Chapter 2
Kinetic Equations and Particle Collisions

The inner magnetospheric plasma is a unique composition of different plasma particles and waves. It covers a huge plasma energy range, with spatial and time variations of many orders of magnitude. In such a situation, the kinetic approach is the key element, and must be the starting point of a proper theoretical description of these inner magnetospheric plasma phenomena. The highly nonequilibrium inner magnetosphere particle and energy flows are characterized by large temperature differences between the interacting components and the flow conditions changing from collision-dominated to collisionless regimes. That is why the questions of how to handle the different kinds of kinetic equations, and the kinetic description of the particle collisions are very important. Therefore, it is instructive to trace the derivations of the different kinetic collision terms in order to understand the strengths and limitations of different approaches.

2.1 Kinetic and Maxwell Equations

Practically, the most comprehensive description of plasma is given by the velocity distribution function in the framework of the standard approach based on the Boltzmann–Vlasov kinetic equation. The distribution function $f_a(\mathbf{v}_a, \mathbf{r}_a, t)$ is defined so that the value $f_a(\mathbf{v}_a, \mathbf{r}_a, t)d\mathbf{v}_a d\mathbf{r}_a$ represents the number of particles of the sort a, at time t in a small volume $d\mathbf{r}_a$ with coordinates from $\mathbf{r}_a$ to $\mathbf{r}_a + d\mathbf{r}_a$ and velocities from $\mathbf{v}_a$ to $\mathbf{v}_a + d\mathbf{v}_a$. In other words, the distribution function f represents the density of particles of a given kind in the six-dimensional phase space $(\mathbf{v}_a, \mathbf{r}_a)$.

The distribution function is defined by the following kinetic equation

$$\frac{\partial f_a}{\partial t} + \mathbf{v}_a \nabla f_a + \frac{1}{m_a}\left\{\mathbf{F}_a + e_a\left(\mathbf{E} + \frac{1}{c}[\mathbf{vB}]\right)\right\}\nabla_{\mathrm{v}} f_a = S_a + q_a. \tag{2.1.1}$$

Here, $\mathbf{E}$ and $\mathbf{B}$ are the total electric and magnetic fields, being the sum of the external and self-consistent (internal) parts produced by plasma particles; e_a and m_a

G.V. Khazanov, *Kinetic Theory of the Inner Magnetospheric Plasma*, 11
Astrophysics and Space Science Library 372, DOI 10.1007/978-1-4419-6797-8_2,

are the charge and mass of particles of the sort a; $\mathbf{F}_a$ are forces of nonelectromagnetic nature; c is the speed of light; and $\nabla_{\mathbf{v}}$ is the gradient in the velocity space. The distribution function variations caused by particle collisions are formally described by the terms in the right-hand side of the kinetic equation denoted by S_a. These terms are called *collision integrals* and, in turn, depend on the distribution function. The last term in the kinetic equation, q_a, is the production of particles of the sort a.

The kinetic equation (2.1.1) is only complete if the electric and magnetic fields that exist in the plasma are known. Therefore, the Maxwell equations for electromagnetic field must be solved along with the kinetic equation (2.1.1). In plasma, these equations are given by

$$\begin{aligned} &\nabla \cdot \mathbf{E} = 4\pi(\rho + \rho_0), \quad \nabla \cdot \mathbf{B} = 0, \\ &\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{B} = \frac{4\pi}{c}(\mathbf{j} + \mathbf{j}_0) + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}, \end{aligned} \tag{2.1.2}$$

where ρ_0 and $\mathbf{j}_0$ are the external charge density and current; ρ and $\mathbf{j}$ are the charge density and current produced by plasma particles and defined through the distribution function

$$\rho(\mathbf{r}, t) = \sum_a e_a \int f_a(\mathbf{v}_a, \mathbf{r}_a, t) d\mathbf{v}_a. \tag{2.1.3}$$

$$\mathbf{j}(\mathbf{r}, t) = \sum_a e_a \int \mathbf{v}_a f_a(\mathbf{v}_a, \mathbf{r}_a, t) d\mathbf{v}_a. \tag{2.1.4}$$

The summation in (2.1.3) and (2.1.4) extends over all kinds of charged particles present in plasma.

The kinetic equation is simplified substantially when particle collisions can be neglected. In that case, setting $S_a = 0$ in (2.1.1), we arrive to a collisionless approximation. The collisionless kinetic equation together with (2.1.2)–(2.1.4) was first considered by Vlasov (1938), and is called the kinetic equation with self-consistent field or the Vlasov equation. It is often used in the studies of plasma oscillations and stability (see, e.g., Akhiezer et al. 1975; Alexandrov et al. 1988; Stix 1992).

The use of the kinetic equation (2.1.1) in the study of various processes in plasma encounters major difficulties because of its complexity. That is why the class of problems that could be solved using the kinetic approach is quite limited. Besides, for a large number of problems, the detailed kinetic description is even excessive and only macroscopic parameters such as density, bulk velocity, pressure, viscosity tensor, and heat flux are important. These quantities are defined as corresponding velocity moments of the distribution function. Let us first give the definitions and a brief description of the basic macroscopic parameters of the plasma expressed through the moments of the distribution function.

The zero-order moment is the number density

$$n(\mathbf{r},t) = \int f(\mathbf{v},\mathbf{r},t)d\mathbf{v}. \tag{2.1.5}$$

Here and throughout the book, whenever possible, we will omit the species subscript for simplicity.

