2
Principal Features of the Sky

2.1. Star Patterns: Asterisms and Constellations

2.1.1. Stellar Pattern Recognition

About 15,000 stars are detectable by the human eye, most of them near the limit of visibility. At any one time, we may be able to see a few thousand stars in a dark sky, but we tend to remember only striking patterns of them—asterisms such as the Big Dipper or whole constellations such as Ursa Major (the Big Bear) or Orion (the name of a mythological hunter)—and so it has been for millennia. Today, the entire sky has been divided into constellations; they are not defined according to appearance alone but according to location, and there are no boundary disputes. The modern names and locations are more or less those of Argelander (1799–1875) for the Northern Hemisphere and John Herschel (1824–1896) for the Southern, but the present divisions of the constellations[1] were adopted by the International Astronomical Union (IAU), the chief authority on such matters as astronomical nomenclature, in 1930. The IAU has established 88 constellations in the sky; many reflecting an ancient heritage.

The names of the constellations recognized in antiquity were based on

- Mythological figures
- Animals or inanimate objects as perceived in the sky
- Geographical or political analogues
- Associations with seasonal phenomena, or some other basis

As we will show in later chapters, non-Western traditions have perceived a rich variety of star patterns; some include the *absence* of stars, the "dark constellations."[2] Chinese constellations were different from and far more numerous than were those of the Mediterranean area. As far as we are aware, the oldest extant Chinese star chart on paper is contained in a 10th-century manuscript from Dunhuang, but there is far older evidence for sky charting from this area of the world (see §10 and §2.2.3); a compilation by Chhien Lu-Chih listed 284 constellations containing a total of 1464 stars and is said to be based on a Han catalogue (see §10.1.2.3; and Yi, Kistemaker, and Yang (1986) for new maps and a review of historical Chinese star catalogues).

Western constellations in current use largely derive from ancient Mediterranean sources, mainly the Near East and Greece, as we show in §7. The earliest surviving detailed description of the Greek constellations is in the poem *Phaenomena* by the Greek poet Aratos (*Aratus* in the Roman sources), ~250 B.C. (Whitfield 1995, p. 23). The constellations portrayed in the poem derive from a work also called *Phaenomena*, which has not survived, by the Greek astronomer Eudoxos (or Eudoxus) (4th century B.C.). One of the later sources that discusses this work is that of the sole remaining manuscript of Hipparchos (~150 B.C.), one of the greatest astronomers of antiquity. Many of the constellations can be seen as raised images on the Farnese Globe, the oldest extant celestial globe, dated to the 2nd century B.C., but representing a copy of an older work. Aratos mentioned 47 constellations, whereas Claudius Ptolemy (~150 A.D.), the source of much of our knowledge about Hipparchos, referred to 48 in the major astronomical work that we know today as the *Almagest*.

In ancient Greek usage, the constellations *were* the figures. For example, in the constellation of Cassiopeia, the star ζ Cassiopeiae (abbreviated ζ Cas) is described as "the star on the head"; α Cas, as "the star in the breast"; and

[1] Boundaries are along coordinates of right ascension and declination referred to the equinox of 1875.0. See sections below for explanations of these terms.

[2] See §14.2.5, for a Peruvian example.

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η Cas as "the star over the throne, just over the thighs." In Perseus, the variable star Algol (β Per) is described as the "bright one" in the "Gorgon's head." Not all naked eye stars fitted neatly into these groupings, so many stars were omitted from the constellations. Those outside the accepted figures were referred to as "unformed" (αμόρφωτοι; our word "amorphous" derives from a related word), or "scattered" (σποράδες, related to the Greek word for seed, σπορά, broadcast during sowing, and our cognate word, "sporadic"). The IAU reorganization created constellation "homes" for these "unformed" stars.

2.1.2. Star Charts

The depictions of the Greco-Roman constellations as they were known in Ptolemy's time (~150 A.D.) were preserved in Arabic sources, one of the best known being that of the astronomer al-Sūfī (10th century). R.H. Allen (1963) states that the sky representations of post-Renaissance Europe derive from those of Albrecht Dürer (1471–1528) of 1515 (Figure 1.1), in which the star positions from Ptolemy's catalogue were set down by another resident of Nürnberg (Nuremberg), a mathematician named Heinvogel. The positions were subsequently improved and more stars added, but the figures of Dürer essentially remained the same through the charts of Bayer (1603), Flamsteed (1729), and Argelander (1843). More details about star charts from 1500 to 1800 can be found in Warner (1979), and an even wider range of charts is found in Stott (1991/1995) and Whitfield (1995).

The representations of the more obvious asterisms differed widely from culture to culture. A familiar example is the Big Dipper, still known in England as the plough, and in Germany and Scandinavia as the *Wagen* (wagon). In the Roman republic, it was the plow oxen. On many pre-19th-century maps and star charts, the term *Septentrion* or some variety of this term appears. The expression became synonymous with the North, or northern regions, but originally meant the seven plow oxen. R.H. Allen (1963) says that the Big Dipper was known as a coffin in parts of the Mideast, a wagon or bear in Greece, and a bull's thigh in pre-Hellenistic Egypt. Systematic attempts were made to rename the constellations at various times. Giordano Bruno (1548–1600) sought to invest the sky with figures representing Moral Virtues. Julius Schiller of Augsburg produced the most widely known type of Bible-inspired charts in 1627. R.H. Allen's (1963) encyclopedic search into the origins of star names and constellations reveals several other European attempts to recast the constellations, although the various sources used by him are not always treated critically.

2.1.3. Modern Nomenclature

Today, *constellations* refer to specified areas on the celestial sphere, whereas an *asterism* is any apparent grouping of stars. Indeed, one could be forgiven for describing the ancient "constellations" as asterisms. With some exceptions, in modern usage, an asterism is usually smaller than a constellation; for example, the Little Dipper asterism is in the constellation of Ursa Minor, the Little Bear, and the Pleiades is a well-known asterism in the constellation Taurus, the Bull. An exception is the Summer Triangle, composed of the bright stars Vega, Deneb, and Altair in the constellations Lyra, Cygnus, and Aquila, respectively. Even a single star may constitute an asterism. The star Spica, for example, the brightest star in the constellation of Virgo, has been envisaged as a spike of wheat.

Modern common names of naked eye stars, derive from European and Arabic usage, as well as proper names devised by Johann Bayer in 1603. The Bayer designations use lowercase Greek letters and, after these are exhausted, small Roman letters, to identify stars in a given constellation, for example, u Herculis or i Bootis. When these were exhausted, capital Roman letters were used. The lettered type of designation was later extended to the Southern Hemisphere by Nicolas Louis de Lacaille (1763) and John Herschel (1847). The Greek letters are universally accepted, but an alternative designation to the Bayer letters for the fainter stars is that of the Flamsteed numbers (Flamsteed 1725, Vol. 3), as, for example, 44 Bootis = i Bootis. Giuseppe Piazzi (1803) also published star catalogues in 1803 and 1814 (see Piazzi/Foderà Serio 1990). The Flamsteed numbers increase with right ascension, a coordinate that increases from west to east (see §2.2.3). Many catalogues of stars and other objects use positional or sequence numbers, usually increasing with right ascension. The best known star catalog of this kind is the Bright Star Catalog (Hoffleit 1982), which uses the positional sequence numbers of the Harvard Revised Photometry Catalog (Pickering 1908); thus, BS 7001 = HR 7001 = α Lyrae.

Usually, the Greek letter designates the relative brightness of the star within the constellation, but occasionally they were assigned to a positional sequence, as in Ursa Major. In the list of modern constellations, Table 2.1, the star names are in Latin, with the historically earliest names referring to Latin forms of Greek originals. The columns contain both nominative and possessive[3] cases of the names, English equivalents, notable stars and other objects, and both modern and ancient asterisms that are within the modern boundaries. Only a few objects that cannot be seen unaided under clear and dark sky circumstances are included.

"Double stars" are stars that appear close to each other in the sky; sometimes they are indeed physically close to each other and interact gravitationally, but not always. The pair of stars Mizar and Alcor (ζ Ursae Majoris and 80 Ursae Majoris, respectively), in the handle of the Big Dipper, is an example of a naked-eye double.

Types of "variable stars" are named after their prototypes, such as delta Cephei or RR Lyrae. In the Bayer designations, no visible star had been assigned a letter later in the alphabet than Q; consequently, Argelander suggested that designations of *R* and later would be used solely for variable stars. This scheme has been followed dogmatically to a logical conclusion ever since. When designations to Z became

[3] The possessive or genitive case is used in formal star names, e.g., α *Canis Majoris*, β *Scorpii*, β *Lyrae*, or *S Doradus*, literally, the stars labeled α of the constellation Canis Major, β of Scorpius, and so on.

Table 2.1. Modern constellations.

Name	Meaning	Possessive[a]	Asterisms/features
Andromeda	Mythological figure (chained lady)	Andromedae	Spiral galaxy M31.
Antlia	Air pump	Antliae	
Apus	Bird of paradise	Apodis (Aps)	
Aquarius	Water bearer	Aquarii (Aqr)	Planetary nebula NGC 7293.
Aquila	Eagle	Aquilae (Aql)	Vultur volans ($\alpha + \beta + \gamma$ Aql); Altair = α Aql part of the "summer triangle".
Ara	Altar	Arae	
Argo[b]	Jason's ship		
Aries	Ram	Arietis	
Auriga	Charioteer	Aurigae	Goat and kids; Capella = α Aur, goat star,
Boötes	Herdsman	Boötis	Arcturus = α Boo, bear keeper, Job's star.
Caelum	Sculptor's chisel	Caeli	
Camelopardalis	Giraffe	Camelopardalis	
Cancer	Crab	Cancri (Cnc)	M44 = the beehive, open star cluster.
Canes Venatici	Hunting dogs	Canum Venaticorum (CVn)	
Canis Major	Big dog	Canis Majoris	Sirius = α CMa, dog star, Isis; M41 open star cluster.
Canis Minor	Small dog	Canis Minoris	Procyon = α CMi.
Capricornus	Ibex/goat-fish	Capricorni	
Carina	Argo's keel	Carinae	Eta Car, unstable variable star & nebula; NGC 2516, IC 2602 star clusters.
Cassiopeia	Mythological figure (lady in the chair, mother of Andromeda)	Cassiopeiae	The "W." Tycho's supernova.
Centaurus	Centaur	Centauri	ω Cen globular cluster.
Cepheus	Mythological figure (king, husband of Cassiopeia)	Cephei	δ Cephei variable star.
Cetus	Whale	Ceti	Mira = o Ceti, variable star.
Chamaeleon	Chamaeleon	Chamaeleontis	
Circinus	Pair of compasses	Circini	
Columba	Dove	Columbae	
Coma Berenices	Berenices's hair	Comae Berenices	Melotte 111, cluster.
Corona Australis	Southern crown	Coronae Australis (CrA)	
Corona Borealis	Northern crown	Coronae Borealis (CrB)	
Corvus	Raven	Corvi (Crv)	
Crater	Cup	Crateris (Crt)	
Crux	Cross	Crucis	Coal Sack (dark nebula); Southern Cross.
Cygnus	Swan, Orpheus	Cygni	Northern Cross; "great rift" (dark nebulae); Deneb = α Cyg, part of "summer triangle."
Delphinus	Dolphin	Delphini	
Dorado	Doradus fish	Doradus	Large Magellanic Cloud; 30 Dor = Tarantula Nebula.
Draco	Dragon	Draconis	
Equuleus	Foal	Equulei	
Eridanus	Mythological river Po River	Eridani	
Fornax	Furnace	Fornacis	
Gemini	Twins	Geminorum	Castor = α Gem, Pollux = β Gem.
Grus	Crane	Gruis	
Hercules	Mythological figure (kneeler, son of Zeus)	Herculis	"keystone"; M13, globular cluster.
Horologium	Clock	Horologii	
Hydra	Water snake	Hydrae (Hya)	
Hydrus	Small water snake	Hydri (Hyi)	
Indus	North American Indian	Indi	
Lacerta	Lizard	Lacertae	
Leo	Lion	Leonis	Regulus = α Leo regal (kingly) star.
Leo Minor	Small lion	Leonis Minoris (LMi)	
Lepus	Hare	Leporis	
Libra	Balance scale	Librae	
Lupus	Wolf	Lupi	
Lynx	Lynx, tiger	Lyncis	
Lyra	Lyre, harp of Orpheus	Lyrae	Vega = α Lyr, part of "summer triangle."
Mensa	Table	Mensae	
Microscopium	Microscope	Microscopii	
Monoceros	Unicorn	Monocerotis	
Musca (Apis)	Fly (bee)	Muscae	

TABLE 2.1. *Continued.*

Name	Meaning	Possessive[a]	Asterisms/features
Norma	Level, rule	Normae	
Octans	Octant	Octantis	South Celestial Pole.
Ophiuchus	Snake bearer	Ophiuchi	Kepler's supernova.
Orion	Myth. figure (giant hunter)	Orionis	Great Nebula (M42); belt stars; Betelgeuse = α Ori, red, variable.
Pavo	Peacock	Pavonis	
Pegasus	Winged horse	Pegasi	The Great Square.
Perseus	Mythological figure	Persei	(rescuer of Andromeda) χ h Persei Double cluster; Algol = β Per, var. star = head of Medusa, Gorgona.
Phoenix	Myth. bird	Phoenicis (Phe)	
Pictor	Easel	Pictoris	
Pisces	Fishes	Piscium (Psc)	
Piscis Australis (or Austrinus)	Southern fish	Piscis (PsA) Australis (or Austrini)	Fomalhaut = α PsA.
Puppis	Argo's stern	Puppis	M47 open star cluster.
Pyxis	Argo's compass	Pyxidis	
Reticulum	Net	Reticuli	
Sagitta	Arrow	Sagittae (Sge)	
Sagittarius	Archer	Sagittarii	Teapot; M25 open star cluster; M8 nebula; M17 nebula & star cluster.
Scorpius (or Scorpio)	Scorpion	Scorpii	Antares = α Sco; M7, NGC 6231 open star clusters.
Sculptor	Sculptor's studio	Sculptoris (Scl)	
Scutum	Shield	Scuti (Sct)	Ω Nebula; star clouds.
Serpens	Serpent	Serpentis	
Sextans	Sextant	Sextantis	
Taurus	Bull	Tauri	Hyades, Pleiades star clusters; supernova remnant, Crab Nebula near ζ Tau.
Telescopium	Telescope	Telescopii	
Triangulum	Triangle	Trianguli	Spiral galaxy M33.
Triangulum Australe	Southern triangle	Trianguli Australis (TrA)	
Tucana	Toucan	Tucanae	Small Magellanic Cloud; 47 Tuc globular cluster.
Ursa Major	Big bear	Ursae Majoris (UMa)	Big Dipper; horse and rider = ζ + 80 UMa.
Ursa Minor	Small bear	Ursae Minoris (UMi)	Little Dipper; North Star = Pole Star = Polaris = α UMi.
Vela	Argo's sails	Velorum	IC 2391 open star cluster.
Virgo	Young girl	Virginis	Spica = α Vir.
Volans	Flying fish	Volantis	
Vulpecula	Fox	Vulpeculae	

[a] The standard abbreviations are the first three letters; where this is not the case, the abbreviation is given.
[b] Ancient but now defunct constellation, sometimes called Argo Navis, now divided into Carina, Puppis, Pyxis, and Vela.

exhausted, the sequence began again with RR, and proceeded through the sequences, RS, RT, . . . , RZ, SS, . . . , SZ, . . . ZZ, AA, . . . , AZ, . . . , . . . , QZ. At this point, the naming scheme switches to V335, V336, . . . , and so on. See §5.8 for a discussion of the various types of variable stars.

Some asterisms are "nebulae" (clouds) because of their diffuse appearance. A nebula may be a real dust or gas cloud (in space!), a star cluster, or a distant galaxy. Gas and dust clouds, usually illuminated by bright stars embedded in them, are also represented among the asterisms. Examples include the Orion Nebula (M42) and the η Carinae nebula. "Star clusters" are families of stars that were born near the same location in space, travel on parallel orbits around the Galaxy, and generally have similar chemical compositions. There are two types of star clusters: open (also called "galactic") and globular clusters. Open clusters, typically, are located in or near the Milky Way, are irregular in shape, and are composed of hundreds of stars. Examples are the *Pleiades* and the *Hyades* clusters in Taurus and the "Beehive" cluster (also called *Praesepe* or M44) in Cancer. Globular clusters are more widely distributed around the sky, appear spherical in shape, and are composed of hundreds of thousands of stars. Examples are M13 in Hercules, and 47 Tucanae. Finally, there are the galaxies beyond the Milky Way that can be perceived by the naked eye and thus could be considered asterisms, such as "M31" in Andromeda and the *Large* and *Small Magellanic Cloud*s. The "M" designations in some of our examples are entries in the Messier Catalogue, a collection of nonstellar objects compiled by Charles Messier (1730–1817), a noted comet discoverer of his time. The purpose of the compilation was to avoid false identifications of new comets with diffuse-looking objects in the sky, with which they could be confused in small telescopes.

Figures B.1 and B.2 in Appendix B place the modern constellations and asterisms on the sky in a coordinate framework, provided for general reference. Figure B.1 is bisected by the *celestial equator* into northern and southern halves. The chart is a Mercator projection[4] of a variant of the equatorial system, one way of viewing the celestial sphere independently of the observer. Figure B.2 provides views of the regions around the north and south celestial poles.

Star charts, regardless of the superimposed constellation and asterism associations, are most useful when they permit identification of precise positions in the sky. Stott (1991/1995, p. 9) informs us that the first (Western) star atlas with sets of (modern) stellar coordinates was that of Paolo Galluci (from 1588). In this case, the coordinates were with respect to the path of the Sun, the *ecliptic* (see §2.3.3 for a discussion of this system of coordinates). Chinese atlases and charts used measurements somewhat akin to hour angles measured from the beginnings of *xius* (lunar mansions), and polar distance angles much earlier than this (Needham/Ronan 1981 = Needham 1981a, p. 116). Even in the *Almagest*, Ptolemy gives a position of a star in a kind of ecliptic coordinate; referring to the beginning of the first point of a zodiacal sign, he also gives an ecliptic latitude. Moreover, Ptolemy describes a device (see §3.3) with which some coordinates can be measured, and the existence of some kind of spherical coordinates is implied by relatively accurate placements of stars on the external surface of a sphere, such as the Farnese globe (§2.1.1). Yet when Galileo noticed a faint object while studying the satellites of Jupiter, he was unable to track and follow the object because his telescope mounting lacked coordinates to record and rediscover it once Jupiter's relatively large motion had moved away from the field. The faint object was not knowingly discovered until after calculations by John Couch Adams (1819–1892) and Urbain Jean Joseph Leverrier (1811–1877) in the 19th century. The object was the planet Neptune. If Galileo had obtained access to some of the classic instruments of antiquity, he could have replaced a sighting tube with his telescope and been able to record positions relative to the nearby stars.

In the following sections, we will show how coordinate systems enable us to find objects on the celestial sphere, in catalogues, and in the sky.

[4] This is a projection of spherical coordinates onto a cylinder in such a way that lines of latitude and longitude remain perpendicular. It has the property that longitude lines farther from the equator enclose larger areas. The projection is credited to the Flemish cartographer, Gerhardus Mercator (1512–1594).

2.2. The Sphere of the Sky

2.2.1. Daily Sky Motions

Time exposure photography of the sky readily reveals the movement of the sky. Uniform exposures (say, one hour each) under a cloudless sky at each of the cardinal facings will confirm the impression of the unaided eye—that the stars wheel about a hub at constant angular rate. Figure 2.1 shows typical *diurnal* (daily) arcs traced out by stars during such exposures. Traced with a stylus on a graphics tablet, the arc lengths can be shown to be systematically larger with increased angular distance from the center of motion—the *celestial pole*. The longest arcs are 90° from the celestial pole—on what is called the *celestial equator*, which divides the sky into northern and southern halves.

The apparent direction of turning is *counterclockwise*—as we view the North Celestial Pole. It is *clockwise* for Southern hemisphere observers viewing the South Celestial pole. The motions are consistent. As one faces North, the stars rise in arcs from one's right hand and set at one's left hand. Facing South, they rise at the left hand and set at the right hand. The observations imply that either the sky is rotating from East to West above the earth or that the earth is rotating from West to East below the sky.

In antiquity, which condition was true was the subject of much discussion and, in the end, could not be determined definitively. In the absence of a knowledge of the correct physics, misinterpretations of common experience gave many writers the idea that a rotating earth would force unanchored objects to be thrown off (see Chapter 7, especially §7.2).

Although the sense of the turning sky is the same all over the earth, the diurnal arcs have a different character for observers at the equator compared to those nearer the poles. For an observer on the equator, the North and South Celestial poles are on opposite sides of the sky; all stars rise at right angles to the horizon and move across the sky in semicircles, spending half the time above, and half the time below, the horizon. For observers elsewhere, stars that have diurnal circles between the pole and the horizon below it do not rise or set. They are called *circumpolar* stars. Stars equally distant from the opposite pole never appear above the horizon. In modern parlance, these two regions are called the north and south *circumpolar zones*, respectively. The diurnal arcs of stars that rise and set make acute angles (<90°) with the horizon, and this angle becomes smaller with the observer's proximity to the pole. At the North and South Poles, this angle becomes 0°, as the stars move in circles that are concentric with the horizon and are circumpolar. At the equator, it is 90° for all stars, and none are circumpolar.

The notion that the heavens constitute a great sphere surrounding the observer is an ancient one. It seems likely to have been present among the early Pythagoreans. It is associated with the Ionian Greeks, especially Eudoxos of Cnidus who lived in the 4th century B.C. It was known in China by the 2nd century B.C. The heavens were sometimes depicted as an external sphere, such as that shown in the Etruscan depiction of Atlas holding up the sky sphere. Not every culture, however, depicted the sky as a hemispherical bowl;

FIGURE 2.1. Diurnal arcs traced out by stars during a time exposure near the North Celestial Pole. Trails further from the pole appear straighter because the radii of curvature of their diurnal circles is larger. Photo courtesy of T.A. Clark.

in ancient Egypt, the sky was pictured as the body of the goddess *Nut*, for example. The shape of the sky as we perceive it depends on several factors: physiological, psychological, and cultural. We can even measure the perceived shape (see Schlosser et al. 1991/1994, pp. 1–3). For the purposes of locating objects on the sky, however, we use, even today, the concept of the celestial sphere.

