Preface

Fixed point theory is one of the most powerful and fruitful tools of modern mathematics and may be considered a core subject of nonlinear analysis. In recent years a number of excellent monographs and surveys by distinguished authors about fixed point theory have appeared such as, e.g., [2, 4, 7, 25, 31, 32, 100, 101, 103, 104, 108, 155, 196]. Most of the books mentioned above deal with fixed point theory related to continuous mappings in topological or even metric spaces (work of Poincaré, Brouwer, Lefschetz–Hopf, Leray–Schauder) and all its modern extensions.

This book focuses on an order-theoretic fixed point theory and its applications to a wide range of diverse fields such as, e.g., (multi-valued) nonlocal and/or discontinuous partial differential equations of elliptic and parabolic type, differential equations and integral equations with discontinuous nonlinearities in general vector-valued normed spaces of non-absolutely integrable functions containing the standard Bochner integrable functions as special case, and mathematical economics and game theory. In all these topics we are faced with the central problem of handling the loss of continuity of mappings and/or missing appropriate geometric and topological structure of their underlying domain of definition. For example, it is noteworthy that, in particular, for proving the existence of certain optimal strategies in game theory, there is a need for purely order-related fixed point results in partially ordered sets that are neither convex nor do they have lattice structure and where the fixed point operator lacks continuity.

The aim of this monograph is to provide a unified and comprehensive exposition of an order-theoretic fixed point theory in partially ordered sets and its various useful interactions with topological structures. A characteristic feature of this fixed point theory, which is developed in detail in Chap. 2, is that it is based on an abstract recursion principle, called the Chain Generating Recursion Principle, which was formulated in [112, 133], and which is the common source of all the order-related fixed point results obtained in this book. In particular, the developed fixed point theory includes the classical order-theoretic fixed point result established by Knaster in [153], which was later extended by Tarski in [215], as well as the fixed point theorems due to Bourbaki and Kneser (cf. [228, Theorem 11.C]) and Amann (cf. [228, Theorem 11.D]). Surprisingly enough, very recently, the classical and seminal Knaster–Tarski fixed point theorem has been applied to computational geometry in [195]. This unexpected application emphasizes even more the importance of an order-theoretic fixed point theory.

Chapter 1 serves as an introduction to the subject and discusses some simple examples of the order-theoretic fixed point results along with simple applications from each of the diverse fields. This will help the reader to get some idea of the theory and its applications before entering the systematic study. Chapter 3 provides preliminary results on multi-valued variational inequalities regarding the topological and order-theoretical structure of solution sets. This chapter, which may be read independently, is of interest on its own and contains new results. Our main emphasis is on Chaps. 4–8 where we demonstrate the power of the developed fixed point theory of Chap. 2, which runs like a thread through the entire book. Attempts have been made to attract a broad audience not only by the diverse fields of applications, but also by emphasizing simple cases and ideas more than complicated refinements. In the treatment of the applications, a wide range of mathematical theories and methods from nonlinear analysis and integration theory are applied; an outline of which has been given in an appendix chapter to make the book self-contained.

This book is an outgrowth of the authors' research on the subject during the past 20 years. However, a great deal of the material presented here has been obtained only in recent years and appears for the first time in book form.

We expect that our book will be accessible and useful to graduate students and researchers in nonlinear analysis, pure and applied mathematics, game theory, and mathematical economics.

We are most grateful to our friends and colleagues who contributed through joint works and papers to the preparation of this book. Rather than inadvertently leaving someone out, we have not listed the names, but we hope our friends and collaborators will be satisfied with our thanks.

Finally, we wish to express our gratitude to the very professional editorial staff of Springer, particularly to Vaishali Damle for her effective and productive collaboration.

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September 2010

http://www.springer.com/978-1-4419-7584-3

Fixed Point Theory in Ordered Sets and Applications
From Differential and Integral Equations to Game Theory
Carl, S.; Heikkilä, S.
2011, XIII, 477 p., Hardcover
ISBN: 978-1-4419-7584-3