The first moment is the macroscopic (hydrodynamic or directional) velocity

$$\mathrm{V}(\mathbf{r},t) = \frac{1}{n}\int \mathbf{v}f(\mathbf{v},\mathbf{r},t)d\mathbf{v}, \tag{2.1.6}$$

where $\mathbf{v} = \mathbf{V} + \mathbf{w}$ is the total velocity, and $\mathbf{w}$ is intrinsic (chaotic) part of it, which, according to (2.1.6) satisfies the condition

$$\int \mathbf{w}fd\mathbf{v} = \mathbf{0}.$$

The second moment is the stress tensor and is defined as

$$P_{\alpha\beta}(\mathbf{r},t) = m\int w_\alpha w_\beta f(\mathbf{v},\mathbf{r},t)d\mathbf{w}. \tag{2.1.7}$$

Finally, let us define the third-order moment, which is the heat flux tensor

$$S_{\alpha\beta\gamma}(\mathbf{r},t) = m\int w_\alpha w_\beta w_\gamma f(\mathbf{v},\mathbf{r},t)d\mathbf{w}. \tag{2.1.8}$$

The kinetic description of plasma thus could be substituted by the system of equations for the velocity moments. These moment equations are usually called hydrodynamic or transport equations and are presented in Chap. 4 of this book.

2.2 Elastic Collisions

Collisions play a very important role in the kinetics of the inner magnetosphere/plasmasphere/ionosphere coupling processes. They are responsible for the plasma production in these regions as well as for the energy and momentum transfer between the different plasma populations. The collisional processes can be either elastic or inelastic. This section presents a short description of binary *elastic collisions*. In such collisions, the mass, momentum, and energy of the colliding particles are conserved in the collision process.

For binary elastic collisions between a and b particles, the appropriate collision operator is the *Boltzmann collision integral*, which can be presented as

$$S^0_{ab} = \iiint \left[f_a(\mathbf{v}')f_b\left(\mathbf{v}'_1\right) - f_a(\mathbf{v})f_b(\mathbf{v})f_b(\mathbf{v}_1)\right]uI_{ab}(u,\chi)\sin\chi d\chi d\varepsilon d\mathbf{v}_1. \tag{2.2.1}$$

Here $\mathbf{v}'$ and $\mathbf{v}_1'$ are the velocity of the a particle and of the b particle prior to the collision (their velocities after the impact are, respectively, $\mathbf{v}$ and $\mathbf{v}_1$); $I_{ab}(u,\chi)$ is differential scattering cross-section; $u = \left|\mathbf{v}' - \mathbf{v}_1'\right| \equiv |\mathbf{v} - \mathbf{v}_1|$ is the relative speed of the colliding particles a and b. The integration in (2.2.1) is over the velocities of the particle b (we call it particle 1 inside the collision integral) and over the scattering angles $d\Omega = \sin\chi d\chi d\varepsilon$, where χ is the angle between $\mathbf{v} - \mathbf{v}_1$ and $\mathbf{v}' - \mathbf{v}_1'$, or the center-of-mass scattering angle. In (2.2.1) the first term in the brackets corresponds to the particles scattered into a given region of velocity space, $\mathbf{v}$ (production term), and the second term corresponds to the particle scattering out of the same region of the velocity space (loss term).

The Boltzmann collision integral (2.2.1) can be applied to both self-collisions ($a = b$) and collisions between unlike particles. It can be applied to elastic electron–neutral and ion–neutral collisions, and to collisions between different neutral species. In addition, to a resonant charge exchange interaction between an ion and its parent neutral because the charge exchange process is pseudo–elastic. The net energy loss is small.

Although the Boltzmann collision integral (2.2.1) was originally intended to describe a gas of particles interacting through binary collisions, Landau in 1936 applied (2.2.1) to plasma and found the collision integral for the interacting charged particles. This result, known as the Landau equation, was obtained by retaining only the small-angle scattering events in the Boltzmann collision integral (2.2.1) and introducing a cut-off at the Debye shielding distance in the integral over the impact parameters, which would otherwise be logarithmically divergent for the Coulomb scattering low (Landau 1936). The Landau equation can be presented in differential form as (Gurevich 1978)

$$S_{ab} = -\frac{\partial}{\partial \mathbf{v}} \cdot \mathbf{j}_{ab}. \tag{2.2.2}$$

Here, $\mathbf{j}_{ab}$ is the particle flux in the velocity space due to the collisions between a and b particles. It is convenient to express it as

$$\mathbf{j}_{ab} = \frac{A_{ab}}{m_a} \int \left(\frac{\mathbf{I}}{u} - \frac{\mathbf{uu}}{u^3}\right) \left[\frac{1}{m_b} f_a(\mathbf{v}) \frac{\partial f_b}{\partial \mathbf{v}_1} - \frac{1}{m_b} f_b(\mathbf{v}_1) \frac{\partial f_a}{\partial \mathbf{v}}\right] d\mathbf{v}_1. \tag{2.2.3}$$

Here the dyadic notation has been used for the second-rank tensors, with $\mathbf{I}$ being the unit dyadic, equivalent to the Kronecker delta in subscript notation. The following parameters were also introduced

$$A_{ab} = 2\pi e_a^2 e_b^2 \ln\Lambda; \quad \Lambda = \frac{\lambda_D}{b_0}; \quad \lambda_D = \left(\frac{T}{4\pi n_e e^2}\right)^{1/2}; \quad b_0 \equiv \frac{e^2}{3T}, \tag{2.2.4}$$

where $\ln\Lambda$ is the Coulomb logarithm, λ_D is the Debye shielding distance, and b_0 is a typical "distance of closest approach" for a thermal particle.