2.2.2. The Horizon or "Arabic" System

The image of an Earth surrounded by pure and perfect crystalline spheres[5] was emphasized by Aristotle, among others. Astronomers have made continual use of this image for more than two millennia; we refer to a *celestial sphere*, on which all objects in the sky appear, at any given instant, to be fixed. It does not matter in the slightest that such a sphere is borne of perception only, or that it exists only in our imagination. Everything that undergoes diurnal motion is assumed to lie on this sphere; the consequence is that they are assumed to be at the same distance from the observer. This is not strictly true, of course, but for locating very distant objects on the celestial sphere, it is a reasonable approximation. To the naked eye, the Moon is the only one of all the permanent bodies in the sky that seems to shift position among the stars as an observer shifts from one place on Earth to another.[6] For nearer objects, such as the Sun, Moon, and planets, relative motions on the sky can be studied and the predicted positions tabulated for each day, as, for example, in Babylon and Ur (see §7.1). This means that only two coordinates suffice to describe the position of an object on the surface of such a sphere.

On the celestial sphere, we will place the markings of the *horizon* system. We also refer to this system as the *Arab* system, because it was in wide use in the Arab world during the European Dark Ages. Not all the terms currently used in the English description of it stem directly from the Arabic language. Its salient features are indicated and labeled in Figure 2.2, which also includes relevant elements of the equatorial system which is described in §2.2.3.

The highest point, directly overhead, is the *zenith*, a name that reaches us through Spain (*zenit*) and the Arab world of the Middle Ages (*samt ar-ra's*, road (over) the head). Directly below, unseen, is the *nadir* (Arabic *nazir as-samt*, opposite the zenith). The zenith and the nadir mark the *poles* of the horizon system. The *horizon*, which comes from a Greek word meaning to separate, basically divides the earth from the sky. We adopt the modern definition here: The *astronomical horizon* is the intersection with the celestial sphere of a plane through the observer and perpendicular

[5] Indeed, ancient Greek astronomers held that the motions of "wandering stars" or planets could be explained with the turnings of many such transparent spheres. See §7.2.3.

[6] From the place where the Moon appears overhead to the place where it appears on the horizon, the Moon appears to shift by about 1° with respect to the stars. The shift is called the *horizontal parallax*. Parallax shifts are very important in astronomy and are a primary means of determining astronomical distances.

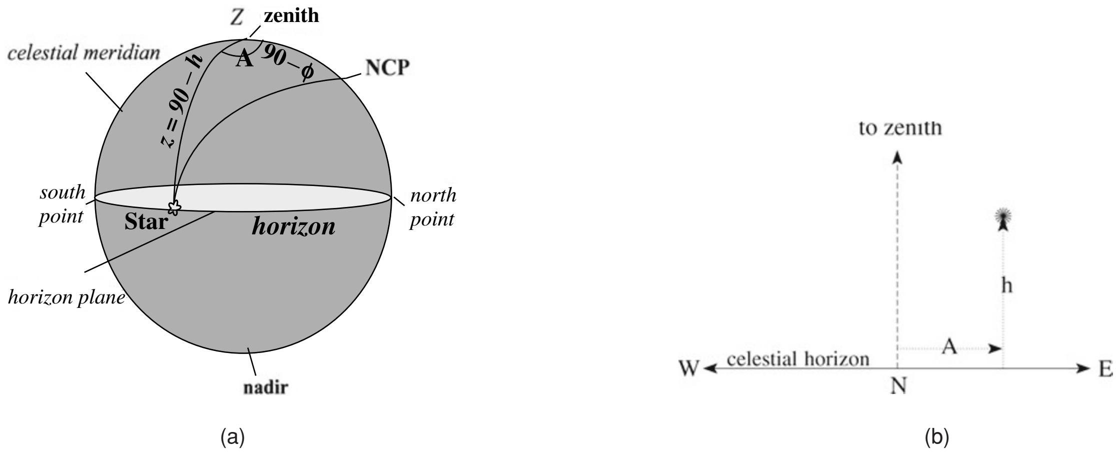


FIGURE 2.2. The horizon system: The main features of the horizon system of spherical astronomical coordinates. (a) The outside-the-sphere view. The azimuth coordinate, A, is represented as a polar angle measured at the zenith; A is measured eastward or clockwise (looking down from outside the sphere) from the *north point* of the horizon. An observer facing any direction on the horizon sees the azimuth increasing to the right. The north point is defined as the intersection of the vertical circle through the north celestial pole, *NCP*, and the horizon. The zenith distance, z, is shown as an arc length measured down from the zenith along a vertical circle through the star; z may be measured also as an angle at the center of the sphere. An alternative coordinate is the altitude, h, measured up from the horizon along the vertical circle. (b) The observer's view. The azimuth also can be measured as an arc along the horizon; it is equivalent to the angle measured at the center of the sphere between the North point of the horizon and the intersection of the horizon and a vertical circle through the star. Drawings by E.F. Milone.

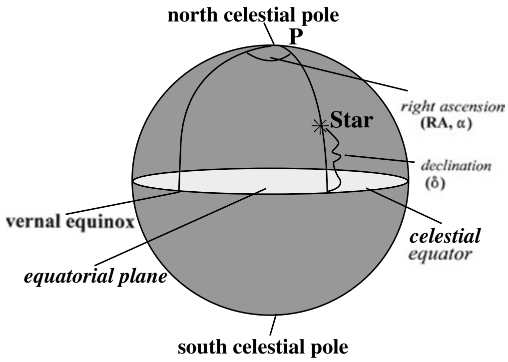
to the line between the observer and the zenith. A family of circles (*vertical circles*) may be drawn through the zenith and the nadir. The centers of these circles must be the sphere's center, where the observer is located (for the time being, we ignore the distinction between the center of the Earth and the observer, i.e., the difference between what modern astronomers call the geocentric and the topocentric systems, respectively). Degrees of *altitude* are measured up from the horizon toward the zenith along a vertical circle to the object. This gives us one of the two coordinates needed to establish a position on the celestial sphere. The other coordinate is called the *azimuth*, a term derived from the Arabic *as-sumut*, "the ways." It is related to the *bearing* of celestial navigation (such as 22°.5 east of North for NNE). Throughout this book, we will use the convention of measuring degrees of azimuth from the North point of the horizon eastward around the horizon to the vertical circle that passes through the star whose position is to be measured.[7] From the use of azimuth and altitude, the horizon system is sometimes called the *altazimuth* system. We will use A for azimuth and h for altitude in formulae, and occasionally, we will refer to the system in terms of this pair of coordinates: (A, h).

The North Point of the horizon is defined as the point of intersection of the horizon with the vertical circle through the North Celestial Pole (*NCP*), the point about which the stars in the sky appear to turn. The opposite point on the celestial sphere defines the South Point. For southern hemisphere observers, the South Point of the horizon is defined analogously with respect to the *SCP*. The visible portion of the vertical circle through the NCP (or SCP) has a special name: It is the *celestial meridian* or simply the observer's meridian. It has the property of dividing the sky into east and west halves. Objects reach their highest altitude (*culminate*) as they cross the celestial meridian in the normal course of their daily motions. Circumpolar objects may culminate below as well as above the pole. At *lower culminations*, the altitudes are lowest, and at *upper culminations*, they are highest. If neither *upper* or *lower* is indicated, the upper is intended in most usages. Another important vertical circle is perpendicular to the celestial meridian. It intersects the horizon at the east and west points. Therefore, a star that is located at the midpoint of a vertical circle arc between the east point of the horizon and the zenith has an azimuth of 90° and an altitude of 45°. Note that no altitude can exceed 90° or be less than –90°, and that the azimuth may take any value between 0° and 360°.

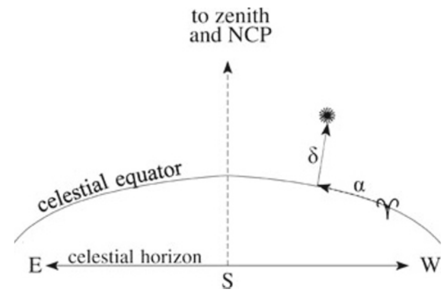
The azimuth coordinate may be considered in any of three ways:

(1) The angle subtended at the center of the celestial sphere between the North point of the horizon and the intersection of the vertical circle through the object and the horizon
(2) The arc length along the horizon subtended by the angle at the center (the observer)

[7] An alternative convention is to measure the azimuth from the South point of the horizon *westward*.



(a)



(b)

FIGURE 2.3. The equatorial or "Chinese" System of spherical astronomical coordinates: (a) The outside-the-sphere view. Note that the *right ascension* (α or *RA*) is measured eastward (counterclockwise as viewed from above the north celestial pole) from the vernal equinox. The declination, δ, is measured from the celestial equator along the *hour circle* through the star. (b) The observer's view. A south-facing observer sees the right ascension increasing along the celestial equator to the left from the Vernal equinox. This is the (RA, δ) version of the equatorial system. Drawings by E.F. Milone.

(3) The polar angle at the zenith, between the vertical circles through the North point and that through the object

The altitude coordinate may be considered in either of two ways:

(1) The angle at the center of the sphere between the intersection of the vertical circle through the object and the horizon
(2) The arc length along the vertical circle subtended by the angle

This second way of considering the altitude, together with the third way of considering the azimuth, permit transformations to be performed between this system and an equatorial system, which we describe below.

The horizon system depends on the observer's location, in the sense that observers at different sites will measure different azimuth and altitude coordinates for the same sky object. One can, however, envisage the sky independent of the observer, so that the stars are fixed in a framework and can be assigned coordinates that may be tabulated for future use. The equatorial and ecliptic systems are examples of such systems.

2.2.3. The Equatorial or "Chinese" System

In ancient China, another system was in use that is similar to the modern equatorial system. The modern equatorial system enables a transient object to be located precisely among the stars at a particular time. The reference great circle in this system (illustrated and labeled in Figure 2.3) is the *celestial equator*, the sky analog of the Earth's equator. It is midway between the poles of the equatorial system, the north and south celestial poles, the sky analogs of the North and South Poles of Earth. This is the system that is traced out by the stars' diurnal circles, which are coaxial with the celestial equator. The angular distance away from the celestial equator and toward the poles is called *declination* (from the Latin *declinatio* or "bending away") and originally referred to the distance from the celestial equator of a point on the ecliptic, the Sun's apparent annual path in the sky. The declination is marked in degrees. The small circles through the object and concentric with the celestial equator are called *declination circles* because each point on such a circle has the same declination. These small circles for all practical purposes trace out the diurnal motions; only the infinitesimally small intrinsic motions of objects on the plane of the sky during their diurnal motions makes this an inexact statement. The centers of all the declination circles lie along the polar axis, and the radius of each declination circle can be shown to be $R \cos \delta$, where R is the radius of the celestial equator (and the celestial sphere), taken as unity, and δ is the declination in degrees of arc. The declination is one of the two coordinates of the equatoral system. It is the analog of terrestrial latitude, which similarly increases from 0° at the equator to ±90° at the poles. Declinations are negative for stars south of the celestial equator. The analog relationship is such that a star with a declination equal to the observer's latitude will pass through the zenith sometime during a 24-hour day.

Great circles that go through the poles in the equatorial system are called *hour circles*. They intercept the celestial equator at right angles and are carried westward by the diurnal motions. The celestial equator rises at the east point of the horizon (and sets at the west point), so that successive hour circles intersecting the celestial equator rise later and later from the east point. A coordinate value may be assigned to each hour circle—indeed, if, as is usually the case, the term is interpreted loosely, there are an infinite number of such "hour" circles, rather than merely 24, each with a slightly different time unit attached. An hour circle can be numbered, as the name suggests, in hours, minutes, and seconds of time in such a way that the number increases, moment by moment, at a given point in the sky, other than exactly at a pole. At any one instant, an hour circle at the celestial meridian will have an associated number 6 hours different than that at the east point, or at the west point. The second coordinate of the equatorial system makes use of the hour circles. There are two varieties of this second coordinate. One variety is called the *right ascension*, and the other is the *hour angle*.

In modern terms and usage, the right ascension is measured from a point called the *vernal equinox*[8] eastward along the celestial equator to the hour circle through the object. The Sun is at the vernal equinox on the first day of spring (in the Northern Hemisphere); from here, the Sun moves eastward (so that its right ascension increases), and for the next three months, it moves northward (so that its declination increases). The term *right ascension* derives from the Latin *ascensio* and from the Greek αναφορα (*anaphora*), a rising or ascension from the horizon. It originally described the time required for a certain arc on the ecliptic (like a zodiacal sign) to rise above the horizon. The time was reckoned by the rising of the corresponding arc of the celestial equator. At most latitudes, in classic phrasing, the risings or *ascensions* of stars were said to be "oblique" because an angle with the horizon made by a rising star's diurnal arc is not perpendicular to the horizon; but, at the equator, where all objects rise along paths perpendicular to the horizon, the celestial sphere becomes a "right sphere" (*sphaera recta*) and the ascension a "right" one.

The right ascension increases to the east (counterclockwise around the celestial equator when viewed from above the north celestial pole), starting from the vernal equinox. Objects at greater right ascensions rise later. The analog of the right ascension in the terrestrial system is the longitude, which may also be expressed in units of time, but may also be given in angular units. The analogy here is imperfect because terrestrial longitude is measured E or W from the Greenwich meridian, but right ascension is measured only eastward from the vernal equinox.

As for the azimuth coordinate in the horizon system, the right ascension can be considered in any of three ways:

(1) As the angle measured at the center of the sphere between the points of intersection with the celestial equator of the hour circle through the vernal equinox and the hour circle through the star
(2) As the arc along the celestial equator between the hour circles through the vernal equinox and that through the star
(3) As the polar angle at the celestial pole between the hour circles

Similarly, as for the altitude coordinate in the horizon system, the declination can be considered in either of two ways:

(1) As the angle measured at the center of the sphere between the celestial equator and the star
(2) As the arc length, along the hour circle through the star, between the celestial equator and the star. This second way of considering the declination and the third way of considering right ascensions permit transformations among the equatorial and other coordinate systems to be made.

The declination is always given in angular measure (degrees, minutes of arc, and seconds of arc). The symbols for right ascension and declination are α and δ, but the abbreviations *RA* and *Dec* are often used.

The celestial equator has a special significance because objects on it are above the horizon for as long a time as they are below the horizon. The word *equator* derives from *aequare*, which means equate. When the Sun is on the celestial equator, therefore, day and night are of nearly equal length.

The equatorial system just outlined is completely independent of the observer—it is not directly tied to the horizon system, but there is another equatorial system that has such a connection. Figure 2.4 shows this observer-related equatorial system. In the ancient world, at least some separations of objects on the sky were measured by differences in their rise times. The modern system that derives from this is identical to the first equatorial system except for the longitudinal coordinate and the reference point. Instead of right ascension, it uses the *hour angle*, an angular distance measured along the celestial equator *westward from the celestial meridian*. The hour angle can be symbolized by H, or HA (we reserve h for the altitude) and usually is also expressed in units of time. It indicates the number of hours, minutes, and seconds since an object was on the celestial meridian. It therefore varies from 0 to 24 hours, but for convenience, it is often taken positive if west of the meridian and negative if east. The connection between the right ascension and the hour angle is the sidereal time (see §4).

Analogously with the azimuth, and the right ascension, the hour angle can be considered in any of three ways. The use of the polar angle between the celestial meridian and the hour circle through the star permits transformations between the horizon and the (H, δ) equatorial system (recall that we sometimes refer to a coordinate system by its coordinates expressed in this way). The transformation equations and procedures are described and illustrated in the next section.

The hour angle is also an analog of terrestrial longitude, in that it is measured along the celestial equator, but, again, the analogy is limited—in this case, because the hour angle is measured only from a local celestial meridian, whereas

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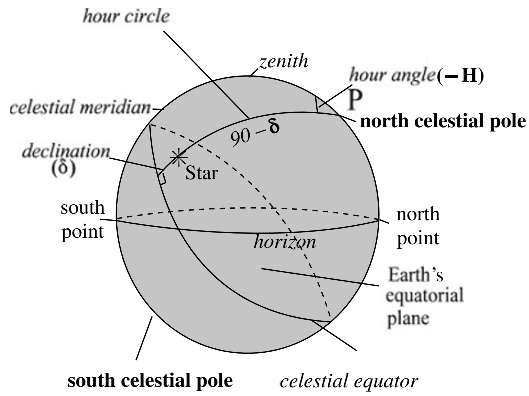
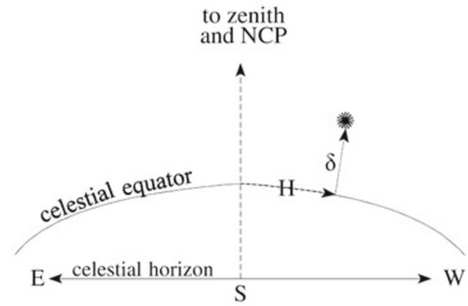
[8] The terms *vernal equinox* and *autumnal equinox* derive from the times of year (in the Northern Hemisphere) when the Sun crosses the celestial equator. "Equinox" is from the Latin *aequinoctium*, or "equal night." The actual point in the sky was called *punctum aequinoctialis*. In modern usage, "equinox" applies to both the time and the point. References to the times of year are more appropriately given as "March" and "September" equinoxes, and "June" and "December" solstices, at least at the current epoch and in the present calendar. In the distant past, this usage could be confusing because historically civil calendars have not been well synchronized with the seasons, and given sufficient time, the month in which the equinox or solstice occurs will change (see §4). We will use the terms as defined for the Northern Hemisphere in their positional meanings generally and in their seasonal meanings only to avoid ambiguity in the distant past.



(a)



(b)

FIGURE 2.4. A variant equatorial system, in which the observer's *hour angle*, H, is used instead of the *right ascension*: (a) The outside-the-sphere view. Note that H is measured westward from the celestial meridian. The declination is defined as in the (RA, δ) system. Note that the altitude of the north celestial pole is equal to the latitude, and the limiting (minimum) declination for circumpolar objects is $90 - \phi$. (b) The observer's view. A south-facing observer sees the hour angle increasing to the right. This is the (HA, δ) system. Drawings by E.F. Milone.

terrestrial longitude is measured from the Prime Meridian, at Greenwich, England.

Note that the connection between the horizon and (H, δ) systems is the celestial meridian, where $H = 0$. Figure 2.4a illustrates how the hour angle and the declination are defined, and how the "declination limit" of circumpolar stars for a given latitude, ϕ, can be determined.

Chinese star maps were commonly laid out in the (α, δ) manner of an equatorial system. Such a chart can be seen, for example, in Needham 1959, Fig. 104, p. 277). In this chart, a horizontal line though the chart represented the celestial equator. A hand-drawn curve arcing above the celestial equator represents the ecliptic or path of the Sun between the vernal equinox and the autumnal equinox. Everything on this chart represents a two-dimensional mapping of the interior of a celestial sphere onto a two-dimensional surface. Such charts have been found from as early as the 4th century A.D. in China. The data in them are older still; polar distances (90° – δ) found in Chinese catalogues have been used to date the catalogues themselves. The coordinates are a kind of hour angle, measured with respect to the edge of a *xiu* or lunar mansion, and a polar angle, a kind of anti-declination (Needham/Ronan 1981, p. 116). It is possible to date such catalogues and charts because the right ascensions and declinations of a star change with time, a phenomenon arising mainly from the *precession of the equinoxes* (see §3.1.6). According to Needham (1981a, p. 115*ff*), the chart has a probable date of ~70 B.C.

The equatorial system became widespread in Europe only after the Renaissance. Figure B.2 shows the polar views of the equatorial system, looking outward toward the north and south celestial poles. The sky centered on the north celestial pole is also depicted in one of the most famous of all historical star charts: the *Suchow* star chart of 1193 A.D. (Figure 10.7). The circle about halfway from the center is the celestial equator, which the inscription that accompanies the chart calls the "Red Road." It "encircles the heart of Heaven. . . ."

2.2.4. Transformations Between Horizon and Equatorial Systems

All students of archaeoastronomy should know how to transform coordinates between systems. It is easy to get equatorial system (H, δ) coordinates from horizon system (A, h) coordinates, given the observer's latitude and some knowledge of spherical trigonometry. Using the "sine law" and the "cosine law" of spherical trigonometry, that are described and illustrated in Schlosser et al. (1991/1994, Appendix A) and basic trigonometric definitions and identities also given there, we depict the appropriate spherical triangle, the "astronomical triangle," in Figure 2.5.

The resulting transformation equations are

$$\sin\delta = \sin\phi\cdot\sin h + \cos\phi\cdot\cos h\cdot\cos A \tag{2.1}$$

from application of the cosine law, and

$$\sin H = \frac{-\cos h\cdot\sin A}{\cos\delta} \tag{2.2}$$

from application of the sine law.

Suppose at a latitude, $\phi = 30°$, the altitude of a certain star, $h = 20°$, and the azimuth, $A = 150°$. From (2.1),

$$\begin{aligned}\sin\delta &= \sin(30°)\cdot\sin(20°)+\cos(30°)\cdot\cos(20°)\cdot\cos(150°)\\ &= 0.50000\cdot 0.34202 + 0.86603\cdot 0.93969\cdot(-0.86603)\\ &= -0.53376\end{aligned}$$

so that $\delta = -32\overset{\circ}{.}260$.

