Chapter 2
Overview of Recent Research in Distributed Multi-agent Coordination

This chapter overviews recent research results in distributed multi-agent coordination. Distributed coordination of multiple autonomous agents, including unmanned aerial vehicles (UAVs), unmanned ground vehicles (UGVs), and unmanned underwater vehicles (UUVs), has been a very active research topic in the systems and controls society. The recent research results in distributed multi-agent coordination are roughly categorized as consensus, distributed formation control, distributed optimization, distributed task assignment, distributed estimation and control, and intelligent coordination. A short discussion is given to propose several future research directions and problems that deserve further investigation.

2.1 Introduction

Control theory can be dated back to the beginning of last century when the Wright brothers made their first flight in 1903. Since then, control theory has received more and more attention, especially during the World War II when control theory has been developed and applied to fire-control systems, missile navigation and control, and various electronic devices. Over the past several decades, modern control theory has been developed due to the booming of spacecraft technology and large-scale systems.

During the development of the control theory, control of a single system has relatively matured and many control methodologies have been developed, such as proportional-integral-derivative (PID) control, adaptive control, intelligent control, and robust control. In the past two decades, the control of multiple interconnected systems has drawn more and more attention because many benefits can be obtained when replacing a solo complicated system with several simple systems. Two approaches are commonly used for the control of multiple interconnected systems: a centralized approach and a distributed approach. The centralized approach is based on the assumption that a powerful central station is available to control a group of systems. Essentially, the centralized approach is a direct extension of the traditional

W. Ren, Y. Cao, *Distributed Coordination of Multi-agent Networks*,
Communications and Control Engineering,
DOI 10.1007/978-0-85729-169-1_2,

single-system based control methodology. Instead, the distributed approach does not require the existence of a central station with a tradeoff that this approach is far more complex than the centralized one. However, the distributed approach is more promising due to inevitable physical constraints, such as limited communication/sensing range, low bandwidth, and large number of systems involved.

Recently, the control of a group of autonomous agents including UAVs, UGVs, and UUVs, has been investigated intensively from different perspectives. The main control objective is to have the agents work together in a cooperative fashion. Here *cooperative* refers to the close relationship among all agents in the team with *information sharing* playing an important role. Distributed coordination of multiple autonomous agents has become an active research topic because many advantages can be achieved accordingly, such as robustness, adaptivity, flexibility, and scalability.

The study of distributed control of multiple autonomous agents was motivated by the work in distributed computing [183], management science [70, 302], and physics [295]. In controls society, the pioneer work was given in [292, 293] where an asynchronous agreement problem was studied for distributed decision making problems. In what follows, [90, 132, 200, 214, 247] studied consensus algorithms under various information flow constraints. Several recent special issues on distributed coordination from 2004 to 2009 include IEEE Transactions on Automatic Control (Vol. 49, No. 9, 2004), IEEE Transactions on Control Systems Technology (Vol. 15, No. 4, 2007), Proceedings of the IEEE (Vol. 94, No. 4, 2007), ASME Journal of Dynamic Systems, Measurement, and Control (Vol. 129, No. 5, 2007), International Journal of Robust and Nonlinear Control (Vol. 17, No. 10–11, 2007), International Journal of Adaptive Control and Signal Processing (Vol. 21, No. 2–3, 2007), IET Control Theory and Applications (Vol. 1, No. 2, 2007), and SIAM Journal on Control and Optimization (Vol. 48, No. 1, 2009).

In this chapter, we overview recent research results in distributed multi-agent coordination from 2006 to 2009.[1] For research results before 2006, the readers are referred to [169, 207, 215, 250]. We roughly categorize the recent research results based on the following directions:[2]

1. Consensus/agreement/synchronization/rendezvous. In this direction, various problems have been investigated towards driving a group of agents to some common state. In many cases, the four words can be used without discrimination.
2. Distributed formation control.[3] Distributed *formation control* refers to the behavior that the agents form a certain geometrical configuration through local interaction with/without a group reference.

[1] Here we primarily focus on the results that appeared in major control/robotics journals although many results might have appeared in other fields or in conferences.

[2] Note that the classification is by no means complete, and the overview will by no means cover all recent research results.

[3] In fact, consensus can be considered a special case of formation control. We have explicitly overviewed consensus because consensus, a fairly basic problem in distributed multi-agent coordination, has received significant research attention, and therefore deserves special attention in the overview.

3. Distributed optimization. As is known to us, optimization always plays an important role in both theoretical study and practical applications. Significant effort has been put into this research topic upon the birth of control theory. Optimization in distributed multi-agent coordination has been studied under both individual and global objectives.
4. Distributed estimation and control. In order to tackle distributed coordination problems, it is, sometimes, assumed that some global information is available to each individual agent. This assumption disobeys the virtue of distributed multi-agent coordination. As an alternative, distributed estimation and control methodologies have been proposed in which some unknown global information can be estimated locally.
5. Distributed task assignment. An interesting problem involved in sensor/robotic networks is to achieve task assignment in a distributed fashion. Examples include task/resource allocation, coverage control, and scheduling.
6. Intelligent coordination. The term *intelligent coordination* refers to the coordinated behavior of a group of agents with intelligence. In this problem, research has been conducted towards introducing intelligent mechanisms into traditional coordination problems or investigating the behavior of a group of intelligent agents from the perspective of coordination.