In some cases it is convenient to express $\mathbf{j}_{ab}$ with the help of functions G_b and H_b, the so-called *Rosenbluth potentials* (Shkarofsky et al. 1966),

$$\mathbf{j}_{ab}(\mathbf{v}) = \frac{c_{ab}}{m_a^2}\left(\frac{m_a}{m_b}\frac{\partial H_b}{\partial \mathbf{v}}f_a - \frac{\partial}{\partial \mathbf{v}}\frac{\partial}{\partial \mathbf{v}}G_b\frac{\partial f_a}{\partial \mathbf{v}}\right), \tag{2.2.5}$$

$$G_b(\mathbf{v}) = \int u f_b(\mathbf{v}_1)d\mathbf{v}_1; \quad H_b(\mathbf{v}) = 2\int \frac{1}{u} f_b(\mathbf{v}_1)d\mathbf{v}_1, \tag{2.2.6}$$

where, as above, $\mathbf{u} = \mathbf{v} - \mathbf{v}_1$, $u = |\mathbf{u}|$.

The collision integrals (2.2.1) and (2.2.2) can be significantly simplified to describe collisions between space plasma electrons ($a = e$) and heavy ionospheric compositions, neutral particles ($b = n$) and ions ($b = i$). In an elastic impact of an electron against the heavy atom or molecule, the velocity of the electron can readily change direction, but the absolute value of the velocity or energy of the electron can change only insignificantly. Recognizing that the electron velocity is much larger than that of the atom or molecule, and assuming the Maxwellian background of the neutral atmospheric particles

$$f_n = n_n\left(\frac{M_n}{2\pi T_n}\right)^{3/2}\exp\left(-\frac{M_n \mathrm{v}^2}{2T_n}\right), \tag{2.2.7}$$

the Boltzmann collision integral that describes elastic electron–neutral particle collisions can be presented as (Khazanov 1979)

$$\begin{aligned} S_{en}^0(E,\theta,\varphi) = n_n \mathrm{v}\Bigg\{&\frac{2m}{M_n}\frac{1}{E}\left[E^2\sigma_{en}^{(1)}\left(f + T_n\frac{\partial f}{\partial E}\right)\right]. \\ &+ \int I_{en}(E,\chi) f(E,\theta',\varphi')d\Omega' - \sigma_{en}^{(0)} f(E,\theta,\varphi)\Bigg\}. \end{aligned} \tag{2.2.8}$$

To present this collision term, the spherical coordinate system in the velocity space has been introduced, $\mathbf{v} = \{\mathrm{v},\theta,\varphi\}$, with a polar axis along the magnetic field direction, and the connection between the modulus of electron velocity and its energy, $E = m\mathrm{v}^2/2$. In this coordinate system, the angle θ is the electron pitch angle ($\mu = \cos\theta = \mathbf{v}\cdot\mathbf{B}/\mathrm{vB}$), and angle φ is the azimuthal angle of the rotation around the magnetic field line. In this coordinate system, the $\mu_0 \equiv \cos\chi = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\phi - \phi')$ relation between the angles in (2.2.8) holds. In addition, the following notations have been used for the transport ($\sigma_{en}^{(1)}$) and total ($\sigma_{en}^{(0)}$) cross-sections of the scattering of the electron on neutral particle

$$\sigma_{en}^{(1)}(E) = \int I_{en}(E,\chi)(1 - \cos\chi)d\Omega; \quad \sigma_{en}^{(0)}(E) = \int I_{en}(E,\chi)d\Omega. \tag{2.2.9}$$

Now the Landau equation (2.2.2) and expression (2.2.5) are used to simplify the description of the electron ion collisions. Assuming the Maxwellian background of the thermal ions and considering that the relative electron ion velocity u is approximately equal to the electron velocity v, Rosenbluth potentials (2.2.6) could be easily calculated and presented as (Shkarofsky et al. 1966)

$$G_i(\mathbf{v}) = \mathrm{v}_i n_i \left[\frac{1}{\sqrt{\pi}} \exp\left(-w^2\right) + \left(w + \frac{1}{2w}\right) erf(w) \right]; \quad H_i(\mathbf{v}) = \frac{2n_i}{\mathrm{v}_i w} erf(w). \tag{2.2.10}$$

Here, $w = \mathrm{v}/\mathrm{v_i}$; $\mathrm{v_i} = \sqrt{2T_i/M_i}$ is the thermal ion velocity, and $erf(w) = 2\Big/\sqrt{\pi}\int\limits_0^w e^{-t^2}dt$ is the probability integral. After substituting (2.2.10) into the Landau collision term, the electron ion collision integral can be written as (Khazanov 1979)

$$S_{ei}^0(E,\theta,\varphi) = An_i\frac{\mathrm{v}}{\mathrm{E}}\left\{\frac{m}{M_i}\frac{\partial}{\partial E}\left(f + T_i\frac{\partial f}{\partial E}\right) + \frac{1}{4E\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{\sin\theta}\frac{\partial^2 f}{\partial\varphi^2}\right]\right\}. \tag{2.2.11}$$

It is also assumed here that $A = A_{ee} = A_{ei}$.

In the case of collisions between electrons, the collision integral is described by (2.2.2) and (2.2.5). It is necessary only to take into account that $a = b = e$. If the electron distribution function has a symmetry with respect to the magnetic field direction, then $f_e \equiv f(\mathbf{v}) = f(\mathrm{v},\theta)$, and the electron/electron collision term can be presented as

$$\begin{aligned} S_{ee}(E,\theta) = An_e\frac{\mathrm{v}}{\mathrm{E}}\Bigg\{&\frac{\partial}{\partial E}\left(A_1 f + A_2 E\frac{\partial f}{\partial E} + \frac{1}{2}A_3\frac{\partial f}{\partial\theta}\right) \\ &+ \frac{1}{4E\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\left(2A_4\frac{\partial f}{\partial\theta} + 2A_3E\frac{\partial f}{\partial E} + A_5\frac{\partial f}{\partial\theta}\right)\right]\Bigg\}. \end{aligned} \tag{2.2.12}$$

