Information-Based Minimal Repair Models

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1 Introduction

Using the minimal repair concept, it is possible to describe in a simple way the fact that many repairs in real life bring the system to a condition which is basically the same as it was just before the failure occurred. Such a repair may be used to model a system where a component of the system is replaced or repaired. Of course, the purpose of the repair action is not to bring the system to the exact same condition. Rather the purpose is to bring the system back to operation as soon as possible. But by looking at the condition of the system after the repair, it is a reasonable assumption to say that the system state has not changed.

To formalize this idea, assume that the system is installed at time $t = 0$ and is subject to failure at a random time T_1. The system has a failure rate (intensity) at time t given by the function $\lambda(t)$, meaning that the probability that the system fails during the interval $(t, t + h)$ is $\lambda(t)h$ if the system has survived until time t. Here h is a small number. Suppose the repair is minimal in the sense that its state or condition is as good as it was immediately before the failure occurred. The minimal repair means that the age of the system is not disturbed by the failures. Consequently, the failure rate at time t is $\lambda(t)$ independent of the number of failures occurred up to time t. We ignore the duration of the repairs. If N_t represents the number of failures in the time period $[0, t]$, it follows that N_t is a non-homogenous Poisson process with intensity function $\lambda(t)$.

This is the basic minimal repair model presented in 1960 by Barlow and Hunter [8]. This model has been extended in many ways since then, see e.g. Aven [1, 2], Aven and Jensen [6, 7], Phelps [16], Bergman [11], Block et al. [12], Stadje and Zuckerman [21], Shaked and Shanthikumar [20], Beichelt [11], Zhang and

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Jardine [23], Finkelstein [14] and Zequeira and Bérenguer [22]. A Bayesian approach is presented and discussed in Mazzuchi and Soyer [15].

Many special cases of the basic minimal repair model have been addressed in the literature. Two of the most frequently studied special cases are:

$$\lambda(t) = \lambda\beta(\lambda t)^{\beta-1} \quad \text{(Power law)}$$
$$\lambda(t) = \lambda e^{\beta t} \quad \text{(Log linear model)}.$$

In this article we will focus on extensions of the basic model, and we will give special attention to the general minimal repair models presented and discussed by Aven [1, 2] and Aven and Jensen [6]. The present paper is partly based on Aven [4], Aven [3] and Sandve and Aven [18].

The minimal repair models are used to describe and predict the performance of repairable systems. A number of applications relate to optimal maintenance and replacement policies, see the above cited references. Two applications are presented in Sect. 4.

2 A Doubly Stochastic Poisson Process Approach

In this section we study the concept of minimal repair as it was introduced by Aven [1] in an optimal replacement analysis context.

Let $X_t, t \geq 0$, be an observable stochastic process, possibly a vector process, representing the condition of the system at time t. We define N_t as the number of failures in $[0, t]$. The failure intensity process, which is denoted λ_t, may depend on $X_s, 0 \leq s \leq t$. Often we can formulate the relation in the following way:

$$\lambda_t = v(X_t),$$

where $v(x)$ is a positive deterministic function. The interpretation of λ_t is that given the history of the system up to time t, the probability that the system shall fail in the interval $(t, t + h)$ is approximately $\lambda_t h$. In other words, λ_t is the expected number of failures per unit of time, given the information up to time t.

If the failure intensity process depends only on the state process X_t and not on the failure process N_t, we can interpret the repairs as minimal: a repair which changes neither the age of the system nor the information about the condition of the system. In this case, the running information about the condition of the system can be thought to be related to a system which is always functioning.

A way of formalizing this is to assume that given the state process X, the failure process follows a non-homogenous Poisson process with intensity $v(X_t)$, i.e. N is a doubly stochastic Poisson process (Cox process) with intensity $v(X_t)$. Hence, if the whole X process is known, the intensity at time t is $v(X_t)$, independent of previous failures. This process models a system which is minimally repaired, as a repair does not change the information about the condition of the system. If we know that

the state is x at time t, the failure intensity is $v(x)$, independent of the history of the failure process.

