including an introduction and summary chapters.

Chapter 2 provides a quick introduction to complex numbers and the rotational qualities of imaginary i. The reader should be comfortable with these objects, as we find imaginary quantities in quaternions and multivectors.

Chapter 3 covers vectors and their products, whilst Chap. 4 describes matrices and their associated algebra. It also explores other relevant topics such as matrix inversion, symmetric and antisymmetric matrices, eigenvectors and eigenvalues.

Chapter 5 covers quaternions and their various forms, but I leave their rotational abilities for Chap. 11. I play down their four-dimensional attributes as I don't believe that this characteristic is too important within this introductory book.

Chapter 6 introduces multivector rotors that are part of Clifford's geometric algebra, and again, their rotational qualities are delayed until Chap. 12.

Chapter 7 covers rotation transforms in the plane and establishes strategies used for transforming points in space, whilst Chap. 8 addresses rotating frames of reference in the plane.

Chapter 9 is an important chapter as it introduces the classic techniques for handling 3D rotations, composite rotations, gimbal lock, and provides a stable technique for extracting eigenvectors and eigenvalues from a matrix.

Chapter 10 develops the ideas of Chap. 9 to explain how coordinates are computed in rotating frames of reference.

Chapter 11 takes quaternions from Chap. 5 and shows how they provide a powerful tool for rotating points and frames about an arbitrary axis.

Chapter 12 takes the multivectors from Chap. 6 and shows how they provide a unified system for handling rotors. Finally, Chap. 13 draws the book to a conclusion.

I would like to take this opportunity to acknowledge the authors of books, technical papers and websites who have influenced my writing over recent years. From these dedicated people I have discovered new writing techniques, how to format equations, and how to communicate complex ideas in an easy manner. Without them this book would not have been possible. However, there is one author that I must acknowledge: Michael J. Crowe. His book *A History of Vector Analysis* [1] is an amazing description of how vectors and quaternions evolved, and is highly recommended.

In particular, I would like to thank Dr Tony Crilly, Reader Emeritus at Middlesex University, who read a draft manuscript and made many important recommendations. Tony read the book through the eyes of a novice and questioned my writing style when clarity started to sink below the surface. Forty years ago, when I was struggling with gimbal lock and Euler transforms, Tony brought to my attention the rotation transform developed by Olinde Rodrigues, who had invented quaternions before Hamilton, but that's another story. I included this transform in my animation software system PICASO, running on a mainframe computer with a 24 KB store! I was very nervous about using it as sines and cosines were evaluated at a software level and extremely slow.

I would also like to thank Prof. Patrick Riley for providing me with a *harmonogram* that has formed the book's cover design, and for his feedback on early drafts of the manuscript when I needed to know whether I was managing to communicate my ideas effectively.

Once again, I am indebted to Beverley Ford, General Manager, Springer UK, and Helen Desmond, Assistant Editor for Computer Science, for their support and reminding me of the importance of deadlines. I would also like to thank Springer's technical support team for their help with $\LaTeX\,2_\varepsilon$.

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