Here, the dimensionless coefficients A_k, following from (2.2.5) and (2.2.6), are determined by the equalities

$$\begin{aligned} &A_1 = -\frac{\mathrm{v}^2}{2n_e}\frac{\partial H_e}{\partial \mathrm{v}}, \quad A_2 = \frac{\mathrm{v}}{n_e}\frac{\partial^2 G_e}{\partial \mathrm{v}^2}, \quad A_3 = \frac{\mathrm{v}}{n_e}\frac{\partial}{\partial \mathrm{v}}\left(\frac{1}{\mathrm{v}}\frac{\partial G_e}{\partial\theta}\right), \\ &A_4 = -\frac{\mathrm{v}}{2n_e}\frac{\partial H_e}{\partial\theta}, \quad A_5 = \frac{1}{n_e}\left(\frac{1}{\mathrm{v}}\frac{\partial^2 G_e}{\partial\theta^2} + \frac{\partial G_e}{\partial \mathrm{v}}\right). \end{aligned} \tag{2.2.13}$$

2.3 Inelastic Collisions

The exact nature of the collision process depends both on the relative kinetic energy of the colliding particles and on the type of the particles. In general, however, for low energies, elastic collisions in the ionosphere dominate, but, as relative kinetic energy increases, inelastic collisions become progressively more important. The order of importance is from elastic to rotational, vibrational, and electronic excitation, and then to ionization as the relative kinetic energy increases. However, the different collision process may affect the continuity, momentum, and energy equations in different ways (see Chap. 4 for details).

Inelastic collisions of electrons with molecules, atoms, and ions resulting in excitation of rotational, vibrational, and optical levels, ionization and recombination, are a very important mechanism of thermalization of the electron gas in the near-Earth space plasma. This section is mostly devoted to the inelastic description of electron–neutral collision processes, leaving some of the details of inelastic ion–neutral interactions for further chapters of this book.

As in a previous section, it is convenient to introduce the spherical coordinate system in the velocity space, $\mathbf{v} = \{\mathrm{v}, \theta, \varphi\}$, with a polar axis along the magnetic field direction, and connect the modulus of electron velocity with its energy $E = m\mathrm{v}^2/2$. To simplify the inelastic collision integrals structure, it is also convenient to transfer from the electron distribution function to the flux

$$\Phi(E, \theta, \varphi) = \frac{2Ef(E, \theta, \varphi)}{m^2}, \tag{2.3.1}$$

determined in such way that $\Phi(t, \mathbf{r}, E, \theta, \varphi)dEd\Omega$ is the quantity of electrons with energy from E to $E + dE$ and the direction of velocity $\mathbf{v}$ within the solid angle $d\Omega = \sin\theta d\theta d\varphi$, which intersect at point $(t, \mathbf{r})$ in a unit of time the unit of area.

When constructing a nonelastic integral of collision, it is convenient to start from an analogy with elastic collisions of electrons with neutral particles, which is described by an integral of collision in the Boltzmann form (2.2.1). The first term of this integral characterizes the entering of electrons in the current phase volume, and the second term characterizes electrons leaving from it. If one considers that the distribution of neutral particles in the velocity space is fixed, then the expression for the integral of collisions that describes the process of excitation by electron of certain level of molecules or atoms with energy E_k from the state that characterized energy $E_j < E_k$, can be presented in the form (Khazanov 1979)

$$\begin{aligned} S_{en}^{\mathrm{I}jk} = n_n^j \Big[\iint W_{jk}\left(E' \to E, \chi, \varepsilon\right)\Phi\left(E', \theta', \varphi'\right) dE' d\Omega \\ - \iint W_{jk}\left(E \to E', \chi, \varepsilon\right)\Phi\left(E, \theta, \varphi\right) dE' d\Omega \Big], \end{aligned} \tag{2.3.2}$$

where n_n^j is the density of neutral particles of the n sort that are in the state characterized by energy E_j; and $W_{jk}(E \leftrightarrow E', \chi, \varepsilon)dE'd\Omega$ is the probability that electrons found in the phase volume close to $E', \theta', \varphi'(E, \theta, \varphi)$ have undergone a nonelastic collision characterized by angles of scattering χ and ε, which scatter the electrons to a current volume (from a current volume) of phase space.

From the expressions of dimensionality and the law of conservation of energy, the probability that characterizes the process of scattering in (2.3.2) can be presented in a form (Khazanov 1979)

$$\begin{aligned} W_{jk}(E' \to E, \chi, \varepsilon)dE'd\Omega = I_{jk}^{en}(E', \chi)\delta\left(E' - E - E_{jk}\right)dE'd\Omega, \\ W_{jk}(E \to E', \chi, \varepsilon)dE'd\Omega = I_{jk}^{en}(E, \chi)\delta\left(E - E' - E_{jk}\right)dE'd\Omega, \end{aligned} \tag{2.3.3}$$

where $I_{jk}^{en}(E, \chi)d\Omega$ is the differential electron cross-section, depending on the energy of the bombarding electron and the polar scattering angle; $\delta(x)$ is the delta function; and $E_{jk} = E_k - E_j > 0$ is the energy of excitation.

