
Preface to the Second Edition

The changes from the first edition to the second have two sources: the many helpful suggestions the author has received from colleagues, reviewers, students and others who took the time and effort to contact me, and the author's experience teaching with this text in the years since the first edition was published.

Though the bulk of the text has remained unchanged from the first edition, there are a number of changes, large and small, that will hopefully improve the text. As always, any remaining problems are solely the fault of the author.

Changes from the First Edition to the Second Edition

- (1) A new section about the foundations of set theory has been added at the end of Chapter 3, about sets. This section includes a very informal discussion of the Zermelo–Fraenkel Axioms for set theory. We do not make use of these axioms subsequently in the text, but it is valuable for any mathematician to be aware that an axiomatic basis for set theory exists. Also included in this new section is a slightly expanded discussion of the Axiom of Choice, and new discussion of Zorn's Lemma.
- (2) Chapter 6, about the cardinality of sets, has been rearranged and expanded. There is a new section at the start of the chapter that summarizes various properties of the set of natural numbers; these properties play important roles subsequently in the chapter. The sections on induction and recursion have been slightly expanded, and have been relocated to an earlier place in the chapter (following the new section), both because they are more concrete than the material found in the other sections of the chapter, and because ideas from the sections on induction and recursion are used in the other sections. Next comes the section on the cardinality of sets (which was originally the first section of the chapter); this section gained proofs of the Schroeder–Bernstein theorem and the Trichotomy Law for Sets, and lost most of the material about finite and countable sets, which has now been moved to a new section devoted

to those two types of sets. The chapter concludes with the section on the cardinality of the number systems.

- (3) The chapter on the construction of the natural numbers, integers and rational numbers from the Peano Postulates was removed entirely. That material was originally included to provide the needed background about the number systems, particularly for the discussion of the cardinality of sets in Chapter 6, but it was always somewhat out of place given the level and scope of this text. The background material needed for Chapter 6 has now been summarized in a new section at the start of that chapter, making the chapter both self-contained and more accessible than it previously was. The construction of the number systems from the Peano Postulates more properly belongs to a course in real analysis or in the foundations of mathematics; the curious reader may find this material in a variety of sources, for example [Blo11, Chapter 1].
- (4) Section 3.4 on families of sets has been thoroughly revised, with the focus being on families of sets in general, not necessarily thought of as indexed.
- (5) A new section about the convergence of sequences has been added to Chapter 7. This new section, which treats a topic from real analysis, adds some diversity to Chapter 7, which had hitherto contained selected topics of only an algebraic or combinatorial nature.
- (6) A new section called “You Are the Professor” has been added to Chapter 8. This new section, which includes a number of attempted proofs taken from actual homework exercises submitted by students, offers the reader the opportunity to solidify her facility for writing proofs by critiquing these submissions as if she were the instructor for the course.
- (7) The notation for images and inverse images of sets under a function, defined in Section 4.2, has been changed from the non-standard notation $f_*(P)$ and $f^*(Q)$ used in the first edition to the standard notation $f(P)$ and $f^{-1}(Q)$, respectively. Whereas the author still finds the notation used in the first edition superior in terms of avoiding confusion with inverse functions, he has deferred to requests from colleagues and reviewers to switch to the standard notation, with the hope that any confusion due to the standard notation will be outweighed by the benefit for students in preparing to read mathematical texts that use the standard notation.
- (8) All known errors have been corrected.
- (9) Many minor adjustments of wording have been made throughout the text, with the hope of improving the exposition.

Errors

Although all known errors from the first edition have been corrected, there are likely to be some remaining undetected errors, and, in spite of the author's best effort, there are likely to be some errors in the new sections and revisions of older material that were written for the second edition. If the reader finds any such errors—which will hopefully be few in number—it would be very helpful if you would send them to the author at bloch@bard.edu. An updated list of errors is available at http://math.bard.edu/bloch/proofs2_errata.pdf.

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