2.2 Consensus

Consider a group of n agents with single-integrator dynamics given by

$$\dot{r}_i(t) = u_i(t), \quad i = 1, \dots, n \tag{2.1}$$

where $r_i(t) \in \mathbb{R}$ and $u_i(t) \in \mathbb{R}$ are, respectively, the state and the control input associated with the ith agent. Here for simplicity of presentation we have assumed that all agents are in a one-dimensional space. However, all results hereafter are still valid for the high-dimensional space by introduction of the Kronecker product. A common consensus algorithm for (2.1) is given by

$$u_i(t) = \sum_{j=1}^{n} a_{ij} \big[r_j(t) - r_i(t) \big], \tag{2.2}$$

where a_{ij} is the (i, j)th entry of the adjacency matrix $\mathcal{A}$ associated with the graph $\mathcal{G}$ characterizing the interaction among the n agents. The objective of (2.2) is to *reach* or *achieve* consensus, i.e., for all $r_i(0)$ and all $i, j = 1, \dots, n$, $|r_i(t) - r_j(t)| \to 0$ as $t \to \infty$. The main idea behind (2.2) is that each agent's state is driven towards the states of its neighbors (see Lemmas 1.3–1.5 for some convergence results on consensus). In the following, we will overview the recent research results in consensus according to different research problems.

2.2.1 Delay Effect

It can be observed that the consensus algorithm (2.2) assumes that each agent can obtain the states of its neighbors without time delay. This assumption poses an obvious limitation because time delay appears in every practical system and, therefore, deserves consideration in the consensus problem. In particular, two types of time delays, i.e., *communication delay* and *input delay*, have been considered in the existing literature. When there exists communication delay, (2.2) becomes

$$u_i(t) = \sum_{j=1}^{n} a_{ij} \left[r_j(t - T_{ij}) - r_i(t) \right], \tag{2.3}$$

where T_{ij} represents the communication delay from the jth agent to the ith agent. When there exists input delay, (2.2) becomes

$$u_i(t) = \sum_{j=1}^{n} a_{ij} \left[r_j(t - T_p) - r_i(t - T_p) \right], \tag{2.4}$$

where T_p represents the input delay. It is worth mentioning that the communication and input delays might be time-varying and there might exist both communication and input delays. In addition to time delay, it is also important to consider packet dropouts when the agents exchange information. Fortunately, consensus with packet dropouts can be considered a special case of consensus with time delay because old information needs to be used in the presence of packet dropouts. The main problem involved in consensus with time delay is to study the effect of time delay in terms of whether consensus can be reached ultimately, also called *consensusability* [186].

In order to study the delay effect on consensus, the authors in [214] present conditions on the maximum allowed time delay without damaging consensus in a continuous-time setting. In particular, it is shown that the maximum allowed time delay is bounded by a threshold determined by the out-degree of the interaction graph. The authors in [309] study the discrete-time case and present necessary and/or sufficient conditions under both fixed and switching interaction graphs. Further studies are given in [310], which shows that bounded communication delay will not affect the consensusability. Different from the analysis in [214, 309, 310] where matrix theory and the properties of row-stochastic matrices are frequently used, the authors in [290] study the effect of both the communication delay and the input delay on the consensusability based on the frequency-domain analysis. It is shown that the communication delay does not affect the consensusability while the input delay does. In a similar manner, consensus with time delay is studied for systems with different dynamics, i.e., (2.1) replaced with other complex system dynamics, under different scenarios [23, 41, 57, 61, 173, 178, 195, 285, 291, 298, 300, 318], including average consensus where a group of agents reaches the average of their initial states [23, 285], consensus over complex networks [41, 173, 300], and robust consensus [178, 291]. The main tools in the stability analysis include Lyapunov functions [41, 173], passivity [318], and contraction [298].

2.2.2 Convergence Speed

Convergence speed is another interesting topic in the study of the consensus problem. Convergence speed is used to characterize how fast consensus is reached. Using (2.2) for (2.1), if the graph $\mathcal{G}$ is undirected, the worst-case convergence speed is determined by [214] as

$$\min_{r \neq \mathbf{0}_n \ \& \ \mathbf{1}_n^T r = 0} \frac{r^T \mathcal{L} r}{\|r\|^2} = \lambda_2(\mathcal{L}), \tag{2.5}$$

where $r \triangleq [r_1, \ldots, r_n]^T$, $\mathcal{L}$ is the Laplacian matrix, and $\lambda_2(\mathcal{L})$ represents the second smallest eigenvalue of $\mathcal{L}$. In order to increase the convergence speed, the authors in [148] propose an iterative algorithm to maximize the second smallest eigenvalue of a state-dependent Laplacian matrix by employing a semidefinite programming solver. In addition to the second smallest eigenvalue of the Laplacian matrix, a commonly used definition of the convergence speed is given by [216, 308]