Substituting this value into (2.2), we find that

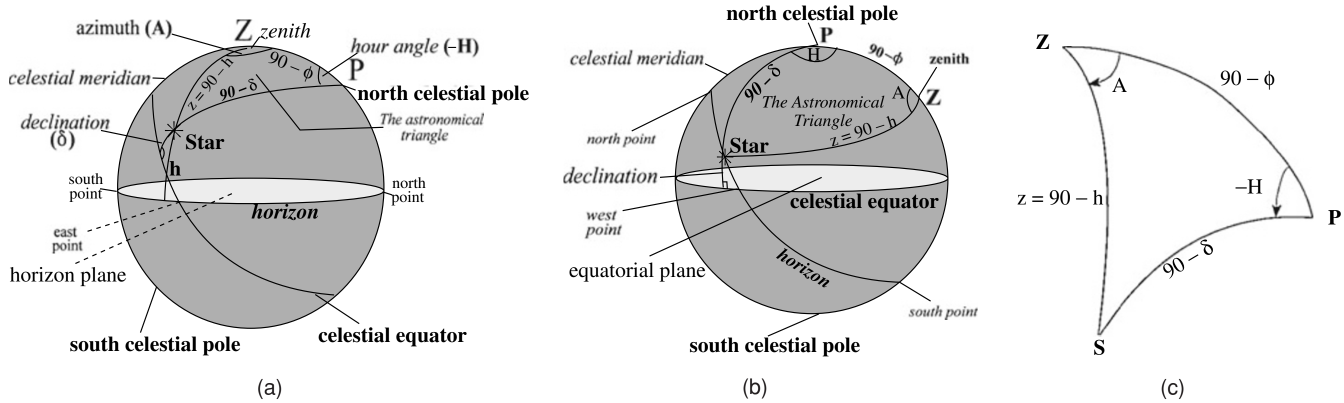


FIGURE 2.5. (a) The horizon system and the hour-angle variant of the equatorial system superposed. The definition of the astronomical triangle for a risen star is illustrated. (b) The equatorial and horizon systems, but now seen from the western side of the celestial sphere and for a slightly different point of view, for a star at the western horizon. (c) The astronomical triangle extracted from its context on the celestial sphere. Drawings by E.F. Milone.

$$\sin(H) = \frac{-\cos(20°)\cdot\sin(150°)}{\cos\left(-32\overset{\circ}{.}260\right)}$$
$$= \frac{-0.93969\cdot 0.50000}{0.84563} = -0.55562$$

so that $H = -33\overset{\circ}{.}753$ or $-33.753/15°/\text{h} \simeq -02^{\text{h}}\ 15^{\text{m}} = 02^{\text{h}}\ 15^{\text{m}}$ east.

These values make sense because of the location of the star, low in the southern part of the sky. Because the sine function is double valued, i.e., two angles have the same function value: $\sin\Theta = \sin(180 - \Theta)$, there is another mathematical solution for H, however. In the above example, therefore, a possible solution is $H = 180° - (-33\overset{\circ}{.}753) = 213\overset{\circ}{.}753$, but this second solution does not make sense physically. The angle is equivalent to $14^{\text{h}}\ 15^{\text{m}}$ west or (noting that $213\overset{\circ}{.}753 - 360° = -146\overset{\circ}{.}247$), $-9^{\text{h}}\ 45^{\text{m}}$, nearly ten hours *east* of the meridian. It is not possible for a visible star so near the southern horizon to be so far from the celestial meridian at this latitude. So the alternative solution can be ruled out "by inspection" in this case. As a rule, however, the other quadrant solution cannot be dismissed without further calculation. To resolve the question of quadrant, $\cos H$ may be computed from (2.5); the actual value need not even be calculated (although a numerical check is always a good idea) because the sign of $\cos H$ alone can resolve the ambiguity.

The cosine function is double valued because, $\cos\Theta = \cos(360 - \Theta) = \cos(-\Theta)$, where Θ is a given angle, but examination of $\sin\Theta$ resolves the ambiguity. The sine function is non-negative in both the first (0° to 90°) and the second (90° to 180°) quadrants, whereas the cosine function is non-negative in quadrants one and four (270° – 360°). In quadrant three (180° – 270°), both are negative. Therefore, the quadrant can be determined by the signs of both functions (see Table 2.2).

From the same spherical triangle and trigonometric rules, it is possible to express the transformation from the equatorial to the horizon system:

TABLE 2.2. Sine and cosine quadrants.

Quadrant	Sign (sin)	Sign (cos)
1	+	+
2	+	−
3	−	−
4	−	+

$$\sin h = \sin\phi\cdot\sin\delta + \cos\phi\cdot\cos\delta\cdot\cos H, \tag{2.3}$$

$$\sin A = \frac{-\cos\delta\cdot\sin H}{\cos h}. \tag{2.4}$$

Solving (2.3) for $\cos H$, we can test our solution:

$$\cos H = \frac{\sin h - \sin\phi\cdot\sin\delta}{\cos\phi\cdot\cos\delta} \tag{2.5}$$
$$= \frac{0.34202 - 0.50000\cdot(-0.53376)}{0.86603\cdot 0.84563}$$
$$= \frac{0.60890}{0.73234} = 0.83144.$$

Note that $\sin H < 0$ and $\cos H > 0$, a condition that holds only in the 4th quadrant (between 270° and 360°, which is equivalent to being between −90° and 0°). Therefore, $H = -33\overset{\circ}{.}753 = -02^{\text{h}}\ 15^{\text{m}}$ is the correct answer.

The quadrant ambiguity also arises in computing the azimuth A from (2.4). The quantity $\cos A$ may be computed from (2.1), and the signs of $\sin A$ and of $\cos A$ from Table 2.2 will decide the quadrant. The numerical values of A computed from (2.1) and (2.4) should agree, and computing them both provides a check on the calculation. A difference between the two values indicates either a miscalculation or lack of precision (insufficient number of digits) used in the calculations. The basic point, however, is that the signs of the sine and cosine functions are sufficient to resolve the quadrant question of both H and A. The same remarks hold for any longitudinal-type coordinate that can range from 0° to 360°.

Consider the reverse of our earlier example. Now we are given the latitude, $\phi = 30°$, the declination, $\delta = -32.263°$, and the hour angle, $H = -33.753°$. Then, from (2.3), we find the altitude, h:

$$\begin{aligned}\sin h &= \sin(30°)\cdot\sin(-32.260°) + \cos(30°)\\ &\quad\cdot\cos(-32.260°)\cdot\cos(-33.753°)\\ &= 0.50000\cdot(-0.53376) + 0.86603\cdot 0.84563\cdot 0.83114\\ &= 0.34202,\end{aligned}$$

from which we get $h = 20.000°$.

Solving (2.4), we can also recover the azimuth:

$$\begin{aligned}\sin A &= \frac{-\cos(-32.260°)\cdot\sin(-33.753°)}{\cos(20.000)}\\ &= \frac{-0.84563\cdot(-0.55561)}{0.93969} = 0.50000,\end{aligned}$$

from which we get either $A = 30.000°$ or $(180° - 30.000°) = 150.000°$.

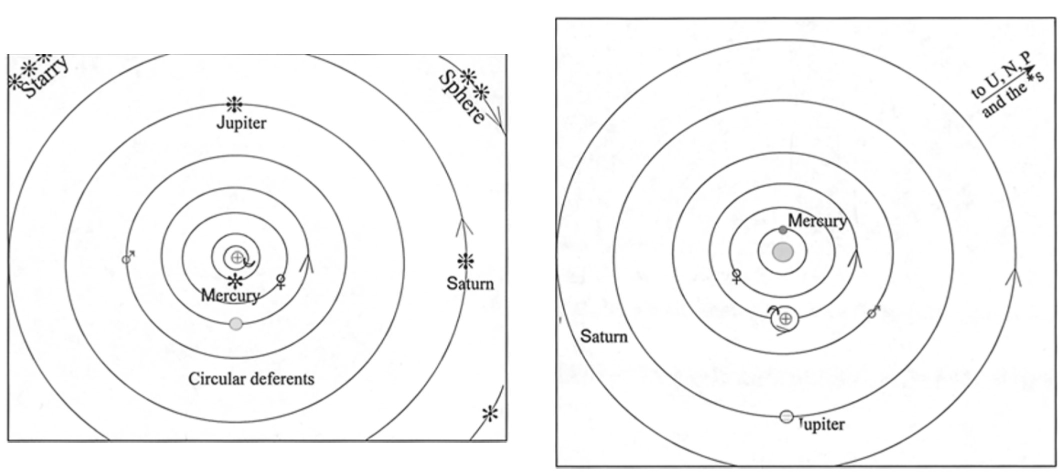
In the present case, we know that the star is near the southern horizon, because the star is south of the equator and more than 2 hours east of the celestial meridian. Therefore, the second answer is correct, $A = 150°$. Many cases are less easy to decide by inspection. Equation (2.1) can be solved for cos A, and the correct quadrant then be determined. This is left to the student as a recommended exercise to gain experience in spherical astronomy! In the chapters to come (especially Chapter 6), we will make frequent use of the transformation relations to explore the possibilities of deliberate astronomical alignments of monuments.

2.3. Basic Motions of the Sun and Moon

2.3.1. The Sun, the Year, and the Seasons

Now we must separate the diurnal motion shared by all objects in the sky from the additional motions of seven distinctive objects known in antiquity: the Sun, Moon, and naked-eye planets. We take for granted that the diurnal motion of everything in the sky is due to the rotation of the earth on which we stand. In the ancient world, this was a radical view, and few astronomers held it. Diurnal motion is perceived moment by moment, whereas the effects of the relative movement of the Sun, Moon and planets with respect to the stars are far more gradual. This made diurnal motion of the fixed stars far more intuitive than any other motion. Nevertheless, an unmoving Earth was not the only option, and ancient astronomers knew it.

Figure 2.6 provides the alternative frameworks for understanding the motions: the Earth-centered and Sun-centered systems. The geocentric perspective has been historically dominant in human cultures, and yet the heliocentric viewpoint leads to a far more economical model to account for the relative motion of the Sun, Moon, and planets in the sky. Prior to the Copernican revolution, and indeed throughout most of known history, the geocentric universe was the accepted cosmic model notwithstanding that the Greek scholar Aristarchus (~320 B.C.) argued for a heliocentric universe and the medieval Islamic scholar al-Beruni (~11th century) said that all known phenomena could be explained either way. Both the constellation backdrop and the direction of the Sun's apparent motion are predictably the same in the two world systems, as Figure 2.6 reveals. In both models, the Sun's annual motion (as viewed from the north celestial pole region) is counterclockwise. We have known



(a) (b)

FIGURE 2.6. The classic cosmological frameworks: (a) Earth-centered and (b) Sun-centered views of the solar system. Drawings by E.F. Milone.

since Newton's time that a less massive object like the Earth is accelerated by the Sun more quickly than a more massive body like the Sun is accelerated by the Earth.[9] Neither the true physical natures of the planets nor the physical principles that ruled their motions were known in the ancient world.[10] The ancient models were designed specifically to predict these apparent motions and were basic to ancient Greek astronomy. The successes and failures of ancient models can be gauged precisely and accurately only if their predictions can be compared with those of modern methods. We start with the most familiar case, the Sun, which undergoes reflexive motions in the sky as the Earth moves.

From the geocentric perspective, that the earth's rotation axis is tilted by 23°.5 with respect to its axis of revolution about the Sun, and that the direction of the rotation axis is fixed[11] while the Earth revolves about the Sun is equivalent to saying that the Sun's path is inclined to the celestial equator by the same 23°.5 angle, so that the Sun's declination varies by about 47° during the year. Except possibly in deep caves and on ocean floors, the effects of the Sun's annual movement are dramatic for life everywhere on Earth. In fact, the large annual variation in declination has profoundly affected development and evolution of life on Earth (especially, if, as is sometimes speculated, the angle of tilt has changed greatly over the age of the Earth).

The obvious diurnal westward movement of the Sun is shared by the Moon, planets, and stars. However, the diurnal westward motions of the Sun, Moon, and planets are different from those of the stars and from each other. The Sun and Moon always move eastward relative to the stars, so that the angular rates of their westward diurnal motions are always less than that of the stars. The planets' apparent motions are more complex, sometimes halting their eastward motions and briefly moving westward before resuming eastward motion. Thus, their diurnal motions are usually slower but sometimes slightly faster than are those of the stars.

To describe the Sun's behavior, we can say that the diurnal motion of the Sun is accomplished in a day and is very nearly parallel to the celestial equator; relative to the stars, the Sun has a slow average motion, $\sim 360°/365\frac{1}{4}\,\mathrm{d} \approx 1°/\mathrm{d}$ eastward, and it requires a year to complete a circuit. Moreover, except for two instants during the year, the Sun's annual motion is *not* strictly parallel to the celestial equator. We can elaborate this motion and then find an explanation for it, or, as the Greeks would put it, "save the phenomenon."

The Sun's eastward motion is easily tracked on the celestial sphere. Figure B1 illustrates the annual path of the Sun (the *ecliptic*) as it appears on an equatorial star chart. The sinusoid shape is the consequence of mapping the path onto a Mercator projection of the equatorial system. The process can be visualized by imagining the celestial sphere cut in two along an hour circle and opened outward. In this projection, in which all hour circles become parallel vertical lines, great circles that intersect the celestial equator at an acute angle appear as sinusoids. The Sun's most northern declination (+23°.5 at present) occurs at the positive maximum of this curve, the June *solstice* (the northern hemisphere's *summer solstice*), at $\alpha = 6^h$, and its most southern declination (−23°.5 at present) at the December *solstice* (the Northern hemisphere's *winter*, and the Southern hemisphere's *summer solstice*), at $\alpha = 18^h$. Like the term *equinox*, *solstice* also has two meanings. It is a positional point on the ecliptic *and* an instant of time when the Sun "stands still" (the literal meaning of the Latin). A solstice, therefore, marks a N/S turning point. At the equinoxes, where (and when) it crosses the celestial equator, the Sun rises at the east point of the horizon, and sets at the west point. The azimuth of rise (or set) of the Sun on any given day depends both on its declination and on the observer's latitude. Solving Eqn. (2.1) for $\cos A$,

$$\cos A = \frac{\sin\delta - (\sin\phi \cdot \sin h)}{\cos\phi \cdot \cos h} \tag{2.6}$$

and on the horizon,[12] h = 0, so that

$$\cos A_{\text{rise/set}} = \frac{\sin\delta}{\cos\phi}. \tag{2.7}$$

At $\delta = 0°$, $\cos A = 0$, so that $A = 90°$ and $270°$, the azimuths of the east and west points of the horizon, respectively. Beginning at (Northern Hemisphere) winter solstice, the Sun rises further to the North each day, with decreasing azimuth, until it reaches summer solstice. At that midsummer[13] date, it has the smallest azimuth of rise (i.e., most northern). It stops decreasing and thereafter rises at greater

[9] Isaac Newton (1642–1727) embodied this idea in the second of his three laws of motion in the *Philosophiae Naturalis Principia Mathematica* (1687). His first law states that an object in motion (or at rest) maintains that state unless acted on by an external force. The second law more fully states that the acceleration of a body is directly proportional to the force acting on it and is inversely proportional to its mass. The third law states that every force exerted by one body on another is matched by a force by the second on the first.

[10] It goes almost without saying, however, that this circumstance does not relieve dedicated students of ancient astronomy from an obligation to obtain at least a rudimentary understanding of the nature and true motions of planetary bodies, so that their relative motions with respect to the Earth can be understood.

[11] Ignoring the long-term phenomena of *precession* (q.v. §3.1.6) and the variation of the *obliquity* (see §2.3.3 and §4.4, respectively).

[12] There is a slight complication in the statement that $h = 0$ indicates an object *on* the horizon. This is true of the astronomical definition of altitude and of the horizon, but the Earth's atmosphere acts as a lens, the refractive properties of which raise both the horizon and the object toward the zenith by an amount that depends on the true altitude and that varies with the temperature and pressure of the atmosphere along the path to the object. Because the light from the object travels a greater path length through the atmosphere, it is lifted higher than the horizon, sometimes dramatically so. Thus, the *apparent* altitude at the astronomical instant of rise is greater than zero; a common value is ~0°.5. See §3.1.3 for further discussion. For the time being, we ignore the effects of atmospheric refraction.

[13] Technically, modern astronomy assigns the *beginning* of the season to the date of solstices and equinoxes, but the older usage is still common. "Midsummer's eve" is the night before the sunrise of the summer solstice. When the terms "midwinter" and "midsummer" are used here, they refer to the dates of the solstices.

azimuths (i.e., more and more to the south). It continues to rise further South each day until it reaches winter solstice again. In the Southern Hemisphere, the Sun rises further to the South each day, its azimuth increasing until summer solstice, and thereafter decreasing again (this follows if the azimuth keeps the same definition we have adopted for the Northern Hemisphere).

Near the equinoxes, the solar declination changes rapidly from day to day so that the points on the horizon marking sunrise and sunset also vary most quickly at those times; at the solstices, the change in declination from day to day is very small, and so is the azimuth change.[14]

The oscillation of its rising (and setting) azimuth on the horizon is one clearly observable effect of the Sun's variable declination during the year. Half of the total amount of oscillation, the largest difference (N or S) from the east point, is called the *amplitude*.[15] We will designate it ΔA. Note that the amplitude of rise is also the amplitude of set. The amplitude depends on the latitude [see Eqn (2.7)]. At the equator, $\phi = 0°$ and $\Delta A = 23.5°$. At any latitude, ϕ, at rise,

$$\Delta A = \text{arc cos}\frac{\sin\delta}{\cos\phi} - 90° . \tag{2.8}$$

Note that the Sun's motion along the ecliptic includes a north/south component that changes its declination, which has been shown to vary the sunrise and sunset azimuth. Because the changing declination of the Sun causes the seasons, the azimuth variation can be used to mark them; a good case can be made that such variation was observed in the Megalithic (§§6.2, 6.3).

The seasonal change in declination also changes the time interval the Sun is above the horizon. This day-time interval is twice the hour angle of rise or set (ignoring, again, the effect of refraction and other physical effects described in §3); so the Sun is above the horizon longer in summer than in winter at all latitudes except the equator. Solving Eqn (2.5) when $h = 0°$, we get

$$\cos H_{\text{rise/set}} = -\tan\phi \cdot \tan\delta. \tag{2.9}$$

At the equinoxes, when $\delta = 0°$, $H_{\text{rise/set}} = 90°$ and $270°$, equivalent to 6^h and 18^h (-6^h), the hour angles of set and rise, respectively. At such a time, the Sun is above the horizon half the day, so that the numbers of daylight and night-time hours are about equal, hence, the Latin *aequinoctium*, whence *equinox*. At winter solstice, the Sun spends the smallest fraction of the day above the horizon; and its noon altitude (its altitude on the celestial meridian, where $H = 0$) is the lowest of the year. At summer solstice, the Sun spends the largest fraction of the day above the horizon and its noon altitude is the highest of the year. The symmetry in the last two sentences mimics the symmetry of the Sun's movements over the year. The larger fraction of the Sun's diurnal path that is below the horizon at winter solstice is the same fraction that is above the horizon at summer solstice. That the ancient Greeks worked with chords subtended at the centers of circles rather than with sines and cosines did not deter them from discovering and making use of these wonderful symmetries, as we show in §7.3.

A high declination object has a larger diurnal arc above the horizon than below it, and by a difference that increases with latitude (see Figure 2.7). The result of the low altitude of the winter Sun means that each square centimeter of the ground receives less solar energy per second than at any other time of the year, as Figure 2.8 illustrates, resulting in lower equilibrium temperatures. In practice, the situation is complicated by weather systems, but the seasonal insolation of the Sun, as the rate of delivery of solar energy to a unit area is called, is usually the dominant seasonal factor. The effects of seasonal variations and the association of these changes with the visibility of certain asterisms (especially those near the horizon at sunrise or sunset) was noticed early. This association may have been a crucial factor in the development of ideas of stellar influences on the Earth. The changing visibility, ultimately due to the orbital motion of the Earth, shows up in the reflexive motion of the Sun in the sky. The Sun's motion among the stars means that successive constellations fade as the Sun nears them and become visible again as it passes east of them.

References to seasonal phenomena are common in the ancient world. From Whiston's Josephus,[16] writing about the followers of the high priest John Hyrcanus, who was besieged by the Seleucid king Antiochus VII:

> [T]hey were once in want of water, which yet they were delivered from by a large shower of rain, which fell at the setting of the Pleiades. *The Antiquities of the Jews*, Book XIII, Ch. VIII, paragraph 2, p. 278.

Whiston's footnote to the line ending with the "Pleiades" reads:

> This helical setting of the Pleiades was, in the days of Hyrcanus and Josephus, early in the spring, about February, the time of the latter rain in Judea; and this is the only astronomical character of time, besides one eclipse of the moon in the reign of Herod, that we meet with in all Josephus.

The "helical" (heliacal) setting (see §2.4.3) indicates a time when the Pleiades set just after the Sun. Due to the phenomenon of precession (see §3.1.6), the right ascension of the Pleiades in the time of John Hyrcanus, ~132 B.C.,

[14] This can be seen by taking the rate of change of azimuth due to a change in declination, in Equation 2.7:

$$\sin A_{\text{rise/set}} \cdot dA_{\text{rise/set}} = \frac{-\cos\delta \cdot d\delta}{\cos\phi},$$

so that

$$dA_{\text{rise/set}} = \frac{-\cos\delta}{\cos\phi \cdot \sin A_{\text{rise/set}}} \cdot d\delta.$$

Near the equinoxes, $\delta \approx 0$, so that $\cos\delta \approx 1$ and near the solstices, $\delta \approx \pm 23\overset{\circ}{.}5$, so that $\cos\delta \approx 0.9$. Moreover, when $\cos\phi$ is small, $\sin A$ is large and vice versa, so that dA changes proportionally with $d\delta$, but with opposite sign, at all times of year. Near the solstices, when $d\delta \approx 0$, $dA \approx 0$ also, so that the Sun is at a standstill, roughly keeping the same azimuth from night to night for several nights.

[15] Not to be confused with the term as used in variable star astronomy, where *amplitude* refers to the range of brightness variation. See §5.8.

[16] More properly, "The Works of Flavius Josephus."