Then, substituting (2.3.3) in (2.3.2) and integrating according to dE', we find

$$S_{en}^{\mathrm{I}jk} = n_n^j \left[\int \Phi\left(E + E_{jk}, \theta', \varphi'\right) I_{en}^{jk}\left(E + E_{jk}, \chi\right) d\Omega - \sigma_{en}^{jk}(E)\Phi(E, \theta, \varphi) \right]. \tag{2.3.4}$$

Here, $\sigma_{en}^{jk}(E)$ is the total cross-section of nonelastic scattering of an electron on a neutral particle of the n type, during which excitation of the latter occurs from the E_j level to the E_k

$$\sigma_{en}^{jk}(E) = \iint I_{en}^{jk}(E, \chi) \sin\chi d\chi d\varepsilon. \tag{2.3.5}$$

In a case of a collision of a second type (de-excitation) of energy of an excited state of an atom or molecule, a bombarding electron can be transmitted $\left(E_{jk} < 0\right)$. Then, presenting arguments similar to the preceding, it is not difficult to find (Khazanov 1979)

$$S_{en}^{\mathrm{II}jk} = n_n^k \left[\int \Phi\left(E - E_{jk}, \theta', \varphi'\right) I_{en}^{kj}\left(E - E_{jk}, \chi\right) d\Omega - \sigma_{en}^{kj}(E)\Phi(E, \theta, \varphi) \right], \tag{2.3.6}$$

where n_n^k is the density of neutral particles of the n sort that are in the state characterized by energy E_k; and I_{en}^{kj}, σ_{en}^{kj} are corresponding cross-sections of inverse transitions involving values I_{en}^{jk} and σ_{en}^{jk} by a principle of detailed equilibrium.

In this way, the integral of collisions that describes the process of excitation and deactivation of energy levels of neutral particles can be presented in the form

$$S_{en}^{*} = \sum_{j,k>j} (S_{en}^{\mathrm{I}jk} + S_{en}^{\mathrm{II}kj}). \tag{2.3.7}$$

Here, a summation of all possible transitions of neutral particles of the n is presented. The integral of collisions with the ions, S_{ei}^{*}, has a similar structure that, in a view of the small concentration of ions, can be ignored.

Now, the ionization collision integral that accounts for the electron–neutral collisions resulting in the ionization of the neutral particles will be constructed. It will be assumed that all such collisions result in single ionization, i.e., that only one electron is removed from the neutral atom or molecule. In this case, both the ejected and the scattered electron must be considered. Presenting considerations similar to those for constructing (2.3.2), we find (Khazanov 1979)

$$\begin{aligned} S_{en}^{+} = n_n \Bigg[& \iint W_1(E' \to E, E_2, \chi_1, \varepsilon_1)\Phi(E', \theta', \varphi')dE'd\Omega_1 \\ & + \iint W_2(E', E_2, \chi_2, \varepsilon_2)\Phi(E', \theta', \varphi')dE'd\Omega_2 \\ & - \iint W(E, E', \chi_1, \varepsilon_1, \chi_2\varepsilon_2)\Phi(E, \theta, \varphi)dE'd\Omega_1 d\Omega_2 \Bigg]. \end{aligned} \tag{2.3.8}$$

Here, $W_1 dE'd\Omega_1$ is the probability of transition of the bombarded electron in the energy interval from E to $E + dE$ (with simultaneous formation close to E_2 of a secondary electron) and scattering of it in the element of the solid angle $d\Omega_1 = \sin\chi_1 d\chi_1 d\varepsilon_1$; $W_2 dE'd\Omega_2$ is the probability of the fact that the bombarding electron formed in the energy interval from E to $E + dE$ a secondary electron with angles of scattering χ_2, ε_2; and $WdE'd\Omega_1 d\Omega_2$ is the corresponding probability of output of the electron from the element of phase volume considered.

Ionizing collision of a neutral particle and electron with energy E is usually characterized by differential cross-section of ionization, as a result of which, the bombarded electron is scattered in the element of the solid angle, $d\Omega_1$, and the secondary electron is formed with the energy of E_2 in the element of $d\Omega_2$ (Khazanov 1979)

$$I^{(1,2)}(E, E_2, \chi_1, \chi_2)dE_2 d\Omega_1 d\Omega_2. \tag{2.3.9}$$

Here, as in the case of nonelastic scattering (2.3.3) resulting in excitation, let us assume an angular dependence only on the polar angle of scattering.

Considering dimensionality of S_{en}^{+} and the laws of conservation of energy, the corresponding probabilities of transitions in (2.3.8) can be presented in the form

$$\begin{aligned} W_1 dE'd\Omega_1 &= I^{(1)}(E', E' - E - E^{+}, \chi_1)dE'd\Omega_1, \\ W_2 dE'd\Omega_2 &= I^{(2)}(E', E, \chi_2)dE'd\Omega_2, \\ WdE'd\Omega_1 d\Omega_2 &= I^{(1,2)}(E, E', \chi_1, \chi_2)dE'd\Omega_1 d\Omega_2, \end{aligned} \tag{2.3.10}$$

where E^{+} is the threshold of ionization, and $I^{(1)}$ and $I^{(2)}$ correspondingly equal

$$\begin{aligned} I^{(1)}(E, E_2, \chi_1)dE_2 d\Omega_1 &= \int I^{(1,2)} d\Omega_2, \\ I^{(2)}(E, E_2, \chi_1)dE_2 d\Omega_2 &= \int I^{(1,2)} d\Omega_1; \end{aligned} \tag{2.3.11}$$

where $I^{(1,2)}$ is determined by (2.3.9).