If $X_t = t$ for all t, we are back to the basic minimal repair model, where N_t is a non-homogenous Poisson process.

Example 1 Shock model. Assume that shocks occur to the system at random times, each shock causing a random amount of damage, and these damages accumulate additively. At a shock, the system fails with a given probability. A system failure can occur only at the occurrence of a shock.

Let V_t denote the number of shocks in $[0, t]$, and let Y_i denote the amount of damage caused by the ith shock. We assume that V_t is a Poisson process with rate v, and that the Y_is are independent and identically distributed random variables with a distribution $H(y)$. Let X_t denote the accumulated damage in $[0, t]$, i.e.

$$X_t = \sum_{i=1}^{V_t} Y_i.$$

Now, if the system is active before time t, and the accumulated damage equals x, and a jump of size y occurs at t, then the probability of failure at this point is $p(x + y)$, where $0 < p(x) < 1$ for all x.

This model is a special case of the general set-up described above. The failure intensity process of the counting process N_t equals

$$v \int_0^\infty p(X_t + y) dH(y).$$

For a formal proof of this result, see [6, 7]. These references also include extensions of this model, by allowing the probability p to depend on the number of failures occurred. However, the repairs are then not minimal repairs.

Example 2 Monotone system. Consider a binary, monotone system ϕ of n independent components. We refer to [7, 9] for the definition of such a system. Let $N_t(i)$ denote the number of failures of component i in $[0, t]$, and N_t the number of system failures in the same interval. The counting process $N_t(i)$ is assumed to have an intensity process $\lambda_t(i)$. Hence the failure process of the system N_t has an intensity λ_t given by

$$\lambda_t = \sum_{i=1}^{n} \lambda_t(i) X_t(i)(1 - \phi(0_i, \mathbf{X}_t))\phi(\mathbf{X}_t), \tag{1}$$

where $\phi(\cdot_i, \mathbf{x}) = \phi(x_1, \ldots, x_{i-1}, ; x_{i+1}, \ldots, x_n)$.

Observe that $X_t(i)(1 - \phi(0_i, \mathbf{X}_t))\phi(\mathbf{X}_t)$ is either 0 or 1, and equals 1 if and only if the system is functioning, component i is functioning and the system fails if component i fails.

Assume now that if the system fails, it is minimally repaired in the following sense: If a component fails and causes system failure, then this component is minimally repaired in the traditional sense. A component which fails without causing system failure is not repaired.

We assume that component i when not being repaired has a lifetime R_i with distribution function $F_i(t)$ and failure rate equal to $r_i(t)$. The n components are assumed to be independent.

It follows that we have a special case of the general set-up with λ_t having the form (1) with $\lambda_t(i) = r_i(t)X_t(i)$. The process $X_t(i), t \geq 0$, is in this case either identical to one, or one up to R_i and then zero. If component i is in series with the rest of the system, then $X_t(i) \equiv 1$.

3 A General Point Process Approach

In this section we extend the analysis of the previous section to a general point process, using the setup of Aven and Jensen [6, 7].

Let $(T_n), n \in \mathbb{N}$, where $\mathbb{N} = \{1, 2, \ldots\}$, be a point process on a basic probability space $(\Omega, \mathscr{F}, P)$ describing the failure times at which instantaneous repairs are carried out, i.e. (T_n) is an increasing sequence of positive random variables which may also take the value $+\infty : 0 < T_1 \leq T_2 \leq \ldots$. The inequality is strict unless $T_n = \infty$. We always assume that $T_\infty = \lim_{n\to\infty} T_n = \infty$. The corresponding counting process $N = (N_t),\ t \in \mathbb{R}_+$,

$$N_t(\omega) = \sum_{n \geq 1} I(T_n(\omega) \leq t),$$

where $I(\cdot)$ denotes the indicator function, represents the number of failures up to time t. Here $\mathbb{R}_+ = [0, \infty)$.

