# Preface

The increasing application of information technology in a variety of fields has led to a high degree of diversification, to the extent that it is difficult to clearly delimit the scope of this discipline and to establish its distinctive characteristics. Nevertheless, it is well recognized that *signals* are salient features of this discipline and have a paramount influence on all the related fields, such as physics, astronomy, biology, medicine, oceanography and meteorology, among others, that take advantage of the electronic and the digital revolutions. The fact that signals are the greatest protagonists of this evolution was clear from the beginning of the electronic era. In fact, recalling the definition of electronics as the production, transmission and use of *information* (Everett, 1948) and considering that signals are the physical carriers of information, we arrive at the conclusion that signals play a fundamental role in every field related to information technology. As a natural consequence, it follows that the enormous growth of information technology, and its diversification, are regularly transferred to the discipline that specifically deals with signals, that is, Signal Theory.

The idea of a Unified Signal Theory (UST) stems from the requirement that the large variety of signals (continuous-time, discrete-time, aperiodic, periodic, one-dimensional, two-dimensional, etc.) proposed over the last few decades can be treated efficiently and with conceptual economy. The target of the UST is a unified introduction and development of signal operations, such as convolution, filtering, Fourier transformation, as well as system formulation and analysis. This approach is rather atypical, with respect to standard signal theories, where different definitions and separate developments are provided for each specific class of signals.

**Philosophy of the UST** The key to this unification was my decision to treat the signal domains as *Abelian groups*, which have an appropriate mathematical structure that permits a unified introduction of the fundamental operations. I used the notation $s(t)$, $t \in I$, to emphasize that function $s$ (the signal) is defined on the Abelian group $I$ and realized, by inspection of all the signal classes of interest in applications, that every signal can be modeled in this unique and unified form. This remark may appear to be trivial, but it is essential for the unification and permits realizing the leitmotif *e pluribus unum* for signals. Note the fortunate circumstance that the domains of the Fourier transforms are Abelian groups, too, so that we can write every Fourier transform as $S(f)$, $f \in \widehat{I}$, where $\widehat{I}$ is the frequency domain, again an Abelian group.

A further requirement for this unification is a linear functional that permits the development of a coherent Signal Theory architecture. This is given by an *integral* introduced in 1933 by the Hungarian mathematician Alfred Haar, a student of Hilbert, which provides the second fortunate circumstance, because the Haar integral, applied to each specific case, produces all the integrals and the summations usually encountered in the standard signal theories.

In conclusion, the UST is built on two mathematical notions, Abelian groups and the Haar integral, which again is atypical for signal theory.

Using these notions, it is possible to treat the unified signal $s(t)$, $t \in I$, as a single abstract object and to introduce it in the basic definitions and developments. Once the unified architecture is completed, specific results for any given class of signals are simply obtained by particularizing the signal domain, as one-dimensional continuous, one-dimensional discrete, two-dimensional continuous, and so forth.

**Originality** This atypical formulation leads unavoidably to originality in the development of specific topics within the framework of the unified approach. The idea of unification itself is original, but so are several topics within the UST. The most important ones include: representation of Abelian groups by base–signature pairs; cells (as a generalization of the unit cells used for lattices); general definition of periodicity, formulation of signal symmetries; impulses (as a generalization of the concept of the Dirac delta function); multirate systems defined over structured groups without domain normalizations; ideal and elementary transformations; duality theorem for ideal transformations; band and bandwidth generalizations; unified sampling theorem; multidimensional polyphase decomposition in the signal domain (usually formulated in the $z$-domain). In my own opinion, the most profound result of the UST is the Duality Theorem on ideal transformations, which collects and unifies a dozen known results. I have published only very few of these original results, the reason being that their formulation would have required several pages of UST preliminaries, which would be far too long and not suitable for a paper.

**Mathematical Level** The mathematical level is perhaps a little high for engineers (and certainly too low for mathematicians), but it is appropriate for a graduate level in the area of information engineering. The main problem is concerned with the treatment of topological groups, the Abelian groups on which the Haar integral is introduced. Considering that the field of topology is very abstract and difficult for a fully mathematical development, I adopted the compromise of *using the results of Topology, without introducing topological details*. This is a typical "engineering compromise", as is usually made in engineering oriented books on probability and random processes, whose theoretical background is fully anchored on Measure Theory, but leaving out the mathematical details of this discipline.