Substituting (2.3.10) in (2.3.8) and conventionally assuming that the energy of the primary electron E_1 is larger than the energy of the secondary electron (Khazanov 1979), we find

$$S_{en}^{+} = n_n \left\{ \int\limits_{E+E_n^+}^{2E+E_n^+} \left[\int\limits_0^{2\pi} \int\limits_0^{\pi} I_n^+ \left(E', E'-E-E_n^+, \chi_1\right) \Phi(E', \theta', \varphi') \sin\chi_1 d\chi_1 d\varepsilon_1 \right] dE' \right.$$
$$\left. + \int\limits_{2E+E_n^+}^{\infty} \left[\int\limits_0^{2\pi} \int\limits_0^{\pi} I_n^+ (E', E, \chi_2) \Phi(E', \theta', \varphi') \sin\chi_2 d\chi_2 d\varepsilon_2 \right] dE' - \sigma_n^+(E) \Phi(E, \theta, \varphi) \right\}. \tag{2.3.12}$$

Here,

$$\sigma_n^+(E) = \int\limits_0^{\left(E-E_n^+\right)/2} I_n^+(E, E_2) dE_2 \tag{2.3.13}$$

is the total cross-section of ionization, and $I_n^+(E, E_2)$ is the corresponding differential cross-section

$$I_n^+(E, E_2) = \int I^{(1,2)} d\Omega_1 d\Omega_2. \tag{2.3.14}$$

In the case where, during ionization, an excitation of the ion occurs, the integral of collisions S_{en}^+ can be rewritten similarly to (2.3.12), introducing partial values that characterize excitation of the l level of the ion $I_{nl}^+, \sigma_{nl}^+, E_{nl}^+ = E_n^+ + E_{nl}$ (E_{nl} is the energy of excitation of the l level of the ion from its basic state), and making a summation of all the l levels possible in its state.

Finally, for integrals of collision, which describe the process of recombination of electrons and ions, S_{ei}^r, which only removes electrons from the current phase volume, the expression (Khazanov 1979)

$$S_{ei}^r = -n_i \sigma_i^r(E) \Phi(E, \theta, \varphi), \tag{2.3.15}$$

is used, where $\sigma_i^r(E)$ is the total recombination cross-section for an ion of the i sort.

2.4 Plasma Source

Let us now establish a clear form of the last component of the kinetic equation (2.1.1), q, for the case of electrons. The basic processes leading to formation of the ionosphere plasma at altitudes $h > 100$ km are processes of ionization of the L- and

K-shells of neutral components of the upper atmosphere by solar radiation with wavelengths $\lambda<1{,}026$ A (middle latitudes) and electrons of magnetospheric origin poured out with energies $E \geq 1$ KeV (high latitudes).

The number of electrons with energies from E to $E + dE$ at point s of the geomagnetic field line that has formed during ionization of the external L-shells of neutral atoms and molecules of the upper atmosphere n_n by a solar source with wavelength λ and intensity Φ_λ per unit of volume per second, with the direction of velocity within the solid angle $d\mu d\varphi$, can be defined by

$$dq(s,E,\mu,\varphi) \equiv q(s,E,\mu,\varphi)dEd\mu d\varphi \\ = dEd\mu d\varphi \sum_{n,l} n_n(s)\Phi_\lambda\left(s,E+E_{nl}^+,\mu',\varphi'\right)I_{nl}^\lambda\left(E+E_{nl}^+\right)d\mu_0 d\varepsilon_0. \tag{2.4.1}$$

Here, $I_{nl}^\lambda\left(E+E_{nl}^+\right)$ is the differential cross-section of photoionization of photons with energies $hc/\lambda = E + E_{nl}^+$, as a result of which an ion forms at the l energy level; μ_0, ε_0 are angles that characterize the directions of motion of a photon and knocked-on electron; $\Phi_\lambda(\mu,'\varphi') = \bar{\Phi}_\lambda \delta(\mu' - \mu_\upsilon)\delta(\varphi' - \varphi_\upsilon)$ is the unidirectional flux of photons; and $\theta_v = \arccos \mu_v, \varphi_v$ are angles that characterize its motion in relation to the magnetic field.

If the process of ionization is isotropic ($I_{nl}^\lambda = \sigma_{nl}^\lambda/4\pi$ for O_2 and N_2), then, on the basis of (2.4.1), the source of photoelectrons can be presented in the form

$$q(s,E) = \frac{1}{4\pi}\sum_\lambda \sum_n \sum_l n_n(s)\sigma_{nl}^\lambda\left(E+E_{nl}^+\right)\bar{\Phi}\left(s,E+E_{nl}^+\right), \tag{2.4.2}$$

where σ_{nl}^λ is the partial cross-section of photoionization.

For the atomic oxygen, according to the data of Kennedy and Manson (1972), when E > 5 eV, the differential cross-section of ionization has an angular dependence

$$I_{nl}^\lambda\left(E+E_{nl}^+\right) = \frac{\sigma_{nl}^\lambda\left(E+E_{nl}^+\right)}{4\pi}[1 - P_2(\mu_0)]. \tag{2.4.3}$$

From the connection of angular variables in (2.4.1) with consideration of (2.4.3), we find the following expression for the source of photoelectrons during ionization of the external shell of the atomic oxygen

$$q(s,E,\mu) = \frac{[1 - P_2(\mu_v)P_2(\mu)/2]}{4\pi} n(O)\sum_\lambda \sum_n \sum_l \sigma_{nl}^\lambda\left(E+E_{nl}^+\right)\bar{\Phi}\left(s,E+E_{nl}^+\right). \tag{2.4.4}$$

Here, $\mu_v \equiv \cos\theta_v = -\sin\delta\cos(J+\Lambda) + \cos\delta\sin(J+\Lambda)\cos\Phi$; and δ, J, Λ, and Φ are solar declination, magnetic inclination, latitude, and hour angle, respectively.

The second source of ionosphere electrons whose calculation can be made similarly to (2.4.1) involves ionization of the K-shells of the atom. The internal vacancy formed here has a very small lifetime ($\tau \approx 10^{-14}$ s) (Avakyan et al. 1977) and is filled by an electron with more external shells. Here an X-ray quantum can be emitted or radiationless transition can occur with emission of an electron from one of the external electron shells (Auger effect), which carries off energy stored by the atomic system during primary ionization of the K-shell of the atom (Avakyan et al. 1977).

