Preface

More than 50 years ago when I was studying to become an electrical engineer, I came across complex numbers, which were used to represent out-of-phase voltages and currents using the j operator. I believe that the letter j was used, rather than i, because the latter stood for electrical current. So from the very start of my studies I had a clear mental picture of the imaginary unit as a rotational operator which could advance or retard electrical quantities in time.

When events dictated that I would pursue a career in computer programming—rather than electrical engineering—I had no need for complex numbers, until Mandlebrot's work on fractals emerged. But that was a temporary phase, and I never needed to employ complex numbers in any of my computer graphics software. However in 1986, when I joined the flight simulation industry, I came across an internal report on quaternions, which were being used to control the rotational orientation of a simulated aircraft.

I can still remember being completely bemused by quaternions, simply because they involved so many imaginary terms. However, after much research I started to understand what they were, but not how they worked. Simultaneously, I was becoming interested in the philosophical side of mathematics, and trying to come to terms with the 'real meaning' of mathematics through the writing of Bertrand Russell. Consequently, concepts such as i were an intellectual challenge.

I am now comfortable with the idea that imaginary i is nothing more than a symbol, and in the context of algebra permits $i^2 = -1$ to be defined. And I believe it is futile trying to discover any deeper meaning to its existence. Nevertheless, it is an amazing object within mathematics, and I often wonder whether there could be similar objects waiting to be invented.

When I started writing books on mathematics for computer graphics, I studied complex analysis in order to write with some confidence about complex quantities. It was then that I discovered the historical events behind the invention of vectors and quaternions, mainly through Michael Crowe's excellent book "*A History of Vector Analysis*". This book brought home to me the importance of understanding how and why mathematical invention takes place.

Recently, I came across Simon Altmann's book "*Rotations, Quaternions, and Double Groups*" which provided further information concerning the demise of quaternions in the 19th century. Altmann is very passionate about securing recognition for the mathematical work of Olinde Rodrigues, who published a formula that is very similar to that generated by Hamilton's quaternions. The important aspect of Rodrigues' publication was that it was made three years before Hamilton's invention of quaternions in 1843. However, Rodrigues did not invent quaternion algebra—that prize must go to Hamilton—but he did understand the importance of half-angles in the trigonometric functions used to rotate points about an arbitrary axis.

Anyone who has used Euler transforms will be aware of their shortcomings, especially their Achilles' heel: gimbal lock. Therefore, any device that can rotate points about an arbitrary axis is a welcome addition to a programmer's toolkit. There are many techniques for rotating points and frames in the plane and space, which I covered in some detail in my book "*Rotation Transforms for Computer Graphics*". That book also covered the Euler–Rodrigues parameterisation and quaternions, but it was only after submitting the manuscript for publication, that I decided to write this book dedicated to quaternions and how and why they were invented, and their application to computer graphics.

Whilst researching this book, it was extremely instructive to read some of the early books and papers by William Rowan Hamilton and his friend P.G. Tait. I now understand how difficult it must have been to fully comprehend the significance of quaternions, and how they could be harnessed. At the time, there was no major demand to rotate points about an arbitrary axis; however, a mathematical system was required to handle vectorial quantities. In the end, quaternions were not the flavour of the month, and slowly faded from the scene. Nevertheless, the ability to represent vectors and manipulate them arithmetically was a major achievement for Hamilton, even though it was the foresight of Josiah Gibbs to create a simple and workable algebraic framework.

In this book I have tried to describe some of the history surrounding the invention of quaternions, as well as a description of quaternion algebra. In no way would I consider myself an authority on quaternions. I simply want to communicate how I understand them, which hopefully will be useful for you. There are different ways to represent a quaternion, but the one I like the best is an ordered pair, which I discovered in Simon Altmann's book.

This book divides into eight chapters. The first and last chapters introduce and conclude the book, with six chapters covering the following subjects. The second chapter on number sets and algebra reviews the notation and language relevant to the rest of the book. There are sections on number sets, axioms, ordered pairs, groups, rings and fields. This prepares the reader for the non-commutative quaternion product, and why quaternions are described as a division ring.

Chapter 3 reviews complex numbers and shows how they can be represented as an ordered pair and a matrix. Chapter 4 continues this theme by introducing the complex plane and showing the rotational features of complex numbers. It also prepares the reader for the question that was asked in the early nineteenth century: could there be a 3D equivalent of a complex number?

Chapter 5 answers this question by describing Hamilton's invention: quaternions and their associated algebra. I have included some historical information so that the reader appreciates the significance of Hamilton's work. Although ordered pairs are the main form of notation, I have also included matrix notation.

To prepare the reader for the rotational qualities of quaternions, Chap. 6 reviews 3D rotation transforms, especially Euler angles, and gimbal lock. I also develop a matrix for rotating a point about an arbitrary axis using vectors and matrix transforms.

Chapter 7 is the focal point of the book and describes how quaternions rotate vectors about an arbitrary axis. The chapter begins with some historical information and explains how different quaternion products rotate points. Although quaternions are readily implemented using their complex form or ordered-pair notation, they also have a matrix form, which is developed from first principles. The chapter continues with sections on eigenvalues, eigenvectors, rotating about an offset-axis, rotating frames of reference, interpolating quaternions, and converting between quaternions and a rotation matrix.

Each chapter contains many practical examples to show how equations are evaluated, and where relevant, further worked examples are shown at the end of the chapter.

Writing this book has been a very enjoyable experience, and I trust that you will also enjoy reading it and discover something new from its pages.

I would like to thank Dr Tony Crilly, Reader Emeritus at Middlesex University, for reading a draft manuscript and correcting and clarifying my notation and explanations. Tony performed the same task on my book *Rotation Transforms for Computer Graphics*. I trust implicitly his knowledge of mathematics and I am grateful for his advice and expertise. However, I still take full responsibility for any algebraic *faux pas* I might have made.

I would also like to thank Professor Patrick Riley, who read some early drafts of the manuscript and posed some interesting technical questions about quaternions. Such questions made me realise that some of my descriptions of quaternions required further clarification, which hopefully have been rectified.

I have now used LaTeX 2_ε for three of my books, and have become confident with its notation. Nevertheless, I still had to call upon Springer's technical support team, and thank them for their help.

I am not sure whether this is my last book. If it is, I would like to thank Beverley Ford, Editorial Director for Computer Science, and Helen Desmond, Associate Editor for Computer Science, Springer UK, for their professional support during the past years. If it is not my last book, then I look forward to working with them again on another project.

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