**Organization of the Book** The book is (conceptually) subdivided into three parts.

Part I: *Classical Theory*. Since the UST may appear to be a difficult topic to anyone who is not already familiar with signals, I have introduced a preliminary chapter where the fundamentals of continuous-time and discrete-time signals are developed according to the traditional (not unified) approach, including several illustrations and examples. I hope that with these preliminaries, the book can profitably be read even by readers who do not possess elementary notions on signals.

Part II: *Unified Theory*. The UST itself is developed in six chapters, in *no more than 130 pages*, where sections are explicitly marked **UT** for clarity.[1] The first two chapters deal with UST fundamentals, that is, with Abelian groups and the Haar integral. Then, the unified approach is developed in both the signal domain and the frequency domain (Fourier transform). Finally, systems (conventionally called *transformations*) are developed, concluding with the formulation of the unified sampling theorem. Throughout its development the UST is illustrated in some detail with examples of one-dimensional and two-dimensional applications.

Part III: *Specific Classes of Signals and Applications*. The UST is general and not specific to any particular application. However, in the final nine chapters, we have some real-world applications, namely implementation of the fast Fourier transform (FFT), both one- and multidimensional, sampling and reconstruction of signals, multicarrier modulation system (OFDM), wavelets, image scanning, in particular, television scanning, image compression and tomography (Radon and Hankel transforms).

The last two chapters develop some advanced topics of the UST, with applications to spatio-temporal systems.

**Suggested Paths** The book could be used by both undergraduate and graduate students, and also by researchers, following three distinct paths.

Undergraduate students should begin with the Classical Theory, presented in Chap. 2. When studying the UST part, they should take in the statements and conclusions, without dwelling on finer mathematical proofs and justifications, and concentrate on one-dimensional signals in Part III.

Graduate students can avoid a detailed study of the Classical Theory, limiting themselves to a fast reading. But they should pay great attention to the mathematical formulation (both one-dimensional and multidimensional) in order to develop the attitude that problems related to signal theory can be approached from a general viewpoint, not merely confined to a specific problem. For graduate students, parts of the book will also be useful for future studies. I suggest that graduate students omit, at first reading, some mathematical details, explicitly indicated with the "jump" symbol $\Downarrow$.

Researchers could follow the path of graduate students, early concentrating their attention on the mathematical fundamentals and on the advanced applications they are considering in their professional activity.

[1] Some sparse contributions of UST are introduced also in Part III.

In this regard, I wish to add some personal considerations derived from my experience (and also from that of my colleagues). I have taught the UST for more than 20 years and realized that students never had conceptual difficulties in understanding the general fundamentals, they rather showed their enthusiasm for the compactness and generality of the formulation (saving their memory). At a first glance, the mathematical fundamentals might discourage a reader, but, depending on the teacher's sensibilities, mathematical details can be adjusted and adapted. In fact, Abelian groups (not so topological groups) represent a very elementary concept. Also, the Haar integral may simply be viewed as a formalism that has exactly the same properties as the ordinary integral on the real line. In conclusion, this book is intended for people who may never have studied signal theory before, as well as for experienced people. It is proposed as a panacea that satisfies everybody, even if it carries the risk of satisfying nobody.

**Examples and Problems. Solutions to All Problems** I have introduced several examples to illustrate the UST during its development. The final chapters, dedicated to specific classes of signals, may be viewed as extended illustration examples. I have also suggested several problems at the end of each chapter; problems are marked by one to three asterisks indicating the degree of difficulty. The examples and problems were tested on graduate students over the course of several years.

The solutions to all the problems proposed in the book are available on the Springer website www.springer.com/978-0-85729-463-0.

**Manuscript Preparation** To prepare the manuscript, I have used LaTeX, supplemented with a personal library of macros. The illustrations too are composed with LaTeX, sometimes with the help of `Mathematica`©.

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