During ionization of the K-shells of atmospheric gases of nitrogen and oxygen in the initial system, radiationless Auger transitions occur basically and the output of X-ray fluorescence is very insignificant (Avakyan et al. 1977). Specifically, N_2 Auger electrons form a series of peaks from 315 to 367 eV, atomic oxygen forms Auger peaks from 474 to 509 eV, and the O_2 Auger electron peaks range from 456 to 507 eV.

If one assumes that angular distribution of the Auger electrons has an isotropic character, and filling of the appropriate energy intervals by the electrons occurs uniformly, then the expression for the source of electrons as the result of the Auger transition can be presented in the form

$$q(s,E) = \frac{1}{4\pi}\sum_{\lambda}\sum_{n} n_n(s)\sigma_{nl}^{\lambda}\left(E+E_{nl}^{+}\right)\bar{\Phi}\left(s,E+E_{nl}^{+}\right)\frac{\theta(E-E_{1n})\theta(E_{2n}-E)}{E_{2n}-E_{1n}}, \tag{2.4.5}$$

where $\theta(x)$ is the unit function

$$\theta(x) = \begin{cases} 0, if\ x<0, \\ 1, if\ x \geq 0, \end{cases}$$

and E_{1n} and E_{2n} are the lower and upper boundaries of the energy intervals where formation of the Auger electrons takes place.

Altitude variations of photons in (2.4.2), (2.4.4), and (2.4.5) are determined by (Schunk and Nagy 2000)

$$\bar{\Phi}_{\lambda}(h) = \bar{\Phi}_{\lambda\infty}\exp\left[-\sum_{n}\sigma_{n}^{\lambda}\int_{h}^{\infty} n(h')ch(\chi_o,h')dh'\right], \tag{2.4.6}$$

where $\bar{\Phi}_{\lambda\infty}$ is the flux of photons with wavelength λ outside the atmosphere; σ_n^{λ} is the cross-section of absorption, and $ch(\chi_o,h)$ is the Chapman function. Note that, when calculating electrons along the geomagnetic field line, (2.4.6) usually is recalculated for the appropriate coordinates s of the magnetic force line.

If ionization occurs with ultraviolet and X-ray radiation of the sun, then the primary energy spectrum of the photoelectrons actually reflects the spectral distribution of solar radiation slightly distorted by the presence of numerous thresholds of ionization in atmospheric gases. Figure 2.1 illustrates the energy distribution of

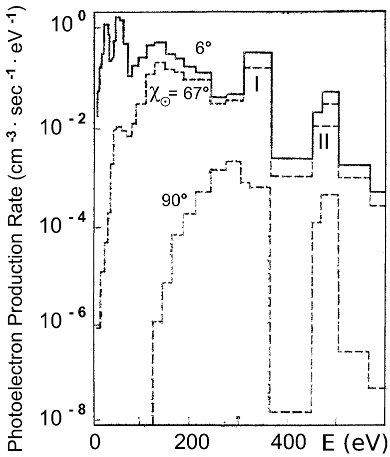

Fig. 2.1 Energy distribution of the photoelectron source at the altitude 125 km with different zenith angles of the Sun

the electron source at an altitude of 125 km with different zenith angles of the sun. The character of the spectrum above 100 eV only slightly changes in the range of zenith angles from 0° to 70°. There is a broad maximum at 110–150 eV caused by the intense flux of solar in the region 75–100 A, then, right up to 600 eV, one observes a constant drop on which there are two intensive peaks: I is the Auger electrons from a molecule of nitrogen and II is from an atom and molecule of oxygen. It is apparent from Fig. 2.1 that when $\chi_o = 90°$, peak I blends at the base on the entire spectrum with peak 245–330 eV, and peak II increases greatly in relation to the remaining part of the spectrum. This is due to the strong absorption of the X-ray radiation, which is more intense in the range of wavelengths 31–20 A; at the same time, radiation with wavelengths <20 A (which the Auger transitions excite in oxygen) and above 31 A (giving the base of the peak 245–300 eV) is even weaker because the corresponding cross-sections of photoabsorption of nitrogen and oxygen are fairly small.

If ionization of neutral component of the upper atmosphere occurs by flux of the precipitated electrons of magnetospheric origin, $\Phi_e(s, E, \mu)$, then the source of the secondary electron can be presented in the form (Khazanov 1979)

$$q(s, E, \mu) = \sum_n \sum_l n_n(s) \int_0^{2\pi} \int_{-1}^{1} \int_{2E+E_{nl}^+}^{\infty} \Phi_e(s, E', \mu') I_{nl}^+(E', E, \mu_\chi) dE' d\mu_\chi d\varepsilon, \tag{2.4.7}$$

where I_{nl}^+ is the differential cross-section of ionization with electron impact of the L-shell of the atom or molecule.

For calculation of the spectrum of the secondary electrons, it is necessary to have the altitude distribution of precipitated electrons. This should be found from the solution of an appropriate electron kinetic equation. Some aspects of secondary electron production and calculations of secondary electrons distribution functions are discussed in Chaps. 5 and 7.