$$\rho \triangleq \lim_{t \to \infty \ \& \ r(t) \neq r^\star} \left[\frac{\|r(t) - r^\star\|}{\|r(0) - r^\star\|} \right]^{\frac{1}{t}}, \tag{2.6}$$

where $r^\star \in \mathbb{R}^n$ represents the final consensus equilibrium, which is given by $\sigma \mathbf{1}_n$, where σ is a constant real number. To achieve the fastest convergence speed, the corresponding optimization problem becomes $\max_{u_i(t)} \rho$. In [308], the authors cast the problem of finding the fastest convergence speed into a semidefinite programming problem. In [150], the authors study the problem of reaching the fast average consensus, i.e., $r^\star = \frac{r^T(0)\mathbf{1}_n}{n} \mathbf{1}_n$ in (2.6). In particular, the authors proposed two numerical solutions: the qth-order spectral norm minimization and gradient sampling. The convergence speed defined in (2.6) is studied in both the deterministic and stochastic settings. In the deterministic setting, [5, 6, 216] study the convergence speed and present the estimate or the lower bound of the convergence speed. On the other hand, [3, 115, 331] study the convergence speed in a stochastic setting. In particular, the authors in [331] study the per-step convergence factor, which can be considered the measurement of the convergence speed.

2.2.3 Stochastic Setting

Existing research on the consensus problem is mainly conducted under the assumption that the interaction graph is deterministic. However, due to the existence of communication failures, packet dropouts, and unstable communication channels, it is of great importance to study the consensus problem in a stochastic setting where the interaction graph evolves according to some random distributions, for example, binomial distribution.

In the deterministic setting, consensus is reached if all agents ultimately reach an agreement on some common state. In the stochastic setting, consensus is reached almost surely (respectively, in the mean square sense or with probability one) if all agents reach an agreement on some common state almost surely (respectively, in the mean square sense or with probability one). Consensus over a stochastic network is first studied in [115]. Sufficient conditions on the interaction graph is given to guarantee consensus with probability one and the rate of convergence is also studied. The authors in [3, 128, 185, 232, 286, 305, 328] continue the study of the consensus problem over a stochastic network in different settings. In particular, more general results on consensus in the stochastic setting are given in [3, 232, 286, 305]. The authors in [286] present necessary and sufficient conditions to guarantee consensus almost surely. Note that the condition in [286] is analogous to that in [200, 247] with the exception that the conditions and results are in the stochastic setting. Note that the properties of the row-stochastic matrices play a crucial role in the convergence analysis.

2.2.4 Complex Systems

In addition to the study of the consensus problem for systems with simple dynamics, for example, single-integrator dynamics, double-integrator dynamics, or general linear systems [172, 294], consensus for complex systems is also an interesting topic and has received significant research attention. Here we use the term *consensus for complex systems* to refer to the study of the consensus problem when the system dynamics are nonlinear [15, 55, 60, 64, 66, 71, 76, 77, 123, 177, 208, 258, 267, 277, 320, 321, 329, 330, 333] or the consensus algorithm itself is nonlinear [67, 129, 130]. The main system dynamics studied in the consensus problem include oscillators [60], complex networks [321, 329], nonholonomic mobile robots [76], passive systems [333], and rigid bodies [15, 64, 208, 239, 258]. Similar to the consensus algorithms proposed for systems with simple dynamics, the consensus algorithms proposed in these papers are also based on the state differences with an exception that some additional terms are required to ensure consensus. Note that although the objective is also to guarantee the agreement on the final states, the problem is more complicated due to the nonlinearity of the closed-loop systems. In addition, the properties of row-stochastic matrices might not be applied to the convergence analysis. The main control techniques and approaches used in the stability analysis include adaptive control [329], pinning control [55], dissipativity theory [277], nonsmooth analysis [66, 76, 129], and Lyapunov functions [15, 60, 64, 76, 208, 258].

2.2.5 Quantization

Consensus under quantization has been studied recently with the motivation from digital signal processing. Here *quantized consensus* refers to consensus when the measurements are digital rather than analog. Therefore, the information received by each agent is digital. In [143], a quantized gossip algorithm is proposed and the convergence analysis is studied. In particular, the bound of the convergence time for a fully connected undirected graph is shown to be polynomial in the number of agents. In [48], the authors introduce coding/decoding strategies in quantized consensus and show that the convergence rate depends on the accuracy of the quantization but not the coding/decoding strategies. In [165, 166], quantized consensus is studied via gossip algorithms under an undirected connected interaction graph. In addition, both the lower and upper bounds of the convergence time are investigated.

2.2.6 Sampled-data Setting

Consensus in a sampled-data setting, here called *sampled-data consensus*, has been investigated recently with the motivation from the fact that the system dynamics are normally continuous while the measurements and control inputs might only be made in a discrete-time setting. Sampled-data consensus is mainly investigated in [33, 36, 99, 101, 116, 312, 313]. Consensus for single-integrator dynamics is studied in a sampled-data setting under both fixed and switching interaction graphs in [312, 313] where necessary and/or sufficient conditions are presented to guarantee consensus. Consensus for double-integrator dynamics is studied in a sampled-data setting under both fixed and switching interaction graphs in [33, 36, 99, 101, 116]. Various approaches, including Lyapunov theory [116], matrix theory [33], infinite product of row-stochastic matrices [36], and linear matrix inequalities [99, 101], have been used to determine necessary and/or sufficient conditions to guarantee consensus.