The information up to time t is represented by the pre-t-history (σ-algebra) $\mathscr{F}_t$, which contains all events of $\mathscr{F}$ that can be distinguished up to and including time t. The filtration $\mathbb{F} = (\mathscr{F}_t),\ t \in \mathbb{R}_+$, is assumed to follow the usual conditions of completeness and right continuity. The main assumption now is that the failure counting process is integrable and admits an $\mathbb{F}$-intensity $\lambda = (\lambda_t)$, i.e. a decomposition

$$N_t = \int_0^t \lambda_s ds + M_t, \tag{2}$$

with a mean zero martingale $M = (M_t)$. The question now is which counting processes can be classified as minimal repair processes (MRPs) and which cannot.

Different types of repair processes are characterized by different intensities λ. The repairs are minimal if the intensity λ is not affected by the occurrence of

failures or in other words, if one cannot determine the failure time points from the observation of λ. More formally minimal repairs can be characterized as follows.

Definition 1 Let (T_n), $n \in \mathbb{N}$, be a point process with an integrable counting process N and corresponding $\mathbb{F}$-intensity λ. Suppose that $\mathbb{F}^{\lambda} = (\mathscr{F}_t^{\lambda})$, $t \in \mathbb{R}_+$, is the filtration generated by $\lambda : \mathscr{F}_t^{\lambda} = \sigma(\lambda_s,\ 0 \le s \le t)$. Then the point process (T_n) is called a minimal repair process (MRP) if none of the variables T_n, $n \in \mathbb{N}$, for which $P(T_n < \infty) > 0$, is an $\mathbb{F}^{\lambda}$-stopping time, i.e. for all $n \in \mathbb{N}$ with $P(T_n < \infty) > 0$ there exists $t \in \mathbb{R}_+$ such that $\{T_n \le t\} \notin \mathscr{F}_t^{\lambda}$.

This is a rather general definition which comprises a lot of special cases.

It is easily verified that the non-homogeneous Poisson process with a time-dependent deterministic function $\lambda_t = \lambda(t)$ is an MRP, because here $\mathscr{F}_t^{\lambda} = \{\Omega, \emptyset\}$ for all $t \in \mathbb{R}_+$, so clearly the failure times T_n are not $\mathbb{F}^{\lambda}$-stopping times.

If the intensity is not deterministic but a random variable $\lambda(\omega)$ which is known at the time origin (λ is $\mathscr{F}_0$-measurable) or more general $\lambda = (\lambda_t)$ is a stochastic process such that λ_t is $\mathscr{F}_0$-measurable for all $t \in \mathbb{R}_+$, i.e. $\mathscr{F}_0 = \sigma(\lambda_s, s \in \mathbb{R}_+)$ and $\mathscr{F}_t = \mathscr{F}_0 \vee \sigma(N_s,\ 0 \le s \le t)$, then the process is called a doubly stochastic Poisson process or a Cox process. The failure (minimal repair) times are not $\mathbb{F}^{\lambda}$-stopping times, since $\mathscr{F}_t^{\lambda} = \sigma(\lambda) \subset \mathscr{F}_0$ and T_n is not $\mathscr{F}_0$-measurable.

In the following we give another characterization of an MRP.

Proposition 1 *Assume that $P(T_n < \infty) = 1$ for all $n \in \mathbb{N}$ and that there exist versions of conditional probabilities $F_t(n) = E\left[I(T_n \le t) | \mathscr{F}_t^{\lambda}\right]$ such that for each $n \in \mathbb{N}, (F_t(n)), t \in \mathbb{R}_+$, is a $(\mathbb{F}^{\lambda} - progressive)$stochastic process.*

1. *Then the point process (T_n) is a MRP if and only if for each $n \in \mathbb{N}$ there exists some $t \in \mathbb{R}_+$ such that*

$$P(0 < F_t(n) < 1) > 0.$$

2. *If furthermore $(F_t) = (F_t(1))$ has P-a.s. continuous paths of bounded variation on finite intervals, then*

$$1 - F_t = \exp\left\{ -\int_0^t \lambda_s \, ds \right\}.$$

See Aven and Jensen [6] for the proof.