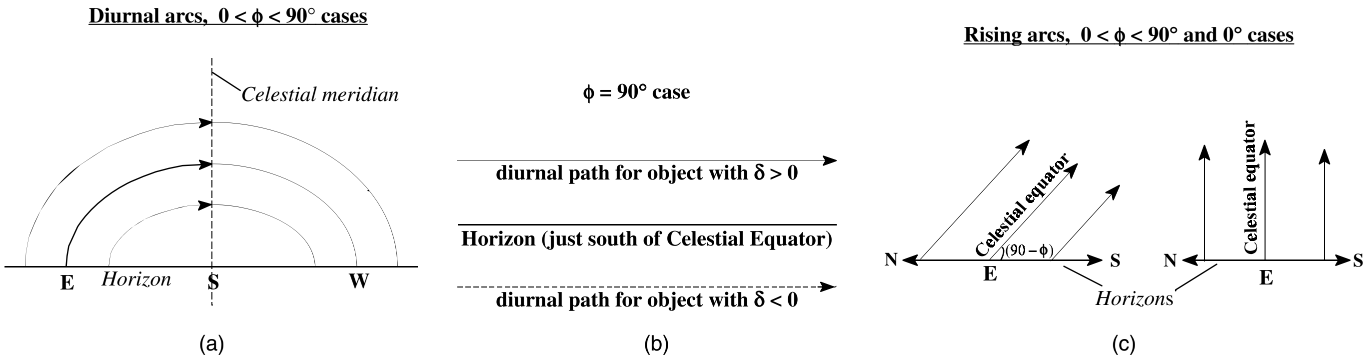


FIGURE 2.7. Horizon sky views of diurnal arcs as a function of declination and latitude (a) as seen at intermediate northern latitudes, looking south; (b) at the North Pole; (c) as seen at intermediate northern latitudes (left) and at the equator (right), looking east. Drawings by E.F. Milone.

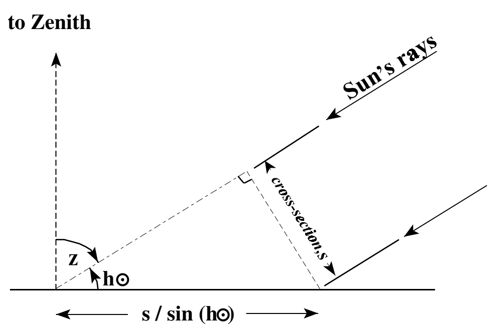


FIGURE 2.8. The effects of solar altitude on ground warming. Note that a given cross-section of sunlit area is spread out on the ground by a factor that increases with solar zenith distance. Drawing by E.F. Milone.

would have been ~01^h45^m, which is ~2 hours smaller than today, and its declination would have placed it further south, at $\delta \approx 15°$ compared with about 24° today. Thus, the Sun would need to be east of the vernal equinox and just west of the Pleiades for the Pleiades to be seen setting just after the Sun. As a rule of thumb, stars as faint as the Pleiades are required to be ~5° or more above the horizon to be seen clearly by the naked eye in an otherwise dark sky because of the dimming of star light by the long sightline through the atmosphere of an object near the horizon. The Sun must be sufficiently below the horizon (~10°) for these relatively faint naked eye stars to be seen above the twilight. See §3.1 for discussions of the visibility of astronomical objects and particularly §3.1.2.2 on atmospheric extinction and §3.1.2.5 on sky brightness and visibility. A simulation of the sky (Figure 2.9) shows that these conditions would last apply in early-to-mid-April, and thus early spring, as indicated by Whiston, but not in February!

From Hesiod's *Works and Days*, (8th century B.C.), we find the use of seasonal signs among the stars:

When first the Pleiades,
Children of Atlas,
arise,
begin your harvest;
plough,
when they quit the skies,

In West's (1978) translation. We can see that these verses provide calendrical references: the visibility of well-known asterisms at important times of day, typically sunup or sundown. Two and a half millennia ago, the Pleiades had a right ascension, $\alpha \approx 1^h\ 15^m$, nearly two and a half hours less than it has today. However, we must ask what Hesiod meant by the first rising of the Pleiades. If they were seen to rise as the Sun set, an "acronychal rising" as we call this phenomenon,[17] the Sun must have been almost opposite in the sky or at $\alpha \approx$ 13–14 h, and this implies the time of year—about a month past the Autumnal equinox, a suitable enough time for harvesting, one might think. Then when the Sun approached the Pleiades so closely that they were no longer visible, and they disappeared before the end of evening twilight ("heliacal" or "acronychal setting"), the Sun's RA must have been ~1–2 hours; so the time of year must have been early spring, not an unsuitable time for planting. If a heliacal rising is intended, then the Sun must be least 10–20 minutes further east than the Pleiades, and so at $\alpha \approx 2^h$; this places the time of year a month after vernal equinox, in late April or early May. However, a contrary reading of the "begin your harvest" passage is possible and turns out to be more likely (see Pannekoek 1961/1989, p. 95; and Evans 1998, pp. 4–5), *viz*, that the heliacal rising of the Pleiades signifies the season for *harvesting* a *winter* wheat crop. Moreover, if "plough when they quit the skies" implies that the Pleiades set as the Sun rises, the autumn planting of a winter wheat crop would have been implied. It is well known that

[17] See §2.4.3 for a full discussion of the terms "heliacal" (referring to a rise/set close to the Sun), "acronychal" (associated with the setting sun), and "cosmic" (connected with the rising Sun).

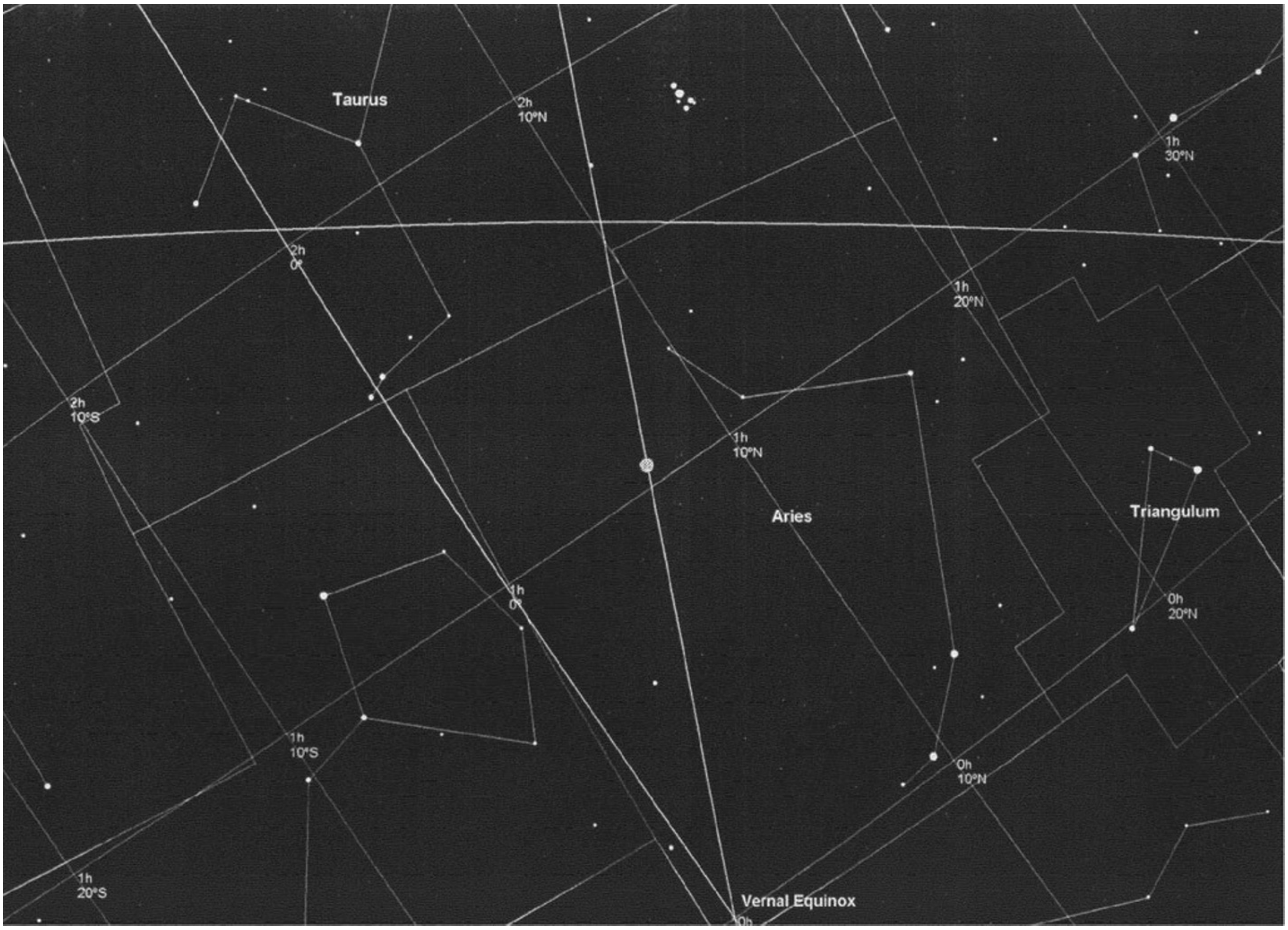


FIGURE 2.9. The heliacal setting of the Pleiades in Jerusalem in 132 B.C. would have occurred no later than ~April 10, according to the Red Shift planetarium software package (Maris, London). The simulation sky map of that date shows the Pleiades to be 4° to 5° above and the Sun ~9° below the western horizon at ~6:40 p.m., Local Time.

winter wheat was grown in the ancient world, even though at some point summer wheat was also (see, e.g., Pareti, Brezzi, and Petech 1965, p. 385). Hesiod instructs his brother, "Plough also in the Spring," and in a later passage, he cautions against waiting until the Sun reaches its "winter turning point," thus resolving the issue for the main planting time.

Another passage from the same work,[18] indicates an important late-winter/early-spring activity:

When from the Tropic, or the winter's sun,
Thrice twenty days and nights their course have run;
And when Arcturus leaves the main, to rise
A star shining bright in the evening skies;
Then prune the vine.

Here, the season and time are delineated, and we can interpret the comment directly. The Sun has now and had then a right ascension of ~18 h at winter solstice, and moves ~2 h east each month; thus, 60 days after the solstice, $\alpha_\odot \approx 22$ h. As the Sun sets, Arcturus (currently $\alpha \approx 14^h\ 16^m$, $\delta \approx +19.^\circ 2$; 2500 years ago, $\alpha \approx {\sim}12^h\ 18^m$, $\delta \approx {\sim}{+31.^\circ 3}$) rose in the east; in the Mediterranean region, it could well have arisen from the sea. Here, Arcturus's higher declination in the past would have caused it to rise earlier than it does today at a site with the same latitude.

A late-night talk-show host in the 1990s garnered a number of laughs by showing through interviews how few students understood the astronomical cause of the seasons (hopefully they were not astronomy students!). Most thought that the varying distance of the Earth from the Sun was the primary cause. Had they lived in the Southern hemisphere, they could have been forgiven for this incorrect view, because the Earth is closest to the Sun in January, but they still would have been wrong. The varying distance does have an effect on the seasons, but it is a secondary one (it would have a greater effect if the Earth's orbit were more eccen-

[18] Translation by T. Cooke, cited in R.H. Allen 1963 ed., p. 95.

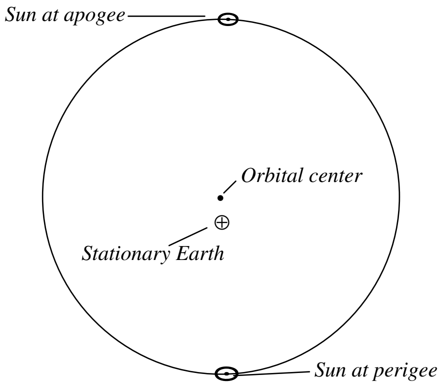


FIGURE 2.10. The off-center circle Hipparchos model for the eccentric solar orbit.

TABLE 2.3. Changes in lengths of the seasons over millennia.

Date	Spring	Summer	Autumn	Winter	Year length[a]
2001 B.C	$94^{d}.29$	$90^{d}.77$	$88^{d}.39$	$91^{d}.80$	365.25
1 B.C.	$93^{d}.97$	$92^{d}.45$	$88^{d}.69$	$90^{d}.14$	365.25
2000 A.D.	$92^{d}.76$	$93^{d}.65$	$89^{d}.84$	$88^{d}.99$	365.24

[a] The year lasts slightly more than 365 civil days (the numbers of days as recorded by civil authorities), requiring the inclusion of an extra day almost every 4 years to keep the calendar in step with the astronomical seasons.

tric than it is). The main cause is that the Sun does not travel along the celestial equator but along the ecliptic. Its declination changes with season and, consequently, so do the midday altitude and the length of time spent above the horizon and so does the insolation, as we have shown. The distance of the Sun from the Earth does indeed vary around the year, but at the present time the Earth's passage through *perihelion*, or nearest point to the Sun, occurs during the Northern Hemisphere winter.

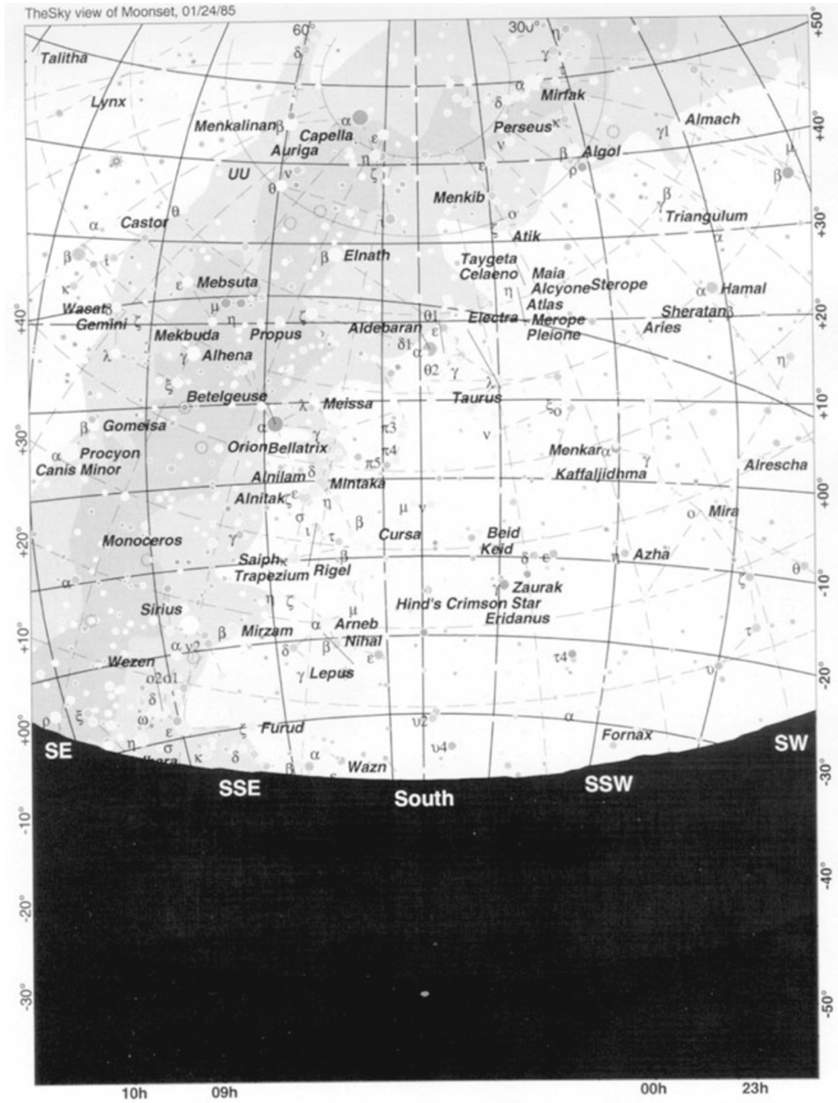
The primary and secondary causes for seasonal effects were understood in antiquity. Ptolemy correctly defines the equinoxes and solstices with respect to the relations between the ecliptic and the celestial equator. He also states (*Almagest*, Toomer tr., 1984, p. 258) that both Sun and Moon vary in distance, and he proceeds to calculate their parallaxes (shift in position as viewed, for example, by observers at different places on Earth). That the Sun's motion on the ecliptic is not uniform throughout the year was also known, and this was modeled in terms of the varying distance of the Sun from Earth. Hipparchos detected the inequality of the seasons and deduced that the Sun moves slower in some parts of its path than it does in others. Because in keeping with all ancient Greek astronomers he believed that planetary bodies moved on circular paths, he had to devise a way to explain why the rate should be different from season to season. His explanation was that the Earth did not lie at the center of the Sun's orbit. As viewed from the Earth, therefore, the Sun's orbit, although circular, appeared eccentric. Such an orbit was referred to as an *eccentre* (or sometimes by the adjective form, *eccentric*). The model is illustrated in Figure 2.10. Hipparchos's observation was correct, and his explanation was a reasonable approximation for his time.

The lengths of the seasons vary slightly from year to year as the Earth's orbit slowly rotates. Meeus (1983b) has tabulated the lengths of the seasons for each millennium year beginning with –3000 (3001 B.C.), when autumn was the shortest season, and notes that winter has been the shortest only since the year 1245. The lengths of the (Northern Hemisphere) seasons for three important epochs among others tabulated by Meeus (1983b) are shown in Table 2.3.

Now we can tie in the motions of the Sun to the seasonal visibility of asterisms. Because the Sun must cover 360° in the course of a year, it must move eastward at slightly less than 1°/day on average. As a consequence, the groups of stars that can be seen during the night, change slowly from night to night, amounting to an angular displacement of about 1/12 of the sky's circumference or 30° in a month. Suppose a particular group of stars on the celestial equator will be seen to rise at sunset; 10 days later, another group of stars about 10° to the east will appear to rise at that time. In the same interval, the stars in the westernmost 10° will disappear in the evening twilight. Figure 2.11 compares the constellations on the meridian at evening twilight, but two months apart. Over the course of a year, Hesiod's seasonal signs follow. The Egyptians used asterisms to keep track of hours, days, months, and, indeed, years! (See §4.) These decans[19] were about 10° apart.

The seasonal variation of the Sun in both right ascension and declination creates an interesting pattern in the sky over the course of the year. The Sun's eastward motion, combined with its apparent northern motion from winter to summer (and southern motion from summer to winter), appears to spiral through the sky; some cultures saw the weaving of a pattern. With sufficient patience and endurance, it can be demonstrated! A camera recording the noon position of the Sun a regular number of days apart over the course of a year will produce a figure-eight pattern called an *analemma*. This figure marks the variation in the Sun's instant of arrival at the meridian and its variation in declination, and so it is a marker of the seasons and of solar time. It will be discussed in later chapters (e.g., §4.1.1.2) for both reasons. For many cultures, from Britain to Egypt, the return of the Sun from its winter quarters and its return from darkness every morning were direct analogs of an endless cycle of death and rebirth. As such, they became mystical, religious events to be observed and celebrated and, in the highest plane of the human spirit, appropriated.

[19] The decans were depicted as two-legged beings, sentries guarding the portals of the night. From the tomb of Seti I (~1350 B.C.) (Neugebauer and Parker, 1969, plate 3).

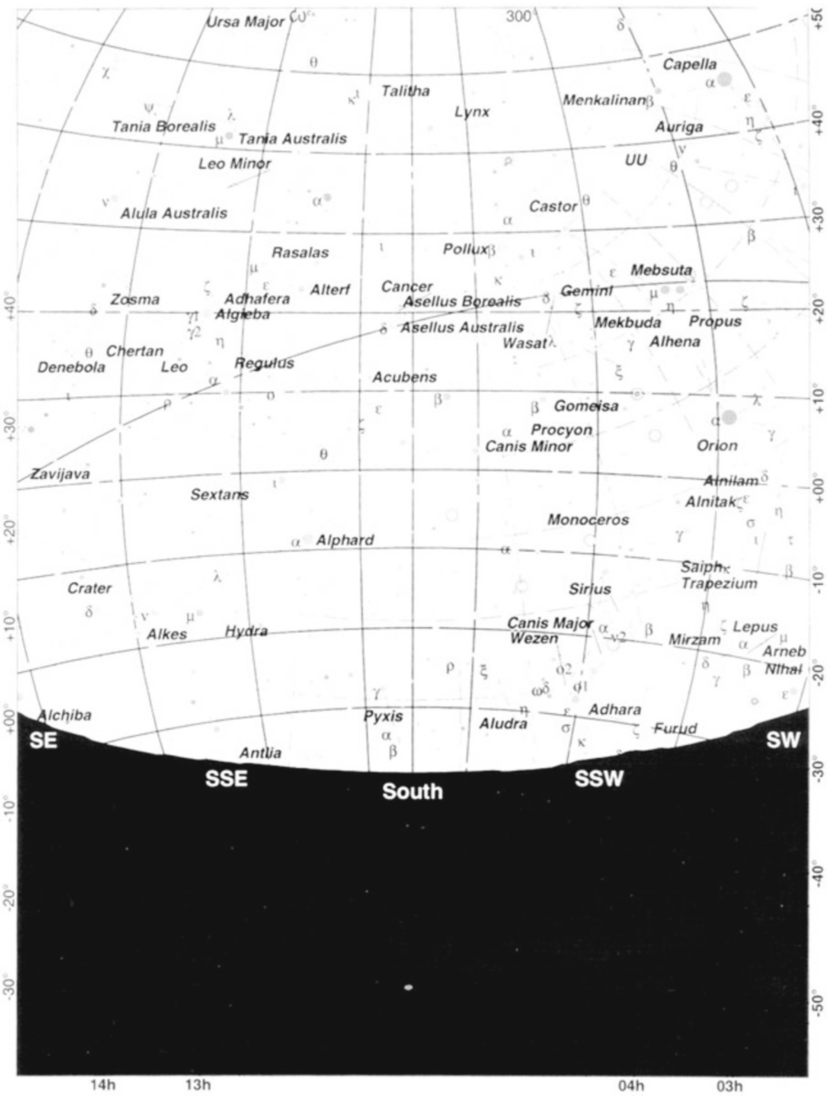


(a)

FIGURE 2.11. Simulation of constellations centered on the meridian at the same mean solar time in the evening (2100 MST), but two months apart: (a) Jan. 24, 1985, (b) Mar. 24, 1985, as seen from Calgary, Alberta. The equatorial grid is shown with the solid line, with declinations indicated on the extreme right and a few right ascensions at the bottom. The ecliptic is shown arching across the field. The horizon grid is shown with a lightly broken line, with altitudes indicated on the extreme left and a pair of azimuths marked on the vertical circle arcs radiating from the zenith. Produced by E.F. Milone with TheSky software package (Software Bisque, Golden, CO).