2.2.7 Finite-time Convergence

Reaching consensus in a finite time, here called *finite-time consensus*, has been studied recently. For a group of n agents with dynamics given in (2.1), the objective is to design $u_i(t)$ such that $r_i(t) = r_j(t)$ for $t \geq \overline{T}$, where $\overline{T}$ is a constant. Here $\overline{T}$ is also called the consensus time. Finite-time consensus for single-integrator dynamics in a continuous-time setting is solved in [66, 130, 138, 311]. Finite-time consensus for double-integrator dynamics in a continuous-time setting is studied in [297]. It is well known that linear consensus algorithms normally guarantee exponential or asymptotical convergence but not finite-time convergence. Hence, an important characteristic in the proposed finite-time consensus algorithms is the introduction of the signum function.

2.2.8 Asynchronous Effect

In most existing research of the consensus problem, it is assumed that all agents update their states synchronously. Note that the synchronized update requires a synchronized clock for a group of agents. However, the synchronized clock might not exist in real applications. This motivates the study of consensus algorithms with asynchronous updates. That is, each agent updates its state disregard of the update times of the other agents. In [310], consensus for single-integrator dynamics is studied with asynchronous updates and time delays by using the properties of row-stochastic matrices. The authors in [43] solve asynchronous consensus for single-integrator dynamics using matrix theory and graph theory. On the other hand, paracontracting theory is employed in [89] to solve asynchronous consensus for single-integrator dynamics.

2.3 Distributed Formation Control

Formation control has been a very interesting research topic in the controls society where a certain geometric pattern is formed with/without a group reference. The group reference, sometimes also called a leader or a virtual leader, represents the objective of interest for the whole group. Formation control without a group reference, here called *formation producing*, refers to the behavior that a group of agents achieves some geometric pattern in the absence of any group reference. Formation control with a group reference, here called *formation tracking*, refers to the behavior that a group of agents achieves a desired geometric formation and follows the group reference. In the following, we will overview recent research results in formation control, including formation producing, formation tracking, connectivity maintenance, and controllability, in the context of distributed multi-agent coordination with local interaction.

2.3.1 Formation Producing

We overview the existing literature based on different approaches used in the stability analysis.

2.3.1.1 Matrix Theory Approach

Due to the nature of multi-agent systems, matrix theory has been used frequently in the stability analysis of formation producing. In [226], the authors propose a cyclic-pursuit-based strategy and show that different behaviors for a group of agents, i.e., converging to a single point, a circle, or a logarithmic spiral pattern, can be achieved by changing a common offset angle. The stability analysis relies on characterizing the eigenvalues of a circulant matrix in the closed-loop system. Motivated by [226], Cartesian coordinate coupling is introduced to consensus algorithms in [244] to achieve three different collective motions: rendezvous, move on circular orbits, or follow logarithmic spiral curves. In [187], the collective motion for nonholonomic robots is studied for a cyclic pursuit model. In [153], the authors use complex polynomials to represent the space of permutation-invariant multi-robot formations, where the roots of the complex polynomials correspond to the configurations of the robots in the formation. In [273], cooperative multi-agent formation is studied based on parallel estimation-based decentralized control. In addition, necessary conditions on the interaction graph are presented to guarantee the stability of simultaneous parallel estimation and control.

2.3.1.2 Lyapunov-based Approach

Another important approach used in formation producing is the Lyapunov-based approach, where the system stability can be proved by finding a proper Lyapunov function. In [77], the formation feasibility and velocity alignment problem is investigated. In [78], the inverse agreement problem is studied, where the team members are forced to disperse in the workspace. In particular, the minimum distance between every pair of agents is larger than a specific lower bound. In [223], the circular collective motion on a sphere is studied under both fixed and switching interaction graphs. In [69, 170, 202, 289], *flocking* of a group of agents is investigated in different cases under fixed and switching interaction graphs, where a group of agents moves cohesively and the inter-agent collision is avoided. In [97], the Queue-formation structure is investigated in the formation producing problem, where the communication load can be reduced by dividing the information flow into two subgroups: the fast time scale and the slow time scale. In [82], the author studies formation producing with bounded control. In [327], the authors propose control laws to steer particles to an invariant pattern corresponding to a constant orbit value characterizing the curve of the trajectory and constant separations.

2.3.1.3 Graph Rigidity Approach

Graph rigidity has been an important approach in formation producing. For a given number of agents, the edges in the interaction graph are closely related to the shape of the formation. Therefore, distributed controllers can be designed to guarantee desired edge distances. In [213], graph rigidity is used to achieve formation producing for a group of agents under an undirected interaction graph. In [117], the authors study the construction and transformation of two-dimensional persistent graphs, where persistence is a generalization of rigidity in directed graphs. Through primitive operations, the minimally persistent formation can be obtained from any other one while minimal persistence is preserved throughout the reorganization process. Further study on formation producing using graph rigidity and persistence can be found in [157, 319] where a nonlinear control law [319] and a gradient-based control law [157] are designed such that a rigid formation can be obtained.