4 Applications to Optimal Replacement

In this section we consider two applications, the first with minimal repairs at system failures and the second with minimal repairs at component failures.

4.1 Minimal Repairs at System Failures

Consider the setup of Sects. 2 and 3. Suppose now a planned replacement of the system is scheduled at time T, which may depend on the condition of the system, i.e. on the process X_t. The replacement time T is a *stopping time* in the sense that the event $\{T \leq s\}$ depends on the process X_t up to time s. There is no planned replacement if $T = \infty$.

The following simple cost structure is assumed: A planned replacement of the system costs $K(>0)$ and a repair/replacement at system failure costs $c(>0)$.

It is assumed that the systems generated by replacements are stochastically independent and identical, the same replacement policy is used for each system and the replacement and repairs take negligible time.

The problem is to determine a replacement time minimizing the long run (expected) cost per unit time.

Let M^T and S^T denote the expected cost associated with a replacement cycle and the expected length of a replacement cycle, respectively. We restrict our attention to T's having $M^T < \infty$ and $S^T < \infty$. Then using [17], Theorem 3.16, the long run (expected) cost per unit time can be written:

$$B^T = \frac{M^T}{S^T} = \frac{cEN_T + K}{ET}. \tag{3}$$

Using (2) (ref. also [7, 13]), it follows from (3) that

$$B^T = \frac{cE\int_0^T \lambda_t \, dt + K}{E\int_0^T dt}. \tag{4}$$

We note that the optimality criterion is in the same form as analyzed by Aven and Bergman [5]. Below the main results obtained in [5] are summarized.

Introduce $a_t = c\lambda_t$ and assume that a is non-decreasing in t. Define the replacement time T_δ by the first point in time the process a_t exceeds δ, i.e. $a_t \geq \delta$. We assume $ET_\delta < \infty$. It can be seen that T_δ minimizes

$$M^T - \delta S^T = E\int_0^T [c\lambda_t - \delta]dt + K.$$

The results of [7] follow. Let $B(\delta) = B^{T_\delta}$.

The stopping time T_{δ^*}, where $\delta^* = inf_T B^T$, minimizes B^T. The value δ^* is given as the unique solution of the equation $\delta = B(\delta)$. Moreover, if $\delta > \delta^*$, then $\delta > B(\delta)$, if $\delta < \delta^*$, then $\delta < B(\delta)$; $B(\delta)$ is non-increasing for $\delta \leq \delta^*$, non-decreasing for $\delta \geq \delta^*$, and $B(\delta)$ is left-continuous. It follows from (4) and Fubini's theorem that

$$B(\delta) = \frac{c\int_0^\infty E[I(a_t < \delta)]\lambda_t dt + K}{\int_0^\infty EI(a_t < \delta)dt}.$$

Hence if

$$a_t = cv(X_t),$$

where $v(x)$ is a deterministic function in x, and $Q_t(\cdot)$ is the distribution of X_t, we may write

$$B(\delta) = \frac{c\int_0^\infty \int [I(cv(x) < \delta))v(x)Q_t(dx)dt + K}{\int_0^\infty \int I(cv(x) < \delta)Q_t(dx)dt}. \tag{5}$$

Note that if X_t is a vector process, then one of the components of X_t may be the time t.

Examples of applications can be derived from the models presented in Examples 1 and 2 above, ref. Aven [3]. We briefly look at the shock model. In this case the $v(x)$ function is given by

$$v(x) = v\int_0^\infty p(x+y)dH(y).$$

Suppose the parameters of the model are

$$v = 1,\ K = 1,\ c = 2,\ Y_i \equiv 1,\ p(x) = 1 - e^{-x/4}.$$

Hence

$$\lambda_t = 1 - e^{-(X_t+1)/4}.$$

Using formula (5), it is not difficult to find the optimal policy: Replace the system when the number of shocks, V_t, equals 3. The average cost function then equals 1.1; see Aven [3].

