

Preface

The Virasoro algebra is an infinite dimensional Lie algebra which has appeared in several context. For example, E. Cartan [C] in 1909 classified simple infinite dimensional linearly compact Lie algebras over $\mathbb{C}$ which contains the Lie algebra $\overline{W}_m$ of all formal vector fields in m determinates. Around 1940, the Lie algebra of derivations of the group algebra of $\mathbb{Z}/p\mathbb{Z}$ over a field of characteristic $p > 2$ was studied by colleagues of E. Witt, e.g., H. Zassenhaus [Za] showed that this Lie algebra is simple and H.J. Chang [Ch] studied its representations. All these works treated the so-called **Witt algebra**, or the centreless Virasoro algebra. It was only in 1966 that its non-trivial central extension first appeared in the work of R. Block [Bl] over a field of positive characteristic in classifying certain class of Lie algebras. In 1968, I. M. Gelfand and D. B. Fuchs [GF] determined the cohomology ring of the Lie algebra of the vector fields on the circle.

In 1970, the Virasoro algebra appeared in the article [Vir] of M. Virasoro on his work on string theory, hence the name **Virasoro algebra**. See, e.g., [Mand] for an account of this period. A great of impact was made in 1984 by A. A. Belavin, A. M. Polyakov and A. B. Zamolodchikov [BPZ1], [BPZ2] where they treated the phenomena at critical points in 2-dimensional statistical mechanical models.

In the 1980s, the representation theory of the Virasoro algebra was intensively studied. In particular, the structure of its Verma modules and its Fock modules were completely determined by B. Feigin and D. Fuchs [FeFu4]. For a further beautiful historical description, see [GR].

The aim of this book is to describe some fundamental facts about the representation theory of the Virasoro algebra in a self-contained manner. The topics covered in this book are the structure of Verma modules and Fock modules, the classification of (unitarisable) Harish-Chandra modules, tilting equivalence, and the rational vertex operator algebras associated to the so-called BPZ series representations. A detailed description of the contents of this book will be given in the introduction.

In 1995, the first author was proposed by his supervisor M. Jimbo to write a survey about known important facts about the representation theory of the Virasoro algebra as his PhD thesis, since there confusion in the literature as the theory was developed both in mathematics and in physics at the same time. After the PhD thesis of the first author, which had nothing to do with the proposition, he had occasion to take part in a joint project with the second author. Around 2000, we decided to realise the proposition and finally it took us nearly 10 years.

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