2.3.1.4 Receding Horizon Approach

Receding horizon control (RHC), also called *model predictive control* (MPC), has been introduced in the formation producing problem. RHC is essentially a finite-horizon optimization problem. In [86, 87], the authors investigate distributed formation producing via distributed RHC. In [96], a distributed RHC approach is used to solve formation producing in the presence of time delay.

2.3.2 Formation Tracking

Although formation control without a group reference is interesting, it is sometimes more meaningful to study formation control in the presence of a group reference that represents the objective of interest for the whole group. We also overview the existing literature based on the approaches used in the stability analysis.

2.3.2.1 Matrix Theory Approach

In [45, 240], a special case of formation tracking for single-integrator dynamics in the presence of a time-varying group reference is studied in both continuous-time and discrete-time settings. In [233], formation tracking is solved through a two-level consensus approach where agents reach an agreement on the virtual leader's state at one level and are guaranteed to converge to the desired formation about the virtual leader at the other level. In [149], target-capturing formation control based on a cyclic pursuit strategy is proposed and studied for nonholonomic mobile robots. In particular, collision avoidance is shown to be achieved as well. In [236], a general framework is presented to design cooperative control strategies for a group of dynamical systems by studying the properties of the augmentation of reducible and irreducible nonnegative matrices. In addition, the approach can be applied to multiple heterogeneous systems. In [255], formation control with a constant final velocity is studied through ring coupling. In [307], the authors study synchronization of a group of agents on some desired signal that has the same dynamics as the agents and is available to only a portion of the agents.

2.3.2.2 Potential Function Approach

Potential functions have been used frequently in the formation tracking problem, where a controller is designed based on the gradient of the corresponding potential function. By properly choosing the potential function, the desired group behavior can be guaranteed. Motivated by the results in [212], the authors in [280] extend the flocking study in [212] to the case when there exists a group reference. That is, a group of agents moves cohesively with the group reference and inter-agent collision is avoided. In particular, the state information of the group reference is assumed to be available to all agents. In [268], flocking is studied under the assumption that the group reference's acceleration is known to each agent. In [83], formation control of a group of nonholonomic mobile robots is solved by using a bump function and a potential function. In addition, collision avoidance mechanism is introduced without requiring switching control even if the robots have limited sensing ranges.

2.3.2.3 Lyapunov-based Approach

In [299], the authors study conditions for distributed tracking in dynamic networks in the presence of different types of leaders, which has potential applications in biology, e.g., in evolutionary processes and disease propagation. In [84, 85], formation control of multiple nonholonomic mobile robots is solved by model transformation with/without uncertainties. In [104], coordinated path following of a group of agents is studied in the presence of communication losses and time delays. In particular, the authors derive conditions such that the path following errors are driven to a small neighborhood of zero. In [218], three nonlinear leader–follower formation control algorithms based on, respectively, full state feedback, robust state feedback, and output feedback, are proposed to solve the formation control problem for a group of nonholonomic mobile robots. In [180], synchronization of a group of spacecraft on elliptical orbits is solved by using a nonlinear adaptive controller. In [51], a distributed control law is designed for nonhonolomic mobile robots to achieve a circular motion around a stationary or moving beacon. In [224], a distributed coordination algorithm is proposed to guarantee convergence of agents to a set of trajectories that moves along closed curves.

2.3.2.4 Other Approaches

In addition to the aforementioned approaches, there are also some other approaches used to achieve formation tracking. Formation producing and formation tracking are studied in [91] via partial differential equations. In [113], leader-following formation control is solved without the measurement of the leader's velocity. In particular, an observer is designed to estimate the leader's velocity. In [75], formation tracking of nonholonomic mobile robots is solved by using neural networks. The control law is designed by using a backstepping technique and is based on the integration of the signum function. Collision avoidance is considered as well. In [14], formation tracking is solved when the constant velocity of the leader is available to a portion of the followers by using an adaptive control design.

2.3.3 Connectivity Maintenance

In both consensus and formation control problems, it is often assumed that the interaction graph satisfies certain conditions. For example, the interaction graph is connected or has a directed spanning tree. Note that a communication model is often distance-based. That is, two agents can communicate with each other only if their distance is smaller than a certain threshold. In order to guarantee that consensus or formation control can be achieved ultimately, connectivity maintenance mechanism has also been studied recently. The connectivity maintenance mechanism is mainly studied in [98, 133, 269, 279, 282, 324, 325]. The main approach used is to define artificial potentials in a proper way such that if two agents are neighbors at a certain time instant, they will always be neighbors afterwards. In [133], consensus with connectivity maintenance is solved when the weights for the edges of the interaction graph are defined properly. In [98], rendezvous of a group of agents with connectivity maintenance is solved based on a perimeter minimizing algorithm. In [282], a controller based on a properly designed potential function is proposed to solve rendezvous of a group of nonholonomic robots with connectivity maintenance. In [279, 324, 326], connectivity maintenance for flocking of a group of agents is studied based on properly designed potential functions.