2.3.2. The Zodiac

The Sun's annual journey involves visits to successive areas of the sky. Twelve constellations follow one another in a band around the sky. They straddle the ecliptic, the path of the Sun during its annual journey among the stars. The band of constellations is called the *zodiac*, from the Greek ζωδιακὸς κύκλος (zodiacos kuklos), "circle of the animals"). The naming of most of the zodiacal constellations took place in Mesopotamia. According to Neugebauer (1969, p. 102ff), the subsequent assignment of the zodiacal constellation names to a series of 30° segments of the sky along the ecliptic was probably first carried out in the 4th century B.C. (for alternative views, see §7.1.2.3). The uniform lengths of exactly 30° each created a longitude-like coordinate by which positions could be assigned to the stars. These



(b)

FIGURE 2.11. *Continued.*

12 constellations were thus turned into signs.[20] The Greco-Roman zodiac (with the symbol for each sign) is shown with the Mesopotamian and Indian equivalents in Table 2.4, in the order in which they are visited by the Sun during the year. This is also the order in which the constellations rise and the order of increasing right ascension. The series starts with Aries and progresses eastward. Although the attested date of the zodiac's origin is late, the fact that the spring equinox was actually in Aries between 2000 and 100 B.C. provides evidence for a much earlier, if undocumented, usage. The boundaries of the modern zodiacal constellations as established by the International Astronomical Union are not uniform in extent, but the boundaries of the zo diacal *signs* are. Each zodiacal sign is 18° high, centered on the ecliptic. The Greeks fixed the widths of each of the signs at 30°. The zodiac had an important mathematical use in the ancient world: The number of degrees from the beginning of each sign was used to record planetary positions. This measurement scheme, in use in Ptolemy's

[20] A similar shift from 28 (sometimes 27) zodiacal asterisms (representing lunar "houses," "lodges," or "mansions," that is, places for the Moon to "stay" among the stars during its monthly sojourn around the Earth), to 27 *signs*, beginning with the vernal equinox, occurred in India.

TABLE 2.4. Zodiacal constellations.

Latin (Ptolemaic)	Babylonian	Indian	Symbol	Celestial/ ecliptic longitude[a]
Aries	LU.HUN.GA	Mesa	♈	0°
Taurus	MUL	Vrsabha	♉	30°
Gemini	MASH	Mithuna	♊	60°
Cancer	NANGAR	Karka	♋	90°
Leo	UR.A	Simha	♌	120°
Virgo	AB.SIN	Kanya	♍	150°
Libra	zi-ba-ni-tu	Tula	♎	180°
Scorpio	GIR.TAB	Vrscika	♏	210°
Sagittarius	PA	Dhanus	♐	240°
Capricorn	SUHUR	Makara	♑	270°
Aquarius	GU	Kumbha	♒	300°
Pisces	zib	Mina	♓	330°

[a] In ancient use, (celestial) longitude was measured according to placement within each sign, although the 0° longitude origin was not always taken at the western edge (or "first point") of the signs because of the westward shift of the vernal equinox over time, with respect to the stars.

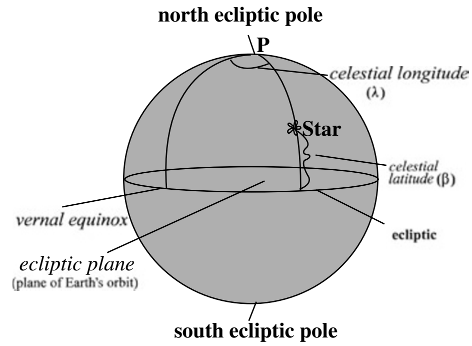


FIGURE 2.12. The ecliptic or "Greek" system of celestial coordinates. The ecliptic is the reference circle, representing the annual path of the Sun. The poles are the north and south ecliptic poles, from which longitude circles radiate. The origin of the coordinates is the vernal equinox, from which the celestial (or ecliptic) longitude increases to the east. The celestial (or ecliptic) latitude is measured positive north and negative south of the ecliptic. Drawing by E.F. Milone.

time,[21] was used in star catalogues well into the 19th century. Subsequently, this expanded into the celestial longitude system,[22] which we discuss next.

2.3.3. The Ecliptic or "Greek" System

The path of the Sun, the *ecliptic*, is the reference great circle for this coordinate system. The ecliptic is the sinusoid crossing the celestial equator in Figure B.1, and in Figure 10.7, it can be seen as the off-center circle crossing the celestial sphere. In ancient China, the ecliptic was known as the "Yellow Road."

The word "ecliptic" (from Latin, "of an eclipse") can be traced back to Greece. It is the path on which eclipses can and do occur, because it is the path of the Sun, and the Moon intercepts this path in two places. Curiously, for the Sun's path, Ptolemy does not use the term ἐκλειπτικός ("eclipticos"—he reserves this term to mean exclusively "having to do with eclipses"), but the phrase "ό λόξος και διὰ μέσων τῶν ζῳδίων κύκλος" ("ho loxos kay dhia menon ton zodion kuklos"; "the inclined circle through the middle of the zodiacal signs") (Toomer 1984, p. 20). Figure 2.12 illustrates the system, showing the north and south ecliptic poles, the secondary circles, called celestial longitude circles, and the coordinates, celestial longitude (λ) and latitude (ß).

Celestial longitude is measured in degrees from the vernal equinox, eastward, along the ecliptic, i.e., counterclockwise as viewed from outside the sphere looking down on the north ecliptic pole. Celestial latitude is measured in degrees north (+) or south (–) of the ecliptic. The terms "longitude" and "latitude" (from the Latin *longitudo*, "length," and *latitudo*, "width") ultimately derive from the Greeks, but Ptolemy uses the term πλάτος ("breadth") for any vertical direction, i.e., declination as well as celestial latitude (Toomer 1984, p. 21). The use of the modern qualifier "celestial" is to avoid confusion with the unrelated terrestrial system, which has a closer counterpart in the equatorial system; "ecliptic longitude" and "ecliptic latitude" are also in current use. Circles parallel to the ecliptic are called celestial latitude circles. They are "small" circles, parallel to, but not concentric with, the ecliptic. An arc contained between two celestial longitude circles is smaller than is the corresponding arc on the ecliptic by the factor $\cos\beta$. As for the other coordinates we have discussed thus far, the quantity celestial longitude can be considered in either of three ways, including a polar angle measured at one of the ecliptic poles. Similarly, the celestial latitude can be considered in either of two ways, including the length of arc between the ecliptic and the object of interest along a longitude circle. With them, we can now consider transformations to and from the (RA) equatorial system. The link between them is the "obliquity of the ecliptic."

Figure B.1 illustrates the ecliptic as it is seen on an equatorial chart. The angle between the celestial equator and the ecliptic is called the *obliquity* of the ecliptic. This is the cause of the seasons, as we have noted above, because when $0 < \delta_\odot \le +\varepsilon$, the Sun's rays fall more directly on the northern latitude zones, and when $-\varepsilon \le \delta_\odot < 0$, they fall more directly

[21] Thus, in the *Almagest* (described in §7.3.2), the longitude of α Ori (Betelgeuse), "The bright, reddish star on the right shoulder" with magnitude "<1," is given as "II [Gemini]" 2°, and its latitude is given as –17°; that of β Ori (Rigel), "The bright star in the left foot, . . ." with magnitude "1," is given as ♉ [Taurus] $19\frac{5}{6}$°, and its latitude $-31\frac{1}{2}$°; and that of α CMa (Sirius), "The star in the mouth, the brightest, which is called 'the dog' and is reddish", of magnitude "1," is given as II [Gemini] $17\frac{2}{3}$°, and its latitude as $-39\frac{1}{6}$°.

[22] The *Berliner Jahrbuch* changed usage in 1829, the British *Nautical Almanac* and the French *Connaissance de Temps* in 1833, to the modern ecliptic system of continuous degrees of celestial longitude from the vernal (March) equinox.

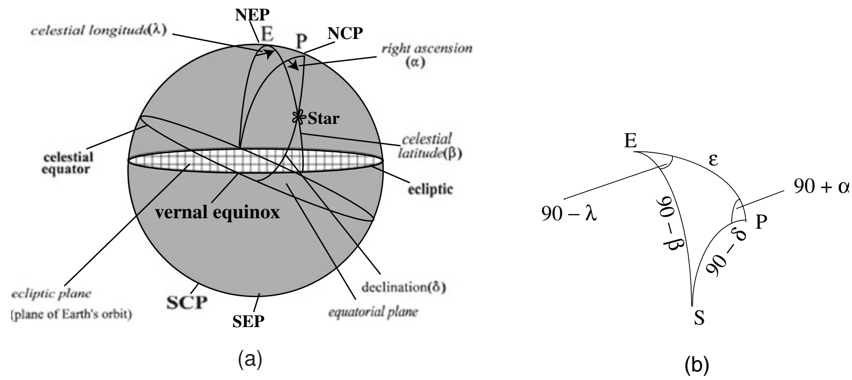


FIGURE 2.13. The equatorial and ecliptic systems (a) superposed on the celestial sphere and (b) the spherical triangle from which the transformations are derived. Drawings by E.F. Milone.

on southern latitude zones. With spherical trigonometry, it can be demonstrated that the maximum and minimum declinations along the ecliptic are $+\varepsilon$ and $-\varepsilon$, respectively. In the IAU 1976 System of Astronomical Constants, the value of the obliquity of the equinox was $\varepsilon = 23°\ 26'\ 21.448'' = 23\overset{\circ}{.}439291$ for the epoch 2000 A.D., but it varies with time [cf. §4.4, (4.22) for the rate of variation of ε].

Figure 2.13 shows both equatorial and ecliptic systems together and the spherical triangle used to transform the coordinates of one system into the other.

The transformation equations may be obtained from applications of the sine and cosine laws of spherical astronomy to yield

$$\sin\beta = \cos\varepsilon\cdot\sin\delta + \sin\varepsilon\cdot\cos\delta\cdot\sin\alpha, \tag{2.10}$$

$$\cos\lambda = \cos\alpha\cdot\frac{\cos\delta}{\cos\beta}, \tag{2.11}$$

$$\sin\delta = \cos\varepsilon\cdot\sin\beta \quad \sin\varepsilon\cdot\cos\beta\cdot\sin\lambda, \tag{2.12}$$

$$\cos\alpha = \frac{\cos\beta\cdot\cos\lambda}{\cos\delta}, \tag{2.13}$$

where α is the right ascension (here, expressed in angular measure: $15° = 1^h$), δ is the declination, β is the celestial latitude, λ is the celestial longitude, and ε is the obliquity of the ecliptic (see §2.4.5 for variations in this quantity over time).

The caution regarding quadrant determination that we urged earlier (§2.2.4) is appropriate here too. Table 2.2 should resolve any ambiguities.

As an example, suppose we wish to find the ecliptic coordinates of an object at $\alpha = 18^h00^m00^s$ or $270\overset{\circ}{.}00000$ and $\delta = +28°00'00''$ or $28\overset{\circ}{.}00000$. At the current epoch, assuming a value $\varepsilon = 23\overset{\circ}{.}441047$, from (2.10),

$$\sin\beta = 0.917470\cdot 0.469472 - 0.397805\cdot 0.882948\cdot(-1.000000)$$
$$= 0.430726 + 0.351241 = 0.781967,$$

so that

$$\beta = \arcsin(0.781967) = 51\overset{\circ}{.}44103 \text{ or } 180° - 51\overset{\circ}{.}44103$$
$$= 128\overset{\circ}{.}55897,$$

from the rules described in §2.2.4. It is obvious that the first value is correct because ß ≤ 90° by definition. From (2.11),

$$\cos\lambda = \frac{0.000000\cdot 0.882948}{0.623320} = 0.$$

Therefore, λ = 90° or 270°.

Because the object is not too far from the celestial equator and $\alpha = 18^h$, 270° is the correct value. If the quadrant were not so obvious, however, one could use (2.12) to resolve the issue:

$$\sin\lambda = \frac{\sin\delta - \cos\varepsilon\cdot\sin\beta}{\sin\varepsilon\cdot\cos\beta}$$
$$= \frac{0.469472 - 0.917470\cdot 0.781967}{0.397805\cdot 0.623320}$$
$$\simeq -1,$$

confirming that λ = 270°.

The use of celestial longitudes spread over 360° is a relatively modern development. The Babylonians and Greeks used degrees of the zodical sign, measuring from the western edge. Ptolemy, for example, gives the position of the star ε UMa as "The first of the three stars on the tail next to the place where it joins [the body]" as ♌ [Leo] $12\frac{1}{6}°$ of longitude, and $+53\frac{1}{2}°$ of latitude (Toomer 1984, p. 34). The equivalent value of celestial longitude is λ = 132°10′. Ptolemy's values differ from current values because of precession (§3.1.6) and the variation of the obliquity of the ecliptic (§4.4), and possibly other factors (see §7.3.2 for an extensive discussion of *whose* data were included in this catalogue).

2.3.4. The Motions of the Moon

The Moon orbits the earth on a path close to, but not on, the ecliptic, changing phase as it does so and basically replicating the motion of the Sun but at a much faster rate, and more variable celestial latitude. The Sun travels its path in a year, and the Moon in a month. In Figure 2.14, the

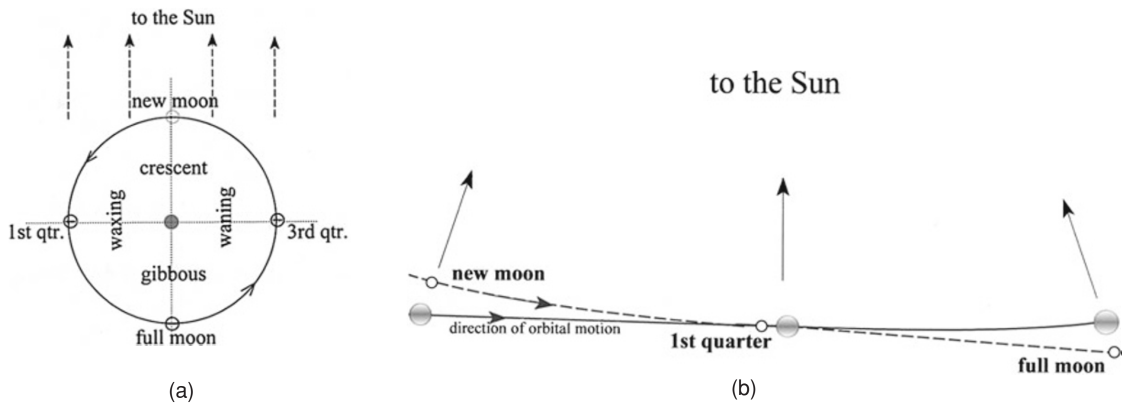


FIGURE 2.14. Lunar phases during the synodic month: (a) The synodic month as viewed geocentrically. (b) Lunar phases for a portion of the month as the Moon-Earth system orbits the Sun. The Moon's orbit is, as the Earth's, concave toward the Sun. Drawings by E.F. Milone.

phase advancement of the Moon during its revolution is chronicled.

Divided vertically, the figure differentiates crescent (less than a quarter Moon) from gibbous phases (more than a quarter Moon). Divided horizontally, it separates waxing from waning phases. The diagram serves to demonstrate the relative motion of the Moon with respect to the position of the Sun in the sky; one full cycle is the *synodic month*, or the month of phases.

As the Moon revolves around the earth, its declination changes in the course of a month, and as it does so, its diurnal arc across the sky changes, just as the Sun's diurnal arc changes over the course of a year. The full Moon, because it is opposite the Sun, rides high across the (Northern Hemisphere) midwinter sky, and low across the midsummer sky. The diurnal arcs of the Moon at other phases can be understood similarly. Although the Sun's rise and set points on the horizon vary slowly from day to day, those of the Moon change much more rapidly from day to day. As the Moon circuits the Earth, the Earth and Moon are circuiting the Sun; in a geocentric context, in the course of a month, the Sun moves East among the stars by ~30°. This means that the synodic month must be longer than the time it takes for the Moon to encircle the Earth with respect to a line to the distant stars. This affects lunar and solar calendars (see §4.2, especially, §4.2.1), and the occurrence of eclipses (§5.2).

The motion of the Moon is even more complex and interesting than that described thus far. For one thing, the Moon's declination is sometimes less and sometimes more than is the Sun's extreme values (±23°.5 at present). This means that the amplitude of its azimuth variation over the month varies from month to month, in an 18.6-year cycle. This fact is of importance in studying alignments to the Moon, as we show throughout §6. For another, the Moon's distance changes during the course of the month by about 10%, and this affects its apparent (angular) size.[23] In addition, the place in the orbit where the Moon achieves its closest point to orbit shifts forward with time. These changes also affect eclipse conditions. To appreciate the full complexity of its behavior in the sky and the roles these play in calendar problems and in eclipse prediction, the moon's orbit must be examined.

2.3.5. Orbital Elements and the Lunar Orbit

In ancient Greece and indeed up to the time of Johannes Kepler [1571–1630], all astronomers assumed the orbital motions of Sun, Moon, planets, and stars either to be circular or a combination of circular motions. Modern astronomy has removed the stars from orbiting Earth and has them orbit the galactic center, which itself moves with respect to other galaxies. The Sun's motion is reflexive of the Earth's and comes close to that of a circle, but not quite. The orbits of the other planets can be similarly described; two of them, Mercury and Pluto, show wide departures and some asteroids and most comets, even more. The combination of a sufficient number of circular terms can indeed approximate the motions, but the physical orbits are more generally elliptical.[24]

One can show from a mathematical formulation of Newton's laws of motion and the gravitational law that in a two-body system, an elliptical or hyperbolic orbit can be expected. If the two objects are bound together (we discuss what this means in §5), the orbit must be an ellipse. Such an ellipse is characterized usually by six unique elements, which we describe and discuss in the next section.

[23] The geometric expression is $r\,\theta = D$, where r is the distance of an object, θ its angular diameter in radian measure ($= \theta° \times \pi/180$), and D is its diameter in the same units as r. This can be called a "skinny angle formula" because it is an approximation for relatively small values of θ. A more general expression would be $D/2 = r \sin(\theta/2)$.

[24] An ellipse can be described geometrically as the locus of all points such that the sum of the distances from the two foci to a point on the ellipse is constant. One may construct an ellipse by anchoring each end of a length of string between two points and, with a pencil keeping the string taut, tracing all around the two points, permitting the string to slide past the pencil in doing so. In orbits, only one focus is occupied, and the other focus and the center are empty.

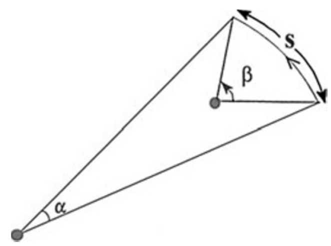


FIGURE 2.15. The relationship between the angular speed of an object and distance. Drawing by E.F. Milone.

As an object moves in an ellipse, its distance from a focus changes. When nearer the Sun, the planets move faster than they do when they are further away from it. These facts are encapsulated in the first two "laws" (limited "descriptions," actually) of the planets' behavior, first formulated by Kepler in 1602. The speed variation arises because the line joining planet to Sun sweeps out the area of the orbit in a uniform way: The areal speed is constant. So the Earth moves faster when it is closer to the Sun, and the Moon moves faster when it is closest to Earth. And this is what seems to occur in the sky: From the Earth, the Sun's motion appears to carry it to the east at a faster rate when it is closer to earth, both because of Earth's orbital motion and because the angular speed of an object moving across our line of sight at a given linear speed increases as the distance to it decreases. The motion of the Moon is also more rapid near perigee. Figure 2.15 illustrates the effect.

In the year 2000, the Earth was at *perihelion* (geocentrically, the Sun was at *perigee*) on Jan. 3 and at *aphelion* (Sun at apogee) on July 4. In the same year, the Moon was at perigee 13 times: Jan. 19, Feb. 17, Mar. 15, Apr. 8, May 6, June 3, July 1, July 30, Aug. 27, Sept. 24, Oct. 19, Nov. 14, and Dec. 12; it was at apogee 14 times, starting on Jan. 4, and ending on Dec. 28. The daily rate of motion of the Sun along the ecliptic was ~1°1′10″ in early January but only ~57′13½″ in early July compared with an average motion of $360°/365^{\mathrm{d}}.24 = 59'8''.3$ (see Section C of the Astronomical Almanac for the year 2000). The Moon's motion is much more rapid, and because the eccentricity is higher than for the solar orbit, the difference in motion is greater from perigee to apogee.

The orbital elements are illustrated in Figure 2.16:

(1) The *semimajor axis*, a, half the major axis, is the time-averaged distance of the orbiter to the orbited. This element defines the size of the orbit and depends on the orbital energy; the smaller the distance, the greater the energy that would have to be supplied for it to escape from the Sun.

(2) The *eccentricity*, e, of the ellipse may be obtained from taking the ratio of the separation of the foci to the major axis, which is just the length of the line joining the perihelion and aphelion. Although a scales the orbit, e defines its shape. From Figure 2.16a, it can be seen that the perihelion distance is $a(1 - e)$ and the aphelion distance is $a(1 + e)$. For the Earth's orbit, $e \approx 0.017$, so that its distance from the Sun varies from the mean by $\pm 0.017a$ or about ±2,500,000 km. The eccentricity of the Earth's orbit is not so important a factor in determining the climate as is the obliquity, but it does cause a slight inequality in the lengths of the seasons, as we noted earlier. The orbit of the moon is sufficiently eccentric that its angular size varies sharply over the anomalistic month (the time for the Moon to go from perigee to perigee; see below).

