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# Operations on Lattices with Mathematica<sup>®</sup>

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# Preface

This manual is a companion of the book

G. Cariolaro, **Unified Signal Theory**, Springer-Verlag 2011.

It deals with the implementation in Mathematica<sup>®</sup> of the fundamental operations on lattices described in Chapter 16 of the book.

The Mathematica procedures are illustrated by demos, which are obtained by combining Mathematica and T<sub>E</sub>X (L<sup>A</sup>T<sub>E</sub>X).

## Manuscript preparation

To prepare the manuscript we used L<sup>A</sup>T<sub>E</sub>X, supplemented with a personal library of macros. The illustrations too are composed with L<sup>A</sup>T<sub>E</sub>X, sometimes with the help of Mathematica.

## Acknowledgements

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The authors will appreciate comments and error detection by the readers.  
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