2.3.4 Controllability

Controllability in distributed multi-agent coordination has been an interesting research topic recently. A multi-agent system is *controllable* if each agent in the system can be steered to a certain position by controlling one agent in the system, which is also called the *leader*. In [288], the author studies the controllability of multi-agent systems in the present of a leader. Necessary and sufficient conditions are presented based on the eigenvalues of a submatrix of the Laplacian matrix. Interestingly, it is further shown that increasing the algebraic connectivity does not necessarily increase the controllability. Further results on controllability of multi-agent systems are presented from a graph-theoretical perspective. In [134, 135], necessary conditions on the controllability are presented. In particular, equitable partitions are introduced in [134] to improve the controllability results presented in [135]. In [237], the authors investigate the relationship between the network symmetry structure and the controllability. Note that [134, 135, 237, 288] focus on the fixed interaction graph case. Different from [134, 135, 237], the authors in [137, 181] study the controllability of multi-agent systems under a switching interaction graph. In particular, the authors in [137] take time delays into account and derive sufficient conditions for controllability.

2.4 Distributed Optimization

Optimization is an important issue in the systems and controls society. The main objective of optimization is to find the optimal strategy under some given cost function. Optimization in distributed multi-agent coordination has been studied recently in two directions, namely, convergence speed and cost functions. One important problem studied in consensus is the convergence speed, which characterizes how fast consensus can be achieved. We refer the readers to Sect. 2.2.2 for the problem. In addition to the fastest convergence speed that is studied as the objective to optimize, various cost functions including both individual cost functions and global cost functions are also studied as the objectives to optimize.

2.4.1 Individual Cost Functions

In this case, the cost function for one agent is defined based on its own and its neighbors' states. In [260, 261], a semi-distributed optimal control problem is studied in the presence of finite-horizon individual cost functions in both leaderless and leader-following cases. In [140], finite-time optimal consensus with input and linear state constraints is solved by using a primal decomposition and subgradient method. In [19], a nonlinear consensus protocol is proposed such that a group of agents can reach an agreement on certain functions of all agents' initial states. Meanwhile, it is shown that the proposed consensus protocol is optimal with respect to certain individual cost functions. In [105], the authors study the coordination problem of a group of robots working under a collision avoidance constraint, where each individual robot strives to optimize its own objective—the elapsed time. The problem is solved based on the notion of *Pareto optimality*.

2.4.2 Global Cost Functions

In this case, the cost function depends on information of the whole group. In [124], the authors study an optimal control problem with free final time and partially constrained final states, which mimics some behaviors in foraging trail optimization. In [188], the authors study optimal sensor placement and motion coordination. The main problem is to maximize a particular class of global cost functions. In [107], mission planning of a group of unhabitated underwater vehicles is solved via a receding horizon mixed-integer constrained quadratic optimal control problem, which is then partitioned into smaller subproblems and solved in a parallel and distributed manner using a distributed Nash-based game approach. In [257], the authors study consensus in terms of the extrema of some global cost function. In particular, consensus and anticonsensus (balance) can be achieved via, respectively, maximizing and minimizing the cost function. In [127], the authors study the optimal coordination problem with formation pattern and collision avoidance constraints by minimizing a global cost function. Through a case study, it is shown that the solution is optimal for sufficiently close starting and final positions. In [26], an optimal distributed control problem is studied via the study of an infinite-horizon *linear-quadratic regulator* (LQR) problem. Then a distributed controller is constructed by analyzing the properties of the local LQR problem. In addition, the relationship among stability, robustness, and the spectrum of a certain matrix is presented as well. In [203], the authors also study an infinite-horizon LQR problem. Different from [26], a special class of operators, called *spatially decaying* operators, is introduced. In [35], the authors study an optimal linear consensus problem from an infinite-horizon LQR perspective. Different from [26, 203], the authors in [35] show that the optimal interaction graph corresponds to a complete directed graph. Different from [26, 35, 203] where an infinite-horizon cost function is used, the authors in [95] propose cooperative control algorithms to minimize a finite-horizon global cost function that includes both the regulation and cooperation objectives. In [142], the authors study a formation controller design so that some desired properties can be optimized. In particular, through the use of a dynamic protocol, formations of real robots are shown to move significantly faster and with greater precision. In [74], minimization of the total travel distance or the minimax distance that the agents must travel is solved using convex optimization.

2.5 Distributed Task Assignment

Distributed task assignment refers to the study of task assignment of a group of agents in a distributed manner, which can be roughly categorized as coverage control, scheduling, and surveillance.