4.2 Minimal Repairs at Component Failures

Consider a monotone system ϕ comprising n independent components, which are minimally repaired at failures. Let $X_t(i)$ be a binary random variable representing the state of component i at time t, $t \geq 0$, $i = 1, 2, \ldots, n$. The random variable

$X_t(i)$ equals 1 if the component is functioning at t and 0 otherwise. Assume $X_0(i) = 1$.

Let $N_t(i)$ denote the number of failures of component i in $[0, t]$, and let $N'_s(i)$ denote the associated process representing the number of failures of component i in $[0, s]$ when time is measured in operating time. We assume that $N'_s(i)$ is a non-homogeneous Poisson process with intensity function $\lambda_i(s)$. Let $\Lambda_i(s) = \int_0^s \lambda_i(u)du$ denote the mean value function of the process $N'_s(i)$. Hence the minimal repairs at the component levels are traditional minimal repairs as defined in Sect. 1.

The setup and analysis can easily be extended to doubly stochastic Poisson processes. We then have to replace $\lambda_i(t)$ by $v_i(U_t)$ where U is the underlying state process representing the condition of the system and v_i is a positive deterministic function, see Sect. 2.

Let $Z_t(i)$ denote the operating time at time t. Then it is not difficult to see that $N_t(i)$ is a counting process with intensity process $\lambda_i(Z_t(i)X_t(i))$.

Let $p_i(t) = 1 - q_i(t) = P(X_t(i) = 1)$. Furthermore let S_{in} denote the nth failure time of component i.

We assume that the repair/restoration times are independent with distribution function $G_i(t)$. Let $\overline{G}_i(t) = 1 - G_i(t)$.

Each component is minimally repaired at failures, which corresponds to the assumption of a non-homogeneous Poisson process of $N'_s(i)$.

The following cost structure is assumed:

- A system replacement cost $K, K > 0$.
- The cost of a minimal repair of component i is $c_i, c_i \geq 0$.
- The cost of a system failure of duration t is $k + bt$.

The system is assumed to be replaced at the stopping time T. After a replacement the system is assumed to be as good as new, i.e. the process restarts itself.

4.2.1 Optimization Function

Let M^T and S^T denote the expected cost associated with a replacement cycle and the expected length of a replacement cycle, respectively. Then again using [17], Theorem 3.16, the long run (expected) cost per time unit can be written:

$$B^T = \frac{M^T}{S^T} = \frac{E\text{costin}[0, \text{T}]}{ET}.$$

It is tacitly understood that the expectations are finite. In a replacement cycle the cost of the replacement and the minimal repairs equals $K + \sum_{i=1}^n c_i N_T(i)$. In addition we have a cost associated with system failures. It is not difficult to see that this cost equals $kN_T + b\int_0^T [1 - \Phi_t]dt$, where N_t represents the number of system failures in $[0, t]$.

It then follows that the cost/optimization function can be written:

$$B^T = \frac{K + \sum_{i=1}^{n} E \int_0^T c_i dN_t(i) + kEN_T + E \int_0^T b(1 - \Phi_t)dt}{ET} \tag{6}$$

Thus (6) expresses the expected cost per unit of time, and the problem of finding an optimal replacement time is reduced to that of minimizing this function with respect to T.