(3) The *inclination*, i or ι, the angle between the reference plane—in the case of the Moon and planets, the ecliptic plane—and that of the orbit, partially fixes the orbital plane in space, but another is needed to finish the job (see Figures 2.16b and c).

(4) The *longitude of the ascending node*, Ω, is measured along the ecliptic from the vernal equinox to the point of orbital crossover from below to above the ecliptic plane. This element, with i, fixes the orientation of the plane in space.

(5) The *argument of perihelion* (for the moon, perigee is used), ω, measured from the ascending node in the direction of orbital motion. This element fixes the orientation of the orbital ellipse within the orbital plane.

(6) The epoch, T_0, or T or sometimes E_0, is the sixth element. In order to predict where the object will be in the future, a particular instant must be specified when the body is at some particular point in its orbit. Such a point may be the perihelion for planets (or perigee for the Moon) or the *ascending node*, where the object moves from south to north of the ecliptic plane; however, it may be an instant when the object is at any well-determined point in its orbit, such as the *true longitude* at a specified instant.

(7) Sometimes a seventh element is mentioned—the *sidereal period*, P_{sid}, the time to complete a single revolution with respect to a line to a distant reference point among the stars.[25] P_{sid} is not independent of a because the two quantities are related through Kepler's third law,[26] but the Sun's mass dominates the mass of even giant Jupiter by more than 1000 : 1. For the high precision required of orbital calculations over long intervals, it is necessary to specify this or a related element (the mean rate of motion).

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[25] A sidereal period usually is not expressed in units of sidereal time; mean solar time units such as the mean solar day (MSD) are used, in general, and the designation is *day* (d, sometimes in superscript). This need not be the same as the local civil day, i.e., the length of a day in effect at a particular place. See §4.1 for the distinctions.

[26] The third law relates the period, P, to the semimajor axis, a. In Kepler's formulation, the relation was $P^2 = a^3$, if P is in units of the length of the sidereal period of Earth and a is in units of the Earth's semimajor axis. In astronomy generally, $a_\oplus$ defines the *astronomical unit*. From Newtonian physics, it can be shown that the constant of proportionality is not 1 and is not even constant from planet to planet: $P^2 = \{4\pi^2/[G(\mathfrak{M} + \mathfrak{m})]\}\ a^3$, where G is the gravitational constant, $6.67\ 10^{-11}$ (MKS units), and $\mathfrak{m}$ and $\mathfrak{M}$ are the masses of the smaller and larger mass bodies, respectively.

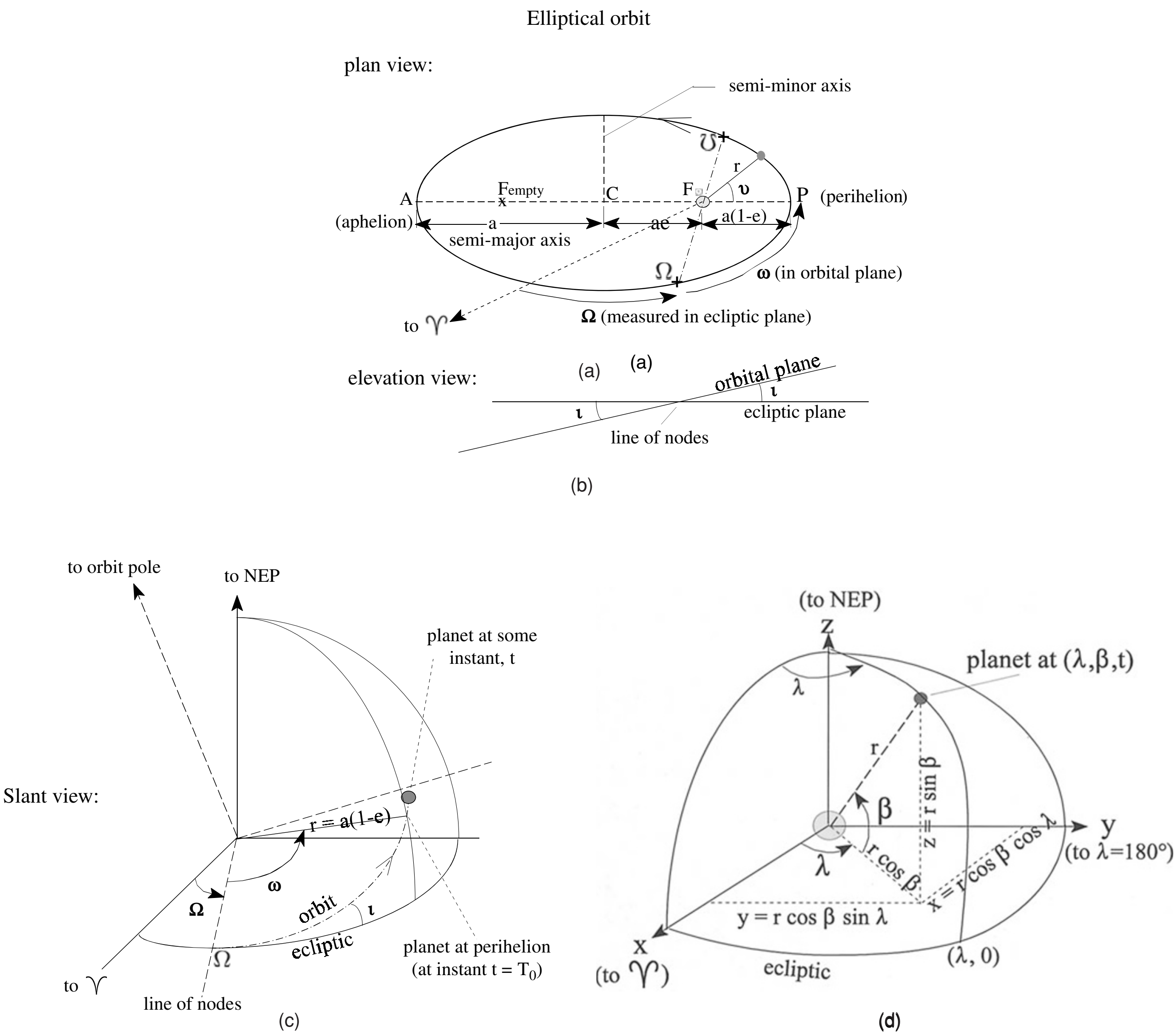

FIGURE 2.16. Elements and other properties of an elliptical orbit: (a) "Plan" and (b) "elevation" views, respectively—The scale and shape of the orbit are established by the semimajor axis, a, and the eccentricity, e. The orientation of the orbital plane with respect to the ecliptic is set by the inclination, i, and the longitude of the ascending node, Ω; the orientation of the orbit within the plane is fixed by the argument of perihelion, ω. The instant of the location of the planet at the perihelion, $r = a(1 - e)$ (or, when $e = 0$, at the ascending node), T_0, is the sixth element; the seventh, the period, P, is not an independent element since it is related to a by Kepler's third law. (c) "Slant view"—The position of the planet in Cartesian coordinates aids the transformation from the orbit to the sky. The relationship between the orbital and ecliptic coordinates are found by successive rotations of the axes shown. (d) The relations between the celestial longitude and latitude and the Cartesian ecliptic coordinates—A further transformation to equatorial coordinates can be carried out through spherical trigonometry or through a transformation of Cartesian coordinates. See Schlosser et al. (1991/1994) for further details. Drawings by E.F. Milone.

The elements of the lunar orbit at a particular date are shown in Table 2.4. Given the elements, one may find, in principle, the position of an object in the orbit at any later time. The angle swept out by the Sun-planet line is called the *true anomaly* (υ in Figure 2.16b). The position of the object in its orbit at any time t since perihelion passage (T_0) can be specified through a quantity called the *mean anomaly*:

$$M = \frac{2\pi}{P}(t - T_0). \tag{2.14}$$

This angle describes the position of a planet that would move at the same average rate as the planet, but in a circular orbit. The mean anomaly is related to the *true anomaly* by the approximation

$$\upsilon - M = 2e - \frac{1}{4}e^3 \cdot \sin M + \frac{5}{4}e^2 \cdot \sin 2M + \frac{13}{12}e^3 \cdot \sin 3M + \cdots. \tag{2.15}$$

This is merely the difference between the actual position of the object in the orbit and the position it would have if it moved at a constant rate. The elements Ω, ω, and T_0 and are sometimes combined with each other or with the true or mean anomaly to produce *longitudes*. For example, the *longitude of the perigee* (or *longitude of the perihelion*),

$$\tilde{\omega} = \Omega + \omega, \tag{2.16}$$

is a very curious angle because it is measured first in the ecliptic, from the vernal equinox to the ascending node, and then in the orbit, in the direction of orbital motion. Another example is the *mean longitude*, ℓ (called in the Astronomical Almanac, L),[27]

$$\ell = \tilde{\omega} + M = \Omega + \omega + n(t - T_0), \tag{2.18}$$

where n is the *mean motion* = 360°/P, and t is the time of observation or calculation. Therefore, the *mean longitude of the epoch*, ε, is merely the value of ℓ when t is T_0 (the instant that defines the epoch):

$$\varepsilon = \ell(t = T_0) = \tilde{\omega} \tag{2.19}$$

(Danby 1962, p. 156). Please note that *this* epsilon is *not* the obliquity of the ecliptic. Another parameter that is sometimes mentioned is the *argument of the latitude*, u, the angle between the ascending node and the object in its orbit, so that we can also express the true longitude in terms of the argument of latitude:

$$L = \Omega + u. \tag{2.20}$$

The mean elements of the Moon's orbit are given in Table 2.5. Only mean or average elements can be given because they vary with time, usually both secularly (rate change with constant sign, i.e., always increasing or always decreasing) and periodically. Danby (1988, App. C, pp. 427–429) provides for higher order terms for the time variation of the elements of the major planets. Now we are in a position to discuss why the elements change with time.

We can approximate the orbits of the Moon or some planet with a set of orbital elements for an instant of time (for some planets, considerably longer), but the elements of the ellipse vary over time because of perturbations of the other bodies (and, especially in the case of the Earth-Moon system, nonuniform mass distributions in the bodies themselves). The fly in the ointment is that the Earth-Moon is really an Earth-Moon-Sun system, a three-body system, for which there is no complete general solution.

[27] Danby uses L to define the "true longitude" of the planet:

$$L = \tilde{\omega} + \upsilon = \Omega + \omega + \upsilon. \tag{2.17}$$

TABLE 2.5. Lunar orbit mean elements (2000.0).

Element	Mean value	Main variation
Semimajor axis (a)	384,400 km	+3 cm/yr
Eccentricity (e)	0.054900489	±0.0117
Inclination[a] (ι)	5°.1453964 = 5°8′43″	±9′
Longitude of ascending node[b] (Ω)	125°.123953	−0°.052 953 76/day
Argument of perigee[b] (ω)	83°.186346	+0°.111 403 55/day[c]
Epoch[d] (T_0)	2000 Jan 19.9583	

[a] With respect to the ecliptic; the inclination w.r.t. the celestial equator varies from 18°.28 to 28°.58. The period of the 9′ variation is 17ᵈ.33.
[b] The value and its variation are correct only for 2000; the current Astronomical Almanac should be consulted for accurate calculation. The major periodic variation of Ω is ±100′.
[c] This also includes the motion of the ascending node and is thus the variation of the *longitude* of perigee. The major periodic variation of ω is ±12°20′.
[d] An instant of perigee during the year 2000.

If an infinitesimally small but fully massive Moon moved around an infinitesimally small but fully massive Earth (i.e., the mass of each body was fully concentrated at its center) and if the effects of the Sun and all other planets could be ignored, the Moon's orbit would be a simple ellipse with the Earth at one of the two focuses of the ellipse. These conditions are not met, and as a consequence, the orbit is anything but simple. Newton used to say that his head ached when he thought about the Moon.

The perturbations on the Moon are particularly great because it moves nearly on the ecliptic, but not exactly on it. That the orbit should be near the ecliptic is curious, because as a satellite of the Earth, we could expect it to move near the plane of the earth's equator, which is the case for most of the other major satellites of the planets. Our satellite is, however, far enough from the Earth at present that its motion is effectively dominated by the Sun, so that the Earth and Moon form a kind of double-planet system. Even so, it is close enough to the Earth to undergo, as well as cause, tidal effects that have slowed the Moon's rotation to equal its orbital period, and to result in an increasing distance from the Earth, and an increasing length of month. Tidal effects on the Earth are resulting in a slowing down of Earth's rotation (see §4.5), which affects the timing of ancient phenomena, such as eclipses (see §5.2, especially, §5.2.1.3).

The solar system is an n-body system, and each object is accelerated by all the other objects. Whereas for a three-body system, a special solution is found for the circular orbit case, when the third body has negligible mass, there is no analytic solution for $n > 3$, and no general solution for $n > 2$. By Newton's gravitational law, the force acting on an object depends on the mass of the perturber and on the inverse square of the distance from the perturber. The acceleration depends on the size of that force and inversely on the mass of the object undergoing the acceleration. The acceleration due to each perturber adds in vector fashion; this means that the direction of each perturber must be taken into account. The net acceleration of the body is slightly different from that due to the Sun alone. In the next instant, the acceleration causes a change in the speed and direction of motion (together called the velocity, a vector). In the next instant, the slightly altered velocity causes a slight shift in position of the object, causing its orbit to change. In this way, the orbit of each object is perturbed away from the elements that characterize it at some particular epoch. In the Earth-Moon system, the Earth may be considered a major perturber of the Moon's orbit about the Sun; the Earth's slightly irregular mass distribution is an additional source of perturbation. There are two types of perturbation effects: those which cause an element to oscillate about a mean value over time, and those which cause a variation of constant sign with time; these are called *periodic* and *secular* variations, respectively. Table 2.5 gives both types, although only the largest of the periodic variations are shown. The perturbations must be taken into account in lunar orbit calculations; without them, the results could be wrong by several degrees.

The average variations in the elements (from Danby 1962, p. 278; 1988, p. 371) are e: ±.0117; ι: ±9 arc min; Ω (variation about its average motion): ±100 arc min; and ω (variation about its average motion): 12°20′. The average motion of the ascending node is about $-19\overset{\circ}{.}35$/y and that of the perigee is about +40°/y. The average rates given in Table 2.5 are appropriate only for the year 2000; for high-precision purposes, data should be taken from the current almanac.

Even though they lacked an adequate physical theory to understand the motions they observed, the astronomers from ancient Greek times to those of the Copernican era were capable of discerning the effects of the perturbations. The variations in some of the elements of the Moon's motion are large enough to have been noticed in the ancient world.

The most important term in the difference between the true and mean anomaly expressed in (2.15) is

$$2e \sin M = (6°17') \cdot \sin M, \tag{2.21}$$

but because of the perturbations in ω and e, an additional term should be included to describe $\upsilon - M$ adequately:

$$(1°16') \cdot \sin(2\omega - 2\lambda_{\odot} + \lambda_M), \tag{2.22}$$

where $\lambda_{\odot}$ and λ_M are the celestial longitudes of the Sun and Moon, respectively. The perturbations in e and in ω are caused by the Sun's position[28] at perigee and apogee. These result in a large perturbation in the Moon's celestial longitude, with an amplitude of ~1°16′ and a period of $31\overset{d}{.}80747$ (Brouwer and Clemence 1961, p. 329). The effect was noted by Ptolemy on the basis of observations by himself and Hipparchos (cf. Toomer 1984, p. 220) and is known as the *evection*.

Another large effect on the Moon's celestial longitude, the *variation*, was discovered by the Danish astronomer Tycho Brahe [1546–1601]. It has an amplitude of 39′ and a period of $P_{syn}/2$; it is maximum at the quadratures (quarter Moon phases) and vanishes at oppositions and conjunctions (full and new Moons) (Brouwer and Clemence 1961, p. 626). The "variation" is large enough to have been detected in the ancient world; yet there is no explicit mention of it by Ptolemy. This has been attributed to the circumstance that the Greeks were working mainly with eclipse data—and therefore with data taken at oppositions (for lunar eclipses) and conjunctions (solar eclipses). In any case, the possibility that Ptolemy discovered this effect has been discounted (cf. Pedersen 1974, p. 198). It is interesting to note that Ptolemy's theory of the Moon's motion predicted a variation in angular size of the Moon that was clearly contradicted by observational data that must have been known to him. See §7.3.2 for a further discussion. We also consider the observability of the variation of the inclination by much earlier observers (megalithic!) in §6.2.

The month is a unit of time associated with the Moon, and we will discuss the month in the context of time and time intervals in §4.14, but there are actually several kinds of months, which help to highlight aspects of the Moon's complex motions. With one exception, they are the periods of the Moon in its orbit with respect to particular reference points or directions:

(1) The *sidereal month* is the orbital or the sidereal period; it is the period of revolution of the Moon around the Earth with respect to a line to a distant star.

(2) The *tropical month* is the interval between successive passages of the Moon through the vernal equinox. Due to *precession* (from the long-term wobbling of the Earth, as the Moon and Sun act to pull the equatorial bulge into the ecliptic plane; see §3.1.3), the vernal equinox is slowly moving westward in the sky at a rate of about 50″/year. Therefore, the tropical period is slightly shorter than the orbital or sidereal period.

(3) The *draconitic*, *draconic*, or *nodal month* is the interval between successive lunar passages through the ascending node. Because the node is regressing at a relatively high rate, the Moon meets it much sooner than it would a line to a distant star. It is therefore much shorter than the sidereal month. Figure 2.17 illustrates the changing appearance of the lunar orbit with respect to the (a) horizon and (b) ecliptic because of the regression of the nodes, and the changing diurnal arcs during the month from major to minor standstill.

(4) The *anomalistic month*, the period from perigee to perigee. The *argument of perigee*, the angle between ascending node and the point of perigee in the orbit, is advancing, i.e. moving eastward in the direction of the Moon's orbital motion, and so the anomalistic month has a longer length than the sidereal month.

(5) The *synodic month* is the month of phases, the interval from new Moon to new Moon. It is the period with respect to a line between the Earth and the Sun. The Earth's motion around the Sun shows up in the eastward shift of the Sun, that is, in the direction of the Moon's orbital motion. The Moon's catching up to the Sun causes the synodic month to be longer than the sidereal month. Because it is not an

28 The Sun takes ~$205\overset{d}{.}9$, not half a tropical year ~$182\overset{d}{.}6$ to move from the longitude of lunar perigee to that at apogee because of the advancement of the apsidal line of the Moon's orbit.

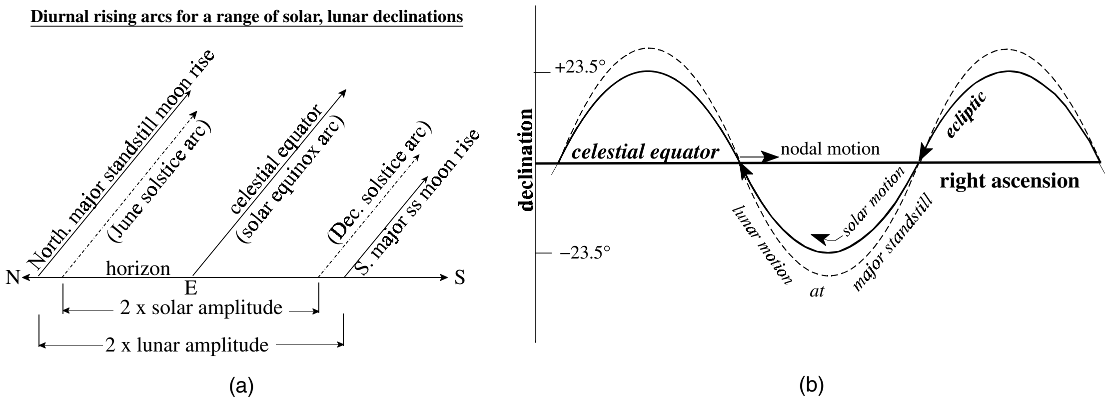

FIGURE 2.17. The changing appearance of the lunar orbit with respect to (a) the horizon and (b) the ecliptic because of the regression of the nodes. For clarity, only the major standstill of the Moon is illustrated. Drawing by E.F. Milone.

TABLE 2.6. Lengths of lunar months.

Type of month	Length
P_{sid}	$27^{d}.321\,662 = 27^{d}07^{h}43^{m}11^{s}.6$
P_{trop}	$27^{d}.321\,582 = 27\ 07\ 43\ 04.7$
P_{drac}	$27^{d}.212\,221 = 27\ 05\ 05\ 35.9$
P_{anom}	$27^{d}.554\,550 = 27\ 13\ 18\ 33.1$
P_{syn}	$29^{d}.530\,589 = 29\ 12\ 44\ 02.9$

integral number of days, this has had important consequences for calendars involving the moon. When we mention the word *lunation*, we usually refer to the synodic month.

(6) The *civil month* is the unit of month in use within a certain political jurisdiction. The modern civil month has 28, 29, 30, or 31 integral days, depending on the particular month and year. It evolved from the synodic month (see §4.2).

The lengths of these various types of months are summarized in Table 2.6; the synodic and civil months will be further discussed in §4.1.4, whereas the implications of the lunar motions for the azimuths of rise and set and the visibility of the Moon will be discussed in §3.2 and for eclipses in §5.2.

The Moon's motions are difficult to follow, and it is to the great credit of the ancient observers that they made as much sense of these motions as they did. In addition, they were able to find several examples of regularity in lunar motion.