2.5.1 Coverage Control

Recently, *coverage control* has been an active research direction in mobile sensor networks. The main objective is to properly assign the mobile sensors' motion in order to maximize the detection probability. Let Q be a convex space with ϕ representing the distribution density function, which denotes the probability that some event takes place over Q [68]. Let there exist a group of n mobile sensors whose locations are given by $P \triangleq [p_1, \ldots, p_n]$, where p_i denotes the location of sensor i. Note that the sensor performance at a point q degrades with respect to the distance $\|q - p_i\|$. Then use a nondecreasing differentiable function f can be used to describe the sensor performance. The coverage control problem is essentially to find a local controller for each mobile sensor such that the cost function

$$J \triangleq \sum_{i=1}^{n} \int f\big(\|q - p_i\|\big)\phi(q)\,\mathrm{d}q$$

is minimized.[4] A complete distributed, scalable coverage control strategy is derived in [68]. In addition, the closed-loop system is adaptive and asynchronous. Several further results about coverage control have also been presented recently. In [131], precise coverage control with collision avoidance is studied under fully and partially connected interaction graphs. In [100], the connection between coverage control and consensus under a cyclic interaction graph is studied. Both the coverage control problem and the average consensus problem can be considered a special class of the distributed optimization problems. In [167], the coverage control of a network of robotic agents with limited-range communication and anisotropic sensing capabilities is studied. By approximating the expected-value objective function, a gradient-based distributed coverage control algorithm is developed. Different from [68, 100, 131, 167], the authors in [204] study the optimal sensor placement problem via minimizing the trace of a weighted covariance matrix. In particular, the optimization problem can be converted to a convex optimization problem.

2.5.2 Scheduling

Distributed scheduling refers to the scheduling of a group of agents in a distributed manner. In [139], the authors study the optimal scheduling sequence to fuel a group of UAVs *via* dynamic programming. In [94], a coordination strategy based on task-load balancing is proposed under a fixed interaction graph. In [197], the distributed adaptive scheduling is solved by choosing the task timings as the consensus variable. In [22], the authors solve task assignment for flocking by using a metric routing algorithm. In [9], the authors study the efficient routing problem when a group of autonomous vehicles must visit multiple targets generated by a random process. Control strategies are presented to minimize the expected time between the time when a target appears and the time when an agent visits the target. Further results are also presented to understand the effect of the inter-agent communication and the knowledge of the stochastic process.

[4] Note that coverage control can be treated as an optimization problem.

2.5.3 Surveillance

Distributed surveillance means the monitoring of a certain area by using a group of agents coordinated in a distributed fashion. In [151], a perimeter surveillance problem is studied and experimental results on multiple UAVs are presented to show the effectiveness. In [198], the authors propose a distributed cooperative control algorithm to drive a group of autonomous vehicles to patrol some area that exceeds the communication and sensing capabilities of the vehicles. In addition, a proper distribution of the vehicles is achieved within a finite time. In [106], a cooperative surveillance problem for a group of UAVs is studied in the presence of unstable communications, time delays, uncertainty in target locations, and imperfect vehicle search sensors. Different from the problems studied in [106, 151, 198], the authors in [227] study a scenario where a group of robots moves towards their individual targets without collision. In [314], the authors study a cooperative search problem where a group of UAVs are used to find the targets in an unknown environment. In particular, an opportunistic-cooperative-learning based distributed strategy is proposed to solve the problem and the bounds of the search time are presented. In [93], the authors study the distribution of a group of heterogeneous vehicles over a certain space that includes several areas in the presence of uncertainty. Scalable allocation strategies are developed to guarantee a desired vehicle distribution in these areas. In [226], different behaviors for a group of vehicles, namely, converging to a single point, a circle, or a logarithmic spiral pattern, are shown to be achieved for a cyclic pursuit model by changing a common offset angle. In addition, by changing the common offset angle based on the locally available information, the paths of the vehicles can be guaranteed to cover a certain area. In [287], the authors study a cooperative sensor placement problem in which a group of mobile sensors is deployed to monitor multiple stationary targets. The cost function used in [287] is nonlinear and nonsmooth.

2.6 Distributed Estimation and Control

Due to the absence of global information that can be used to achieve group coordination, distributed estimation and control has received significant attention recently. Under the distributed estimation and control framework, the first problem is to design distributed local estimators such that some global information can be estimated in finite/infinite time. The second problem is to design distributed local controllers based on the local estimator such that the closed-loop system is stable. It is worthwhile to emphasize that the closed-loop system with both distributed estimators and controllers is much more complicated than that with only distributed controllers.

In [315], the authors present a unified framework of distributed estimation and control to solve a distributed coordination problem. Both proportional-like and proportional-and-integral-like distributed estimation algorithms are proposed and analyzed. In [184], the unified distributed estimation and control framework in [315] is applied to environmental modeling and experimental results are presented as a proof of concept. In [184, 315], it is assumed that no noisy signal exists in the measurements. In [278], the authors study overlapping distributed estimation by using a consensus-like approach in the presence of white noise. The proposed approach is based on a synergy between local *Kalman filters* and a dynamic consensus strategy for the agents. In [28], the authors consider the distributed estimation problem, where each sensor has some noisy linear measurement of some unknown parameter. By using a consensus-like diffusion scheme, the local estimate of each node will finally converge to the true parameter. In [205], the authors study the accuracy of position estimation for groups of mobile robots performing cooperative localization in the presence of white noise. In [332], the authors study cooperative tracking of a type of nonlinear robots. In particular, cooperative sensors are used to estimate the relative posture. In [234], radar position estimation and configuration optimization are studied via the minimization of position errors. In addition to the state estimation in the aforementioned papers, the authors in [73] investigate the estimation of a spatially distributed process via the minimization of expected state estimation errors.