Using that $N_t(i)$ is a counting process with intensity process $\lambda_i(Z_t(i))X_t(i))$ it follows that

$$\sum_{i=1}^{n} E \int_0^T dN_t(i) = \sum_{i=1}^{n} E \int_0^T \lambda_i(Z_t(i))X_t(i))dt. \tag{7}$$

Similarly, we obtain the following expression for the expected number of system failures in a replacement cycle:

$$\begin{aligned} EN_T &= \sum_{i=1}^{n} E \int_0^T [\phi(1_i, \mathbf{X}_t) - \phi(0_i, \mathbf{X}_t)]dN_t(i) \\ &= \sum_{i=1}^{n} E \int_0^T [\phi(1_i, \mathbf{X}_t) - \phi(0_i, \mathbf{X}_t)]\lambda_i(Z_t(i))X_t(i))dt, \end{aligned} \tag{8}$$

where $\phi(1_i, \mathbf{X}_t) - \phi(0_i, \mathbf{X}_t)$ equals 1 if and only if component i is *critical*, i.e. the state of component i determines whether the system functions or not.

Combining (6), (7) and (8) we get

$$B^T = \frac{E \int_0^T a_t \, dt + K}{E \int_0^T dt}, \tag{9}$$

where

$$a_t = \sum_{i=1}^{n} [c_i + k(\phi(1_i, \mathbf{X}_t) - \phi(0_i, \mathbf{X}_t)]\lambda_i(Z_t(i))X_t(i)) + b[1 - \Phi_t]. \tag{10}$$

Observe that $Z_i(t) \approx t$ if the downtimes are relatively small compared to the uptimes.

We see from the above expression for B^T that it is basically identical to the one analyzed in [5]. Unfortunately, a_t does not have non-decreasing sample paths. Hence we cannot apply the results of [5].

In theory, Markov decision processes can be used to analyze the optimization problem. The Markov decision process is characterized by a stochastic process Y_t, $t \geq 0$, defined here by

$$Y_t = (S_t, \mathbf{X}_t, \mathbf{V}_t, W_t),$$

where

S_t = time since the last replacement

$\mathbf{X}_t = (X_t(1), X_t(2), \cdots, X_t(n))$

$X_t(i)$ = state of component$i$ at time t

$\mathbf{V}_t = (V_t(1), V_t(2), \cdots, V_t(n))$

$V_t(i)$ = duration of the downtime of component$i$ at t since the last failure of the component

$\mathbf{W}_t = (W_t(1), W_t(2), \cdots, W_t(n))$

$W_t(i)$ = accumulated downtime of component$i$ at t since last replacement

At each time t, the state Y_t is observed, and based on the history of the process up to time t, an action a_t is chosen. In this case there are two possible actions: "not replace" and "replace".

Here we shall, however, not analyze this approach any further. From a practical point of view the Markov decision approach is not very attractive in this case. The state space is very large and the cost rate function is not "monotone", cf. [19].

Instead, we shall look at a rather simple class of replacement policies: Replace the system at S or at the first component failure after T, whichever comes first. Here T and S are constants with $T \leq S$. We refer to this policy as a $(T,\ S)$ policy. Such a policy might be appropriate if for example the system failure cost is relatively large and a failure of a component often results in other components being critical (this will be the case if the system has minimal cut sets comprising two components).

4.2.2 Replacement Policies (*T, S*)

Let η_T denote the first component failure after T. Then, from (9), it follows that

$$B^{(T,\,S)} = \frac{\int_0^T Ea_t\ dt + \int_T^S EI(t<\eta_T)a_t\ dt + K}{T + \int_T^S P(t<\eta_T)dt},$$

where a_t is defined by (10). To compute $B^{(T,\,S)}$ we will make use of the approximation $Z_t(i) \approx t$. This means that the downtimes are relatively small compared to the uptimes. Using that the structure function of a monotone system can be written as a sum of products of component states with each term of the sum multiplied by a constant, it is seen that

$$a_t \approx \sum_l v_t(l) \prod_{i \in A_l} X_t(i) + \text{ constant},$$

for some deterministic functions $v_t(l)$ and sets $A_l \subset \{1, 2, \cdots, n\}$.