2.5 Superthermal Electron Kinetic Equation Simplification

Superthermal electrons are produced as a result of the ionization of neutral particles by solar radiation (photoelectrons) and by high-energy electrons of magnetospheric origin (secondary electrons). They transform to thermal electrons, thus loosing their energy due to the excitation and ionization of neutral particles, or transferring it to thermal electrons. Superthermal electrons play an important role in the kinetics of microprocesses of ionospheric plasma. They are usually invoked to account for the thermal plasma heating and the intense excitation of energy levels of atoms and molecules, which gives rise to the various atmospheric emissions and to increased rates of some aeronomical reactions. For superthermal electrons, the most obvious simplification in the kinetic description is made by recognizing that the magnetic field dominates the motion of these particles. In a background magnetic field, charged particles gyrate about the magnetic lines of force, and the frequency of this gyration, $\Omega = qB/m$, is very fast for electrons (10^4–10^7 rad s^{-1}). Therefore, the foremost reduction of (2.1.1) is to average over this gyrational motion. In this case, the velocity space reduces to two variables, and instead of following the individual particles in configuration space, the equation solves for the motion of their gyrational guiding center. Yet another simplification that is made possible by the magnetic dominance is that most spatial transport is along the magnetic field lines, and therefore cross-field drifts can be omitted in a first-order calculation. Under these assumptions, (2.1.1) can be rewritten as the field-aligned, guiding-center kinetic equation (Khazanov 1979; Khazanov et al. 1994)

$$\frac{\beta}{\sqrt{E}}\frac{\partial \Phi}{\partial t} + \mu\frac{\partial \Phi}{\partial s} - \frac{1-\mu^2}{2}\left(\frac{1}{B}\frac{\partial B}{\partial s} - \frac{F}{E}\right)\frac{\partial \Phi}{\partial \mu} + EF\mu\frac{\partial \Phi}{\partial E} = \langle S_e \rangle + \langle q_e \rangle, \qquad (2.5.1)$$

which calculates the time-dependent distribution of the differential flux function, $\Phi = 2Ef/m^2$, as a function of time, distance along the field line, energy, and pitch angle. In (2.5.2), t is time; s is the distance along the field line; E is the particle energy; and μ is the cosine of the pitch angle. The inhomogeneity of the geomagnetic field, B, is included, as well as other forces, such as electric fields, in F. $\langle S_e \rangle$ and $\langle q_e \rangle$ are averaged over the azimuth angle; the superthermal electron collision terms, representing interactions with thermal electrons and ions, elastic and inelastic scattering with neutral particles that have been described in Sects. 2.2 and 2.3; and the superthermal electron source presented in the previous section. Combining all collision terms, the kinetic equation (2.5.2) can be written as (Khazanov 1979; Khazanov et al. 1994).

$$
\begin{aligned}
&\frac{\beta}{\sqrt{E}}\frac{\partial \Phi}{\partial t}+\mu\frac{\partial \Phi}{\partial s}-\frac{1-\mu^2}{2}\left(\frac{1}{B}\frac{\partial B}{\partial s}-\frac{F}{E}\right)\frac{\partial \Phi}{\partial \mu}+EF\mu\frac{\partial \Phi}{\partial E}\\
&=An_e\left\{\frac{\partial}{\partial E}\left[\left(1+\frac{m}{M_i}+2\frac{m}{M_n}\frac{\sigma_{en}^{tr}E^2 n_n}{An_e}\right)\frac{\Phi}{E}\right.\right.\\
&\quad\left.+\left(T_e+T_i\frac{m}{M_i}+2\frac{m}{M_n}T_n\frac{\sigma_{en}^{tr}E^2 n_n}{An_e}\right)\frac{\partial}{\partial E}\left(\frac{\Phi}{E}\right)\right]\\
&\quad\left.+\frac{1}{2E^2}\frac{\partial}{\partial \mu}\left[\left(1-\mu^2\right)\frac{\partial \Phi}{\partial \mu}\right]\right\}+n_n\left\{\int_0^{2\pi}\int_{-1}^{1} I_{en}\left(E,\mu_\chi\right)\Phi(\mu')\sin\chi d\chi d\varepsilon\right.\\
&\quad+\sum_{j,k>j}\left[\Delta_n^j\sigma_{en}^{jk}\left(E+E_{jk}\right)\Phi\left(E+E_{jk}\right)+\Delta_n^k\sigma_{en}^{kj}\left(E-E_{jk}\right)\Phi\left(E-E_{jk}\right)\right]\\
&\quad+\int_{E+E_n^+}^{2E+E_n^+} I_n^+\left(E,'E'-E-E_n^+\right)\Phi(E')dE'\\
&\quad+\frac{1}{2\pi}\int_{2E+E_n^+}^{\infty} I_n^+(E,'E)\left[\int_0^{2\pi}\Phi\left(E',\sqrt{1-\mu^2}\cos\varepsilon\right)\right]dE'\\
&\quad\left.-\left[\sigma_{en}+\sum_{j,k>j}\left(\Delta_n^j\sigma_{en}^{jk}+\Delta_n^k\sigma_{en}^{kj}+\sigma_n^++\frac{n_i}{n_n}\sigma_i^r\right)\right]\Phi\right\}+q. \qquad (2.5.2)
\end{aligned}
$$

Here, a summation is made over all types of ions and neutral particles; for shortening of the equation, only the current arguments for all functions have been used, and the following notations are introduced

$$\Delta_n^{j(k)}=\frac{n_n^{j(k)}}{n_n},\quad \sigma_{en}(E)=\int_0^{2\pi}\int_{-1}^{1} I_{en}\left(E,\mu_\chi\right)d\mu_\chi d\varepsilon.$$

When describing Coulomb interaction of superthermal electrons with charged particles, the Landau collision integrals have been presented in a linearized form, assuming that the thermal electrons and ions can be described by the Maxwellian distribution function (see Sect. 2.2). Expressions describing inelastic collisions are also derived using a number of well-known approximations (Khazanov 1979; Khazanov et al. 1994), assuming that the dissipating (initial) electron does not change direction (is forward scattered), and if ionization occurs, that the secondary electron is emitted perpendicular to the velocity of the initial electron before the collision.

Equation (2.5.2) can be further simplified depending on its application to the different space plasma phenomena. This is discussed in the proceeding chapters.

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http://www.springer.com/978-1-4419-6796-1

Kinetic Theory of the Inner Magnetospheric Plasma
Khazanov, G.V.
2011, XX, 584 p., Hardcover
ISBN: 978-1-4419-6796-1