2.7 Intelligent Coordination

In traditional coordination problems, it is often assumed that each agent responds to local information. This assumption is simple and, therefore, the complexity of the closed-loop system is low. Recently, distributed coordination in the presence of intelligence, referred to as *intelligent coordination*, has been studied from different perspectives, especially from economy, social science, and management science. The main feature in intelligent coordination is that each agent is intelligent, and therefore chooses the best possible response based on its own objective. We overview existing results in two aspects: pursuer-invader problem and game theory.

2.7.1 Pursuer–invader Problem

In the *pursuer–invader problem*, there exist a group of pursuers and one invader. The objective of the pursuers is to find and track the invader while the objective of the invader is to escape the pursuers. In [40], the authors study the pursuer–invader problem and presented a five-phase controller to solve this problem. In [25], the authors study the pursuer-invader problem for Dubins-like vehicles when the velocity of the invader is bounded. Similar to [40], the authors propose a five-phase controller to solve the problem. The discrete-time case of the pursuit-evasion problem in [25] is studied in [24].

2.7.2 Game Theory

Recently, *game theory* is also introduced to distributed multi-agent coordination. In [112], formation control is studied via a *linear-quadratic (LQ) Nash differential game* and a RHC-based approach is used. In [296], the authors study the problem of *learning Markov games* using learning automata. In [88], the authors propose and study multi-agent systems with symbiotic learning and evolution (Masbiole) based on symbiosis in the ecosystem. It is further shown that Masbiole can escape from the *Nash equilibria*. In [301], the authors study the role of cooperation in a coupling game. By adding cooperation, it is shown that benefits can be increased. In [21], the authors study consensus with unknown but bounded disturbances. Due to the existence of unknown but bounded disturbances, the local controller under a traditional consensus protocol is bounded. The authors propose a lazy rule, where each agent chooses the minimal control input based on the traditional consensus algorithm. In [20], a consensus-like protocol is derived in noncooperative games. Under the proposed protocol, it is shown that the players converge to the unique *Pareto optimal Nash equilibrium*.

2.8 Discussion

We have reviewed the recent research in distributed multi-agent coordination. The main objective of this overview is to briefly summarize the state-of-the-art in distributed multi-agent coordination. In addition to the aforementioned theoretical results, many experiments are also conducted to validate the theoretical results, for example, [7, 16, 118, 159, 211, 251]. Although the theoretical study and experimental validation have solved many problems in distributed multi-agent coordination, there are still a number of research problems that deserve further investigation. We summarize these problems as follows:

- Quantization effect in distributed coordination problems. Most existing research focuses on the study of distributed coordination problems by assuming that both control inputs and measurements are continuous analog values. However, the use of digital signal processing technique requires digital inputs and measurements. Therefore, it is important and meaningful to investigate the quantization effect in distributed coordination problems. Note that although the quantization effect has been studied in some coordination problems, the quantization effect still deserves further consideration in many other distributed coordination problems.
- Optimization with both individual and global cost functions. The optimization problem in distributed multi-agent coordination has been studied in the presence of either an individual or a global cost function. In real systems, each individual agent has both local and global objectives with corresponding individual and global cost functions. Therefore, optimization of the combined objectives is more realistic and meaningful. Another interesting problem is to investigate the rela-

tionship between the individual cost function and the global cost function. One interesting problem is how to balance the individual cost function and the global cost function.

- Intelligent coordination. Intelligent coordination has potential applications in not only engineering but also in economics, social science, etc. Although several research problems have been studied recently, there are still many open questions, especially the understanding of group behavior in the presence of intelligence. One interesting problem is how we can interpret complex networks and stabilize the complex networks in the presence of intelligence.
- Competition and cooperation. Right now, most research is conducted based on local cooperation but not competition. This poses an obvious limitation because competition also plays an important role in group coordination in reality. For example, due to the lack of competition, the final consensus equilibrium using the traditional consensus algorithms is limited to a weighted average of the initial states. One interesting question is how to introduce competition to distributed coordination to represent more realistic scenarios.
- Centralization and decentralization. Note that decentralization shows obvious benefits over centralization, such as scalability and robustness. However, decentralization also has its own drawbacks. One drawback is that each agent cannot effectively predict the group behavior based on only local information. Accordingly, the group behavior cannot be controlled in some sense. As an interesting example of this drawback, economic crisis can be used to illustrate the disadvantages of decentralization. One interesting question is how we can balance decentralization and centralization to improve the system performance.

2.9 Notes

For further literature review on distributed multi-agent coordination and related problems, see [8, 17, 27, 49, 53, 58, 63, 84, 102, 132, 169, 193, 207, 215, 222, 235, 248, 250, 265, 270, 274, 306, 315] and references therein.

http://www.springer.com/978-0-85729-168-4

Distributed Coordination of Multi-agent Networks
Emergent Problems, Models, and Issues
Ren, W.; Cao, Y.
2011, XVIII, 310 p., Hardcover
ISBN: 978-0-85729-168-4