It suffices therefore to calculate expressions of the form

$$\int_0^T v_l(t) \prod_i p_i(t)dt, \tag{11}$$

and

$$\int_T^S v_l(t) E \prod_i X_t(i) I(t < \eta_T) dt. \tag{12}$$

To compute (11) we make use of the following formula for $q_i(t) = 1 - p_i(t)$:

$$q_i(t) \approx \int_0^t \overline{G}_i(t-y)\lambda_i(y) e^{-(\Lambda_i(t)-\Lambda_i(y))} dy. \tag{13}$$

To establish (13) we note that

$$q_i(t) = \int_0^t P(X_t(i) = 0 | S_{iN_i(t)} = y) H_i(dy, t),$$

where

$$H_i(y,t) = P(S_{iN_i(t)} \le y).$$

It is seen that $P(X_t(i) = 0 | S_{iN_i(t)} = y) \approx \overline{G}_i(t-y)$, and using that

$$H_i(y,t) = P(S_{iN_t(i)} \le y) = P(N_i(i) - N_y(i) = 0) \approx e^{-(\Lambda_i(t)-\Lambda_i(y))},$$

formula (13) follows. The accuracy of formula (13) is studied in [19].

It remains to compute (12). Here we shall present a very simple approximation formula. Observing that $I(t < \eta_T) = 1$ means that there are no component failures in the interval $(T, t]$, and the components are most likely to be up at time T, we have

$$\begin{aligned} E \prod_i X_t(i) I(t < \eta_T) &\approx \text{ P(no component failures in (T, t])} \\ &= \prod_{i=1}^n P(N_t(i) - N_T(i) = 0) \\ &\approx e^{-\sum_{i=1}^n (\Lambda_i(t) - \Lambda_i(T))} \end{aligned}$$

An approximate value of $B^{(T,S)}$ can now be calculated and an optimal policy determined.

In case of doubly stochastic Poisson processes, it suffices to compute

$$\int_0^T E\left[v_l(U_t) E\left[\prod_i X_{t(i)} = 1 | \mathscr{F}_0 \right] \right] dt \tag{14}$$

and

$$\int_T^S E\left[v_l(U_t)E\left[\prod_i X_{t(i)}I(t<\eta_T)|\mathscr{F}_0\right]\right]dt, \tag{15}$$

where v_l are deterministic functions. Conditional on $\mathscr{F}_0$ we can copy the arguments for the non-homogenous Poisson process case, and then by integrating over the distribution of U, we can obtain compact expressions for the optimization criterion. We omit the details.

The (T, S) policy can be improved by taking into account which component fails. Instead of replacing the system at the first component failure after T (assuming this occurs before S), we might replace the system at the first component failure resulting in a critical component, or, wait until the first system failure after T.

In Aven and Bergman [5] (refer previous section) it is shown that the problem of minimizing B^T can be solved my minimizing the function

$$L_\delta^T = M^T - \delta S^T = E\int_0^T [a(t) - \delta]dt + K.$$

If T^* minimizes $L_{\delta^*}^T$, where $\delta^* = \inf_T B^T$, then T^* also minimizes B^T. Hence we can focus on L_δ^T.

It is clear from the expression of L_δ^T that an optimal policy will be greater than or equal to the stopping time

$$T_\delta = \inf\{t : a_t \geq \delta\}$$

Using the optimal average cost $B^{(T,S)}$ as an approximation for δ^* we can obtain an improved replacement policy (T_{δ^*}, S).

An alternative replacement policy is obtained by considering the time points where component failures occur as decision points. Let T_i be the point in time of the ith component failure and let $\mathscr{F}_i$ denote the history up to time T_i. Then, based on $\mathscr{F}_i$ we determine a time $R_i(\in [0, \infty])$ such that the system is replaced at $T_i + R_i$ if $T_i + R_i < T_{i+1}$. The value of R_i is determined by minimizing the conditional expected cost from T_i until the next decision point or replacement time, whichever occurs first, i.e. R_i minimizes

$$g(r) = \int_{T_i}^{T_i+r} E[(a_t - \delta)I(t<T_{i+1})|\mathscr{F}_i]dt.$$

The performance of the above policies are studied in [19].

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