It is sometimes said that the Moon "comes back to the same place" after not one but 3 sidereal months. After one sidereal month, the Moon is at the same position among the stars; so if the phrase has meaning, it must refer to the location in the sky of the observer. After one sidereal month, the Moon will not be at the same hour angle because the sidereal month interval is not an integral number of days. Three sidereal months, however, amount to nearly an integral number of days. From Table 2.6, where the units for all months are in mean solar days (MSDs),

$$3 \cdot P_{sid} = 81^{d}.96499 = 81^{d}23^{h}9^{m}35^{s}.$$

Thus, the Moon will be about an hour east of the meridian after three sidereal months at the same time of night—and in the same constellation—but it will be at a different lunation phase. An interval of three synodic months covers $88^{d}.59177$, at the end of which, the lunar phase is repeated, so that the phase after three sidereal months will be earlier by an angle of roughly

$$\Delta\Phi = \frac{88^{d}.59177 - 81^{d}.69499}{29^{d}.530589} = 0.23355 \text{ lunation.}$$

Thus, if the Moon was initially full, three sidereal months later, it would be in a waxing gibbous phase, just after first quarter, having slipped back not quite a quarter phase. For every subsequent three-month interval, the Moon's phase will slip by an additional 0.234 lunation on average. In 12 sidereal months, [4 × (0.23355) = 0.9342], the phase repeats more closely, but results in a slight phase shift for each such 12-sidereal-month interval.

The movement of the Moon among the stars requires that the Moon traverse a different region each day for the approximately 27 days of its sidereal period. In an anthropomorphic sense, it spends each night in a different "house."[29] The perception and transmission of lunar mansions from one culture to another will be discussed throughout §§6–15 and in some detail in §§7 and 15.

The regression of the nodes of the Moon's orbit has a major consequence for the behavior of the Moon in our sky; one that is spectacular at high latitude locations on Earth. Over an 18.6-year period, the shifting node alters the range in declination achieved by the Moon during the month: from $\sim\pm18\frac{1}{2}°$ to $\sim\pm28\frac{1}{2}°$; this variation changes the azimuthal

[29] Or "lodge" or "mansion."

Table 2.7. The ancient planetary names.

Modern	Greek	Babylonian	Persian	Indian	Chinese
Sun	Helios/Apollo	Shamash	Mithra	Surya	Thai Yang (Greater Yang)
Moon	Selene	Sin	Mâh	Soma	Thai Yin (Greater Yin)
Mercury	Hermes/Apollo	utu	Tîra/Tîr	Budha	Chhen hsing = Hour Star
Venus	Aphrodite	dili-pát Ishtar	Anâhitâ	Śukra	Thai pai = Great White One
Mars	Ares/Herakles	an, ^{d}sal-bat-a-ni Nergal	Verethragna	Kārrtikeya	Ying huo = Fitful Glitterer
Jupiter	Zeus	múl-babbar Marduk	Ahura Mazda/Oromasdes	Brhaspati	Sui hsing = Year Star
Saturn	Kronos/Chronos	genna	Zervan (Zurvan)	Prajapati (Śanaiścara)	Chen hsing = Exorcist

amplitude, resulting in a striking weaving movement of the rise and set points of the Moon on the horizon over an 18.6-year interval. The phenomenon bears strongly on the question of megalithic lunar alignments discussed in §3.2.1 and at length, in applications, in §6.

That there is a difference between the anomalistic and sidereal periods implies the rotation of the orbit of the Moon such that the line of apsides (the major axis) moves forward (i.e., eastward, in the direction of the Moon's motion in its orbit). This motion was also known in ancient China, apparently. Needham (1959, Fig. 180, p. 393) shows a diagram with a series of overlapping orbits called "The Nine Roads of the Moon," which, he writes, are due to apsidal motion, as it was understood in the Han.

The physical appearance of both Sun and Moon over time, eclipse phenomena, and association of these bodies with tides, we leave to later chapters; these phenomena too have had profound effects on the history of astronomy and, indeed, of civilization.

2.4. The Planets

2.4.1. Wanderers

Compared with the constellations and other relatively fixed stars, some are "wandering stars," the translation of the Greek words (αστέρες) πλανῆται or πλάνητες άστέρες (singular: πλανήτης, sometimes πλάνης, or πλάνητος) from which we derive our word *planets*. In Wagner's opera *Die Walküre*, Wotan is called simply "the wanderer," the still powerful, but fatally limited, lord of the heavens. Because they took certain liberties compared with the fixed stars, the astral entities we know as planets appeared to have intelligence. Moreover, they were far above the Earth, apparently immune from local plagues and disasters and, therefore, were of a higher order of being than was mankind. Because they were evidently immortal beings, they were necessarily gods.[30] We know the names of these gods. In the Greek world of the 3rd century B.C., there were seven planetary gods: Selene (the Moon), Hermes (Mercury), Aphrodite (Venus), Helios (the Sun), Ares (Mars), Zeus (Jupiter), and Kronos (Saturn).

In India, there were two dark, and therefore invisible, additional planets—the head and tail of the dragon, *Rahu-head* and *Rahu-tail*, or *Rahu* and *Ketu*. These invisible planets were later interpreted as the ascending and descending nodes of the moon's orbit respectively, which caused eclipses.

The planetary names given by the German tribes can be found in several of the days of the week as expressed in English and in several other languages. The days of the week arise from a scheme for the order of the planetary orbits (cf. §4.1.3). The names by which the planets, or their associated gods (Ptolemy refers to each planet as "the star of . . ."), were known to various cultures can be found in Table 2.7. They include the Sun and Moon, which in antiquity were considered among the planets, because they too wander among the stars.

The Greek list is from late antiquity (after ~200 B.C.). A Hellenistic list dating from the latter part of the 4th century B.C. is given by Toomer (1984, p. 450 fn. 59): Stilbon (Στιλβων) for Hermes; Phosphorus (Φωσφωρως) for Aphrodite; Pyroeis (Πυροεις) for Ares; Phaethon (Φαεθων) for Zeus; and Phainon (Φαινων) for Kronos, at least sometimes identified with Chronos (time) [van der Waerden (1974, pp. 188–197)]. At still earlier times (and in Ptolemy's Almagest), they were called by their *late antiquity* sacred names but with the prefix "star of."

The Persian names are widely attested. The spelling used here is from van der Waerden (1974). See Cumont (1960) (and §§7.3.3 and 15) for further discussion of the role of Mithras.

The Babylonian names, from Neugebauer (1955/1983) and van der Waerden (1974), include both the names of their cuneiform signs first, and, following, the names of the associated gods. The name for Jupiter literally means "star white."

Yano (1987, p. 131) provides parallel lists of the planets ordered in weekday order in Sogdian, and in Indian Sanskrit, from a Chinese text of the 8th century, entitled *Hsiu-yao Ching*. In addition to the known planets, *The Book of Master Chi Ni* (*Chi Ni Tzu*) names an invisible "counter Jupiter," *Thai Yin* (Needham/Ronan 1981, p. 190), which had primarily astrological purposes. The Moon was given this name by the 1st century (Needham/Ronan 1981, pp. 89, 90), but apparently has nothing to do with the invisible planet.

[30] DHK thinks it is truer to say that the characteristics of the planets determined the nature of what came to be called "gods." EFM thinks the point is moot.

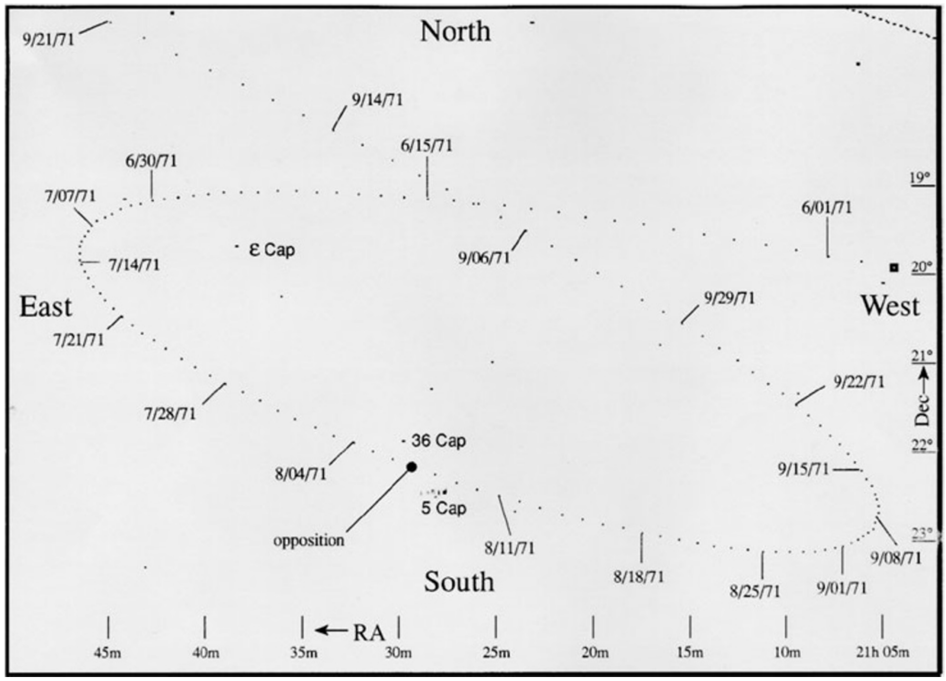

FIGURE 2.18. The movement of the planet Mars, showing its retrograde motion between July 14 and September 10, 1971. The position of opposition is marked. Produced by Bryan Wells with the Voyager II software package (Carina Software).

The wandering of the planets is primarily eastward among the stars, although the eastward motion is less dominant for Mercury and Venus, as those planets pass between the Earth and the Sun moving rapidly westward. The eastward motion is called *direct* motion. The average eastward motion is slower as one descends Table 2.7. For any planet, there are times when the motion is westward, or *retrograde*. In order to accomplish this result, the planet must slow its eastward motion and stop, thereby displaying variable speed across the sky. This behavior was carefully noted by the Babylonian astronomers, and later, by others. The motion of Mercury relative to the ecliptic is depicted as a circle in the *Thu Shu Chi Chhëng* of 1726, as described by Needham/Ronan (1981, p. 189).

An example of retrograde motion for an exterior planet, Mars, is shown in Figure 2.18. The positions of Mars over a 4.5-month interval are shown along with its location at opposition and a few of the stars in the vicinity. The explanation of this motion in the geocentric framework that dominated attention in antiquity required extensive geometrical modeling. Combinations of circular motion succeeded, to various degrees, with the developments of concentric spheres (§7.2.3) and eccentric circles and epicycles. The latter marked the climax of Ptolemy's astronomy (§7.3.2).

2.4.2. Morning and Evening Stars

Any object that rises within a few hours before sunrise will be seen in the eastern, morning sky. Such an object, particularly a bright object, can be called a *morning star*. Similarly, any object setting within a few hours following sunset, and therefore visible in the western, evening sky, can be called an *evening star*. Planets are among the brightest objects in the sky and, because of their wanderings, will noticeably appear and disappear in both roles. Venus is particularly dominant as an evening or a morning star: It can be the brightest object in the sky after the Sun and Moon. Venus can cast shadows in an otherwise dark sky, and it can be seen by a sharp eye sometimes even in daylight. In a twilight sky, it can dominate all other celestial objects. Often in popular and classical literature, and in the arts, "the evening star" refers solely to Venus. In Figure 2.19, the brilliance of Venus in evening twilight shows us why.

In Wagner's epic opera *Tannhäuser*, the goddess of love makes an onstage appearance. Curiously, though, the evening star is not equated with the divine sexpot, but rather with the pure and noble Elisabeth, her opposite pole. The dichotomy is between the beauty and inspiration of the evening star and the lusty Venus of Venusberg, the cause of Tannhäuser's downfall, as it were. For similar reasons, "the morning star" may indicate Venus alone of all potential dawn twilight candidates.

The Greek world identified the two appearances of (Aphrodite): As evening star, it was *Hesperus*, to which our word "vespers" (evensong) is related. In its morning star role, it was known as *Phosphorus*, "bearer of light." It may be startling to some to realize that its Latin counterpart is *Lucifer*, "bringer of light."[31]

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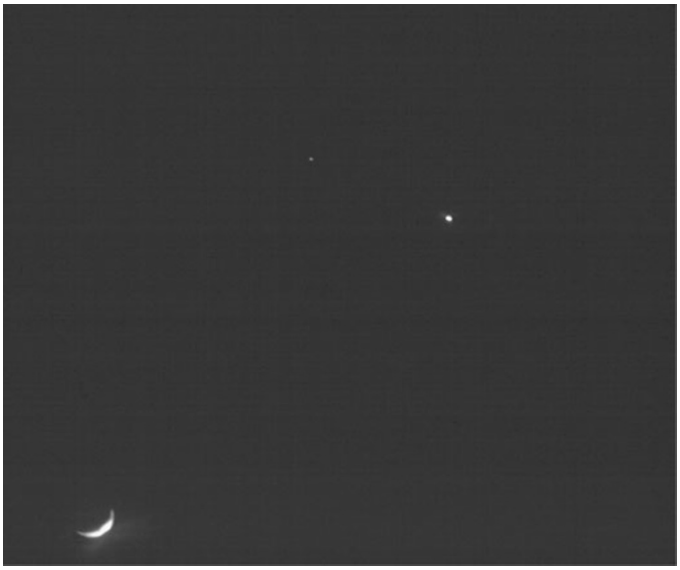
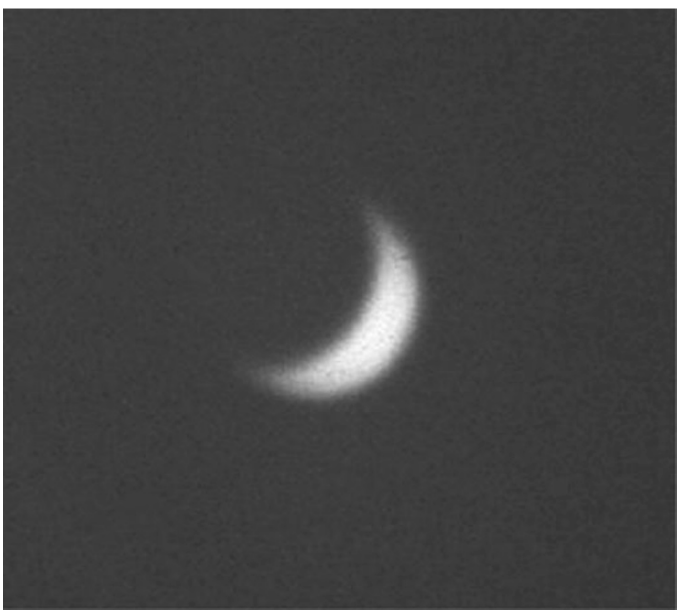
[31] Among others, Gray (1969/1982, pp. 132–133) traces the concept of Lucifer as fallen angel (that of Milton's *Paradise Lost*) to Isaiah



(a)

(b)



(c)

Figure 2.19. At maximum brightness, Venus is the brightest object in the sky after the Sun and the Moon: (a) Venus as an evening star, shown here with the Moon and Mars in a 1-second, 210-mm exposure, Calgary, Jan. 24, 1985. (b) Venus at dusk at the European Southern Observatory, Chile, Jan. 1977. Photos by E.F. Milone. (c) A telescopic (41-cm) view of Venus taken at the RAO at elongation, from the archives (no other details recorded).

Galileo (1610) first observed that "the mother of loves emulates Cynthia" (the Moon) on the basis of his telescopic studies. Venus undergoes changes of phase, angular size and distance, seen by the unaided eye as a waxing and waning of its brightness—like the fortunes of love (see Figure 2.19c). The planet was considered the visible manifestation of the goddess in the Mediterranean region (Roman *Venus*, Greek *Aphrodite*, Babylonian *Ishtar*, etc.) (van der Waerden, 1974, p. 57), but the depiction of Venus as a female deity was not universal. *Athar* was the

(14:12–20): "[12]How art thou fallen from heaven, O Lucifer, son of the morning! *how* art thou cut down to the ground, which didst weaken the nations! [13]For thou hast said in thine heart, I will ascend into heaven, I will exalt my throne above the stars of God" (King James version). According to Gray, this is not a direct reference to Satan, but to the king of Babylon (most likely Sargon II or Sennacherib); Isaiah is referring to a Babylonian myth that describes the attempt of Athar, the Venus god among the Arabs, to take Baal's place while the god was absent. We think that a direct astronomical identification with Venus as evening star becoming morning star is, however, likely here. The subsequent Christian view of Lucifer derives partly from a definition of Satan in the Council of Braga, 563 A.D. (Metzger and Coogan 1993, p. 679).

name of the Semitic Venus god; it was not the only male Venus god.

In Mesoamerica, which also had a male Venus god, the growth of brilliance of Venus as evening star, its eventual decline, and its return as a bright morning star were powerful symbols of struggle, death, and rebirth. There is a possible depiction of the Venus legend of Mesoamerica on the wall of a ballcourt in El Tajin.[32] In Western culture, the analogy between the morning star and resurrection is not as widespread or explicit, but these references to the morning star in the New Testament[33] are metaphors for the second coming:

> For we did not follow cleverly devised myths when we made known to you the power and coming of our Lord Jesus Christ, but we were eye-witnesses of his majesty. For when he received honor and glory from God the Father and the voice was borne to him by the Majestic Glory, 'This is my beloved Son, with whom I am well pleased,' we heard this voice borne from heaven, for we were with him on the holy mountain. And we have the prophetic word made more sure. You will do well to pay attention to this as to a lamp shining in a dark place, until the day dawns and the morning star rises in your hearts. [2 Peter 1:16–19]
>
> Behold, I am coming soon, bringing my recompense, to repay every one for what he has done. I am the Alpha and the Omega, the first and the last, the beginning and the end.
>
> I Jesus have sent my angel to you with this testimony for the churches. I am the root and the offspring of David, the bright morning star. [Revelation 22:14, 16]

The passage from Revelation invokes the completion of a cycle, and the "Morning Star" reference applies the metaphor of the Venus cycle.

The visibility of an object in the evening or morning sky depends mainly on its angular distance from the Sun, but also on the observer's latitude and the time of year. It is reported that Venus was actually observed as an evening star on one evening and as a morning star the next day by observers on the Yucatan peninsula in Mexico. Although unlikely under most circumstances, it does occur. If Venus or Mercury are far north of node while they are passing between the Earth and the Sun, thanks to the tilt of the Sun's diurnal path near the horizon, they can be seen to the north of the Sun just after sundown, and again north of the Sun the following morning. Figure 2.20 illustrates various orientations of the ecliptic and celestial equator to the east and west points of the horizon at the equator and at mid-latitude sites for the important turning points of the seasons: the solstices and the equinoxes. We deal with the related question of the visibility of an object close to the Moon or Sun in §3.1.2.5.

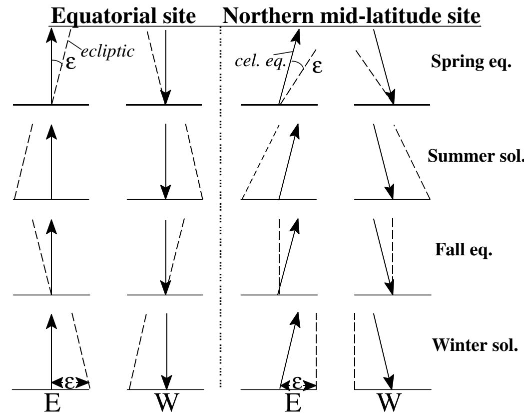


FIGURE 2.20. The orientations of the ecliptic and celestial equator to the horizon near the east and west points of the horizon as seen from the equator and from mid-latitude sites for the important turning points of the seasons: the solstices and the equinoxes. These are views from inside the celestial sphere. Drawn by E.F. Milone.

TABLE 2.8. Planetary phenomena.

Elongation[a]	Phenomenon	Symbol
0°	Conjunction	☌
60°	Sextile	∗
90°	Quadrature	□
120°	Trine	Δ
180°	Opposition	☍

[a] Elongation from the Sun or relative separation between planets.

2.4.3. Planetary Phenomena

Morning and evening stars are only aspects of a more general class of observed events collectively known as *planetary phenomena*. The configurations that the planets achieve with the Sun, stars, or with each other, enabled early observers to keep track of the planets' motions and, from these, to discover periodicities. The phenomena were summarized in terms of elongations or differences in celestial longitude (see Table 2.8). Astrologers make use of all the configurations, but the sextile and trine configurations are not often referenced in modern astronomy. Among other astrological terms that are used to refer to the positions of planets in the sky are *ascendancy* (rising), *descendancy* (setting), *medium caelum* or *mid-heaven* (where the object traverses the celestial meridian),[34] and *imum caelum* or *anti-heaven* (where the object traverses the portion of the celestial meridian below the horizon). Figure 2.21 demonstrates the geocentric planetary configurations, viewed from the north ecliptic pole.

[32] DHK finds this interpretation by C. Cook de Leonard (1975) of the ballcourt panels in this Gulf-coast city of ancient Mexico unconvincing.

[33] See also: Revelation 2:27, based on the Messianic symbolism based on Numbers 24:17 ("A star shall come out of Jacob and a Sceptre shall rise out of Israel"); Matthew 2:2 and 2:10; and our discussion of the Star of Bethlehem in §15. All citations are from the Revised Standard Version (Thomas Nelson and Sons: New York, Edinburgh), 1946.

[34] For a circumpolar object, the "mid-heaven" refers to the upper of the two meridian transits, namely, the *upper culmination*.

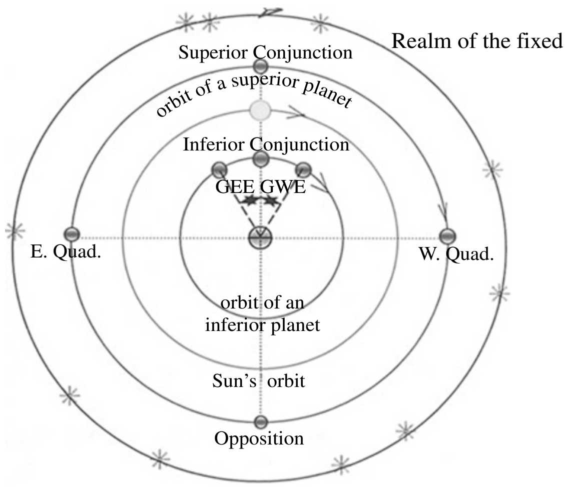


FIGURE 2.21. The geocentric planetary configurations—and cosmology—of antiquity. Drawing by E.F. Milone.

