

Preface

This book is aimed at undergraduates interested in discrete mathematics, enumeration or combinatorics. We have chosen the title *An Introduction to Enumeration* because of a single theme which will run through most of the book. The theme is counting using series: sometimes infinite and sometimes finite.

In the beginning, as children, our first introduction to mathematics is counting. It comes as a surprise to realize that this concept involves functions. Offer four children three chocolate bars and the chances are that one will scream, a one–one function being instinctively understood; enumeration understood. But the bigger surprise is that from counting marbles or chocolate biscuits, which is the discrete world, we can, in one simple step, go to the continuous world, and this new world provides powerful insights into the old world. A further surprise is that group theory can be used to count, and it comes into its own when symmetries are present in the configurations we are interested in enumerating. Because of the use of analysis and group theory we would expect this book to be of interest to anyone who has completed a first-year undergraduate course in mathematics. For background to these topics there are the following books in this series: J. M. Howie [8, 9] and G. Smith & O. Tabachnikova [14]. We should also mention the book by Ian Anderson in this series, [1], which overlaps with our book.

It is the use of generating functions to represent an enumerative sequence which is the link to these other branches of mathematics. A sequence arises naturally when we try to answer a question such as, “Given a particular configuration of some objects, how many such objects are there of size r ?”. We usually suppose that there are u_r of them: the ordered list $\{u_r : r = 1, 2, 3, \dots\}$ is the sequence of interest. Now attach each u_r to the r th power of z , which is z^r . At this stage we leave the question of what z is a little vague. This leads naturally to the series $\sum_{r=0}^{\infty} u_r z^r$. This object, called a power series, is packed full of enumerative information about the sequence $\{u_r\}$. We access this information through a function that the power series defines. The function,

when expanded as a (convergent) Taylor series, has as its coefficients the terms of the sequence $\{u_r\}$. We must be careful about the distinction between the series and the function. There is, however, a formal way of thinking about power series which avoids these issues. There are interesting discussions of these ideas, though from an abstract viewpoint, in D. Zelberger's article in [5], H. S. Wilf [18, 17] and I. Nevin [12]. We do not concern ourselves with such introspection (important though it is): the time for that must follow this book.

One of the ways in which this book differs from many others is that, wherever possible, it uses diagrams that portray subtle and slippery strands of an argument graphically so that they may be immediately grasped.

The first chapter sets the scene by asking in how many ways k objects can be chosen from r objects. The answer is not straightforward and depends on a number of constraints. Is the order in which the objects are chosen significant? In some gambling games this can be crucial. Are the objects distinguishable? Can they be selected more than once – so that there is an inexhaustible supply of them? Such questions perfectly describe the objective of the book: given an object of a certain size and configuration, how many of them are there? What are the relations between objects of different sizes, and how are they related to other objects? We introduce some simple techniques and then explore such objects, chosen from many areas of mathematics – geometry, sets, matrices, functions, groups, symmetries, permutations, paths and partitions. Time and time again, we use the same tools and the same techniques to unearth powerful results.

Chapter 4 introduces the next major theme of the book: group theory and its use in enumeration. Group theory began life as the study of symmetry. So as soon as the counting needs to take into account things which might appear to be the same, group theory naturally arises.

The other chapters take a particular problem and use this to motivate, guide and direct the development of the subject:

- (i) the number of ways of giving change in a particular coinage for the purchase of an item when a given amount is proffered;
- (ii) the number of different ways that a cube can be coloured with three colours;
- (iii) the number of different paths there are in a grid made up from integer coordinate points;
- (iv) the number of ways that a particular score may be gained when multiple dice are thrown;
- (v) the number of ways that subsets of a set can be chosen if the chosen subsets must not contain consecutive elements.

In each case we explore the enumeration of these configurations using tools and techniques that are progressively developed.

Enumeration leads to interesting problems and in this book we develop the theory, linked always to the ideas, tools and techniques on which it relies. We include many exercises (all with full answers) designed to motivate, consolidate and extend mathematical engagement so that the ideas are seamlessly absorbed and mastered.

A fascinating resource for sequences is the web site developed by N. J. A. Sloane [13]. We include in the bibliography some books not mentioned in the text which the reader might find interesting.

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