Note that an object at a conjunction will rise at the same time as the Sun,[35] whereas an object at opposition will be opposite the Sun in the sky and so will set as the Sun rises, and rise as the Sun sets. Planetary phenomena may involve another planet, the Moon, or a star, but in such cases, the other object is always named. The Sun is intended implicitly when no other object is stipulated. Several other terms that depend on sky location are the sextile (separation of 60°), quadrature (90°), and trine (120°). At quadrature, a planet will rise ~6 hours before (if at western quadrature) or after (if at eastern quadrature) the Sun. The sextile and trine are little used in astronomy, but are frequently used by modern astrologers and, more important for us, were extensively used by ancient astrologers.

Several terms are used to describe the visibility of an object. When a star or planet formerly invisible due to proximity to the Sun first becomes visible in the morning sky, it is said to be at *heliacal rising*. When the object is last seen to set in the west after the Sun in the evening sky, it is said to be at *heliacal setting*. Two other pairs of terms are often confused with heliacal risings and settings. Either the rising or setting of a star in the evening, i.e., at or just after sunset, is referred to as *acronychal*[36] and either the rising or setting of a star at sunrise is said to be *cosmical*. Thus, a star that is first seen to rise as the Sun sets is said to be at *acronychal rising*, and if it sets with the Sun, *acronychal setting*; one that sets as the Sun rises is at its *cosmical setting*, and if it rises as the Sun rises, it is at *cosmical rising*. Astronomers do not always follow these definitions strictly, however; so the context must be used to understand what the terms are intended to mean. Parker and Neugebauer (1960, pp. 55, 57, 72) unambiguously identify the term "acronychal setting" to mean setting right after the Sun, i.e., seen in the west just after sunset, in accord with the definitions. In *Sky Watchers of Ancient Mexico*, Aveni (1980, p. 325, *n.* 16) correctly uses the term "cosmic rising" to indicate rising at the same instant as the Sun (and "cosmic setting" to indicate setting at the instant that the Sun sets). However, he also defines "achronic" to indicate rising when the Sun sets (in agreement with the standard definition of "acronychal") but also a setting as the Sun rises (which disagrees). Elsewhere in *Sky Watchers*, the applications of "heliacal rising" and "heliacal setting" are consistent with both our and Aveni's definitions (e.g., pp. 87, 99, 109*ff*), except for one discussion in which "heliacal setting" is used to describe a setting at sunrise in a discussion of the behavior of the Pleiades at Teotihuacan (Aveni 1980, p. 112). Indeed, many authors use this broader usage of "heliacal" to encompass both the restricted sense of the word and the acronychal definition (because they are both, in a sense, heliacal phenomena). However, in the current work, we try to be consistent with the stricter definitions.

As we note in §3.1.5, the hour angle difference from the Sun and the altitude of the object at first and last visibility depend on its brightness and on sky conditions; it is more difficult to see the light of most celestial objects when they near the horizon because the light-scattering path through the atmosphere is the longest at such times. The relationship between the first and last visible phenomena and the true instants when the star/planet and the Sun rise/set together was the topic of a book in the ancient world written by Autolycus of Pitane: *On the Risings and Settings*.

Because the Moon, Mercury, and Venus were considered to be below the orbit of the Sun, they were called *inferior* planets; those beyond the Sun were *superior* planets. Heliocentrically, they are *interior* and *exterior*, respectively, to Earth's orbit. There are important differences between the apparent motions of these two types of planets.

For Mercury and Venus, the elongation reaches maximum values both east and west: the *greatest eastern elongation* (GEE) and *greatest western elongation* (GWE), respectively. When at eastern elongation, the planet is visible east of the Sun, therefore, after sunset and in the western part of the sky. At western elongation, the object is west of the Sun, and there visible before sunrise, and in the eastern part of the sky.

The geometry of the planetary configurations can be understood from Figure 2.22, which, although presented in a heliocentric framework, shows how the planetary configurations are generated relative to the earth.

It will be noticed that only interior planets go through an *inferior conjunction* and only exterior planets can achieve *quadrature* and *opposition*. Both types of planets can go through *superior conjunction*, although in current usage, superior planets are merely said to be at "conjunction" at such times, because this is the only type of conjunction (with the Sun) that they can achieve; i.e., they can never be at inferior conjunction. Exterior planets move eastward through the configurations: superior conjunction, eastern quadrature, opposition, western quadrature, and superior conjunction.

---

[35] Or nearly so: *Conjunction* is sometimes taken to mean identical celestial longitude, and sometimes, right ascension; in either case, if the declination of the two objects is not the same, they will almost certainly rise at slightly different instants of time.

[36] *Or acronycal*. Additional spellings that have been used for this word include *acronical*, *achronical*, and *achronichal*!

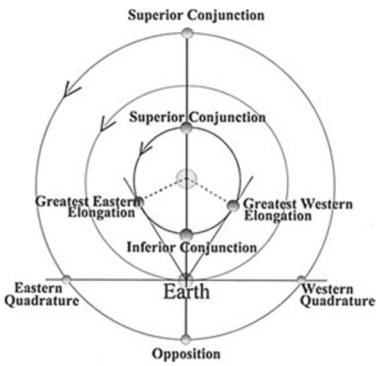


FIGURE 2.22. Successive positions of exterior and interior heliocentric planetary orbits, relative to an arbitrary position of the Earth, and showing how they give rise to the planetary configurations. Drawn by E.F. Milone.

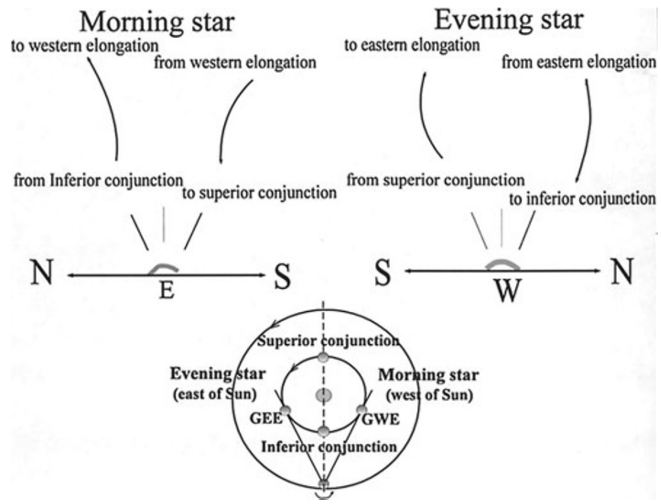


FIGURE 2.23. The motions of an interior planet in the (a) eastern and western skies and (b) in a heliocentric frame of reference. Note the ready explanation in the heliocentric system for the apparent limitation in the motion of an inferior planet. Drawn by E.F. Milone.

Their motion is eastward all the time except during an interval around opposition when they briefly appear to show *retrograde* (westward) motion.[37] Interior planets may be in conjunction with the Sun, but most of the time, they are at some elongation less than GEE or GWE. Interior planets move from superior conjunction through increasing eastern elongations to GEE to decreasing elongations to inferior conjunction to increasing westward elongations to GWE to decreasing western elongations to superior conjunction. Following maxiumum eastern elongation (when they are *evening stars*), Venus and Mercury seem to fall toward the Sun at an increasing rate, and then move rapidly into the morning sky, where they continue westward at a decreasing rate until maximum western elongation is reached. Figure 2.23 illustrates their motions in the western and eastern skies and associated locations in a heliocentric sketch.

The order of the configurations over a synodic cycle, arbitrarily beginning at its heliacal rising, is as follows (with associated phenomena shown below each configuration). For an interior planet,

(1) First visibility in the morning sky (retrograde motion continuing) (heliacal rising, morning star)
(2) Greatest western elongation (onset of prograde motion) (morning star)
(3) Last visibility in the morning sky (prograde motion continuing) (morning star)
(4) Superior conjunction (prograde motion continuing) (rises and sets with the Sun)
(5) First visibility in the evening sky (prograde motion continuing) (heliacal/achronical setting, evening star)
(6) Greatest eastern elongation (onset of retrograde motion) (evening star)
(7) Last visibility in the evening sky (retrograde motion continuing) (evening star)
(8) Inferior conjunction (retrograde motion continuing) (rises and sets with the Sun)
(9) First visibility in the morning sky (retrograde motion continuing) (heliacal rising, morning star)

so that the interior planet moves westward from its GEE evening star appearance (through inferior conjunction) to its GWE morning star appearance; and it moves eastward from GWE (through superior conjunction) to GEE. For an exterior planet, again from heliacal rising:

(1) First visibility in the morning sky (prograde motion continuing) (heliacal rising, morning star)
(2) Western quadrature (prograde motion continuing) (morning star)
(3) First stationary point (beginning of retrograde motion)
(4) Opposition (acronychal rising)
(5) Second stationary point (end of retrograde motion)
(6) Eastern quadrature (prograde motion continuing) (evening star)
(7) Last visibility in the evening sky (prograde motion continuing) (evening star, heliacal/acronychal setting)
(8) Superior conjunction (prograde motion continuing) (rises and sets with the Sun)
(9) First visibility in the morning sky (prograde motion continuing) (heliacal rising, morning star)

Note that the average ecliptic motion of exterior planets is less than that of the Sun and, consequently, get passed by the Sun. The only retrograde motion that these planets undergo is around opposition, when the Earth, in a faster, interior orbit, passes these planets.

[37] It is important to note that in the ancient world, our “direct” or “prograde” (eastward) and “retrograde” (westward) terms for these motions were not in use. Ptolemy uses the term “εἰζ τὰ ἑπόμενα,” “toward the rear,” to mean eastward motion. He uses the term “εἰζ τὰ προηγούμενα,” “toward the front,” to mean westward. To Ptolemy, the “forward” direction was that of the diurnal motion. See Toomer (1984).

The observations of specific configurations, especially of first and last visibility in ancient Mesopotamia, will be elaborated in §7.1.2.1. See Aveni (1980, pp. 109–117) for a similar treatment of configuration visibility in Mesoamerica. The apparent path of a planet in the sky varies from cycle to cycle because of the relative changes in ecliptic latitude as well as in longitude due to orbital inclinations. Thus, for example, the retrograde motion of an exterior planet may be a loop of various degrees of flattening or a zigzag. The looping pattern of an interior planet also varies during its pass through inferior conjunction. The relative periods of motion may be used to determine repetitions of these motion patterns.

2.4.4. Periodicities, Cycles, and Interrelationships

The periodicities in the motions of the planets were studied intently by astronomers from many cultures. Detailed records are available from Mesopotamia, India, China, and Mesoamerica. According to Neugebauer (1969, p. 127), the main interest of the Babylonian astronomers was the first and last visibility of the planets due to their motions and that of the Sun.[38] The earliest observational records from Mesopotamia date from the middle of the second millenium B.C.; from China, they are slightly later. See §§7.1.3 and 10.1.4 for further discussion of these sources.

There are two basic periods by which we characterize the motion of a planet in the sky: the *sidereal* and the *synodic periods*. The modern sidereal period is the time interval between successive passages of the planet through a line between a distant star and the Sun. The synodic period, on the other hand, is the (average) time interval between successive passages of the planet through a Sun-Earth line; it is therefore a relative period. These periods are analogous to the lunar sidereal and synodic months. The difference between the two types of period arises, in the case of an interior planet, from the time required for the interior planet to lap the earth as both revolve counterclockwise around the Sun. In the case of an exterior planet, the Earth moves faster, and the difference arises from the time required for earth to lap the exterior planet. Calculation[39] of the relative rate of motion of a planet in terms of orbital motions of the planet and Earth gives the following expressions for the synodic periods (P_{syn}) of interior and exterior planets, respectively:

$$\text{Interior:} \quad \frac{1}{P_{syn}} = \frac{1}{P_{sid}} - \frac{1}{P_{\oplus}}, \tag{2.23}$$

$$\text{Exterior:} \quad \frac{1}{P_{syn}} = \frac{1}{P_{\oplus}} - \frac{1}{P_{sid}}, \tag{2.24}$$

where P_{sid} is the planet's sidereal period and $P_{\oplus}$ is that of the earth. Note the reciprocal relations among the synodic and sidereal periods. If the periods are taken in units of the Earth's sidereal period of revoution around the Sun, the expressions simplify further.

Neugebauer (1969, p. 172) gives "synodic periods" of Saturn and Jupiter: 28;26,40 and 10;51,40, in the sexigesimal (base-60) notation of the Babylonians used by Neugebauer. These quantities, $28\overset{y}{.}444$ and $10\overset{y}{.}861$ in decimal-based notation, are approximately equal to $P_{sid} - 1$; by setting $P_{\oplus} = 1$ in (2.24), one finds that this quantity is the *ratio* of the two periods, viz., P_{sid}/P_{syn} when they are expressed in units of the Earth's period of revolution. They are not, therefore, the *synodic periods* as usually defined in astronomy. They are, however, very interesting nevertheless.

In an ancient astronomy context, one can draw a distinction between the time interval for a planet to come to the same configuration, e.g., from opposition to opposition, and the time for it to reappear in the same asterism or at the same celestial longitude. The former is the synodic period as defined astronomically, whereas the latter is a kind of sidereal period, although the motion of the earth around the Sun creates a moving platform and the observation therefore suffers from *parallax*. Figure 2.24 illustrates the effect of parallax on the apparent direction to the planet in space.

Even with the complication of parallax, ancient astronomy was capable of giving relatively high precision in the periodicities of the planets; the way they did this was to make use of large numbers of cycles. The number of years required for a planet to reach the same configuration, *in the same star field*, had to be recorded. The number of times the planet moved around the sky through a particular star field provided an integer multiple of the sidereal period. The number of years required for the planet to reach this point in the sky and have the same configuration (with the Sun) is a multiple of the synodic period. The relationship is one of a ratio: $mP_{sid} = nP_{syn} = N$ years. Hence, if m and n are observed, the ratio of the two type of periods follows. For Saturn, we have $m = 9$, $n = 256$, $N = 265$ y; whence, $P_{sid}/P_{syn} = 256/9 = 28.444$. For Jupiter, $m = 36$, $n = 391$, $N = 427$ y, so that $P_{sid}/P_{syn} = 391/36 = 10.861$. Given the total number of years required for the same configuration to be observed[40] at the same place among the stars, we can compute, in theory, both P_{sid} and P_{syn}. For instance, a complete cycle for Saturn would take 265 years. Therefore, $P_{sid} = N/9 = 265/9 = 29.444$ y, and $P_{syn} = N/256 = 265/256 = 1.0352$ y. These results can be compared with the modern values, $P_{sid} = 29.458$ y and $P_{syn} = 1.0352$ y (see below). For Jupiter, $P_{sid} = N/36 = 427/36 = 11.8611$ y, and $P_{syn} = N/391 = 427/391 = 1.0921$ y, compared with modern values, $P_{sid} = 11.8622$ y and $P_{syn} = 1.0921$ y.

The results are excellent for the synodic periods, and the derived sidereal periods are reasonable approximations, but they are not exact. One of the reasons for deviations from modern values is the effect of the shape of the orbit—the orbital eccentricity (others include the accuracy and preci-

[38] The heliacal risings and settings of stars are analogous, but simpler because, unlike planets, their annual changes in position are not detectable to the naked eye.

[39] The difference between the mean angular rates ω_{planet} and $\omega_{\oplus}$ is the relative rate: ω_{rel}. Because $\omega = 2\pi/P$ and $P_{rel} = P_{syn}$, we obtain equations (2.23) and (2.24), after division by 2π.

[40] Or, as in Mesopotamia, *calculated*, based on the differences between observed and exact ecliptic longitudes in near-repetitions of the phenomena.

sion of length of the year, and the use, exclusively, of the ecliptic longitude and exclusion of the ecliptic latitude). The time interval between repetition of celestial longitude coordinate values (and the mean sidereal period) depends on the traveled portion of the orbit of the planet involved: Near perihelion, the interval will be shorter than near aphelion. It also depends on the change in position of the earth in its orbit. The length of the synodic period that is specified in most planetary tables is a period that a planet would have if both it and the earth moved at constant, average rates of motion in their respective orbits. The lap difference involves different portions of the orbit and therefore different velocities, reflected in the change of angular motion of the planet across the sky. Of course, the larger the number of cycles that are involved, the smaller is the effect of the remaining segment of the orbit. The ancients were interested in such problems, and we consider the matter somewhat further in §7. At this point, we need to discuss how to characterize orbits.

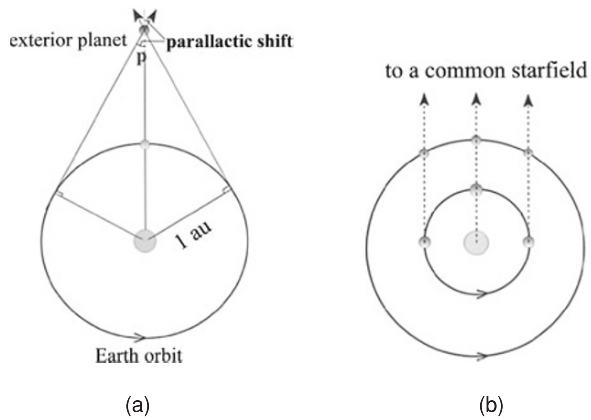


FIGURE 2.24. The effect of parallax on the apparent direction to a planet: (a) The shift of an exterior planet against the starry background. (b) Compensating motions of the planet and Earth may reduce the parallax shift: The positions of alignment of earth and planet to a distant star are not unique but may occur at nearly any planetary configuration. The three positions of the outer planet shown here place it in the same star field. See Figure 2.18 for the positions of Mars near an opposition. Drawn by E.F. Milone.

Table 2.9 lists the mean sidereal and mean synodic periods as well as other orbital parameters for the planets. The sources of the data in Table 2.9 are the Astronomical Almanac for the year 2000 and earlier editions and Allen's Astrophysical Quantities (Allen 1973, pp. 140–141; updated by Cox 2000). The elements refer to the mean equinox and ecliptic for the year 2000. The rates $d\Omega/dt$ and $d\omega/dt$ and the values of the periods are long-term average values. The precision in the elements actually exceeds the number of significant figures that are shown, but because of the gravitational perturbations produced by the other planets, the elements will vary with time. Following the modern planetary names are the adopted symbols, the semimajor axis or mean distance to the Sun in units of the *astronomical unit*, a (and, below, the date of a recent passage through perihelion T_0), the orbital eccentricity e (and, below it, the mean longitude ℓ), the orbital inclination, the longitude of the

TABLE 2.9. Planetary orbital parameters.[a]

Planet/element	a (AU) T_0	e ℓ	ι	Ω $d\Omega/dt$	ω $d\omega/dt$ (″/y)	n (°/d)	$\langle P_{\rm sid}\rangle$ (MSD)	$\langle P_{\rm Syn}\rangle$ (MSD)
Mercury	0.38710	0.20563	7°.0050	48°.33	29°.12	4.09235	$87^{d}.969 = 0^{y}.24085$	$115^{d}.8775$
(☿)	2000 Feb 16	119°.37582		+42.67	+55.96			
Venus	0.72333	0.00676	3.3946	76.68	55.19	1.60215	$224.699 = 0^{y}.61521$	583.9214 ≈ ⊕ +219[d]
(♀)	2000 Jul 13	270.89740		+32.39	+50.10			
Earth[b]	0.99999	0.01670	0.0001	143.9	319.04	0.98562	365.256363 = 1SY	. . .
(⊕)	2000 Jan 3.2	155.16587		. . .	+61.8		$= 0^{y}.99997862$JY	
							= 1.000038804TY	
Mars	1.52376	0.09337	1.8498	49.56	286.54	0.52400	$686.980 = 1^{y}.8809$	779.9361 ≈ 2⊕ +49[d]
(♂)	1998 Jan 7	24.53534		+27.7	+66.26			
Jupiter	5.20432	0.04879	1.3046	100.49	275.03	0.08305	$4332.589 = 11^{y}.8622$	398.8840 ≈ ⊕ +34[d]
(♃)	1987 Jul 10	38.98221		+36.39	+57.98			
Saturn	9.58189	0.05587	2.4853	113.64	336.23	0.03323	$10759.22 = 29^{y}.4578$	378.0919 ≈ ⊕ +13[d]
(♄)	1974 Jan 8	51.87716		+31.42	+70.50			
Uranus[c]	19.22354	0.04466	0.7725	73.98	96.30	0.01169	$30685.4 = 84^{y}.0138$	369.6560
(⛢)	1966 May 20	314.13799		+17.96	+54.			
Neptune[c]	30.0917	0.01122	1.7681	131.79	267.67	0.00597	$60189. = 164^{y}.792$	367.4867
(♆)	1876 Sep 2	305.53768		+39.54	+50.			
Pluto[c]	39.2572	0.24459	17.1533	110.28	113.71	0.00401	$90465. = 247^{y}.685$	366.7207
(♇)	1989 Sep 5	239.27437						

[a] Heliocentric osculating orbital elements, referred to the mean ecliptic and equinox of J2000.0. T_0 is a recent date of passage through perihelion.
[b] Elements are for the barycentre of the Earth-Moon system. SY, JY, and TY are Sidereal, Julian, and Tropical years, respectively, and are given in units of mean solar days (cf. §4.1.2). A Julian year has a length of 365.25 days, exactly.
[c] Years of discovery for Uranus, Neptune, and Pluto, respectively: 1781, 1846, 1930.

Table 2.10. A selection of premodern planetary parameters.[a]

Planet/element	a (AU)	n T_0 = A.D. 1/ T_0 = A.D. 1549	e	ω (°/d)
Sun	1.	0.985635	0.0417	65°60
{Earth}	1.	0.985608	0.0369	211.32
Moon	—	13.176382	0.8281	—
	—	13.176356	0.0237	207.12
Mercury		3.106699	0.0500	188.63
	0.3573	3.106730	0.0736	187.54
Venus		0.616509	0.0208	53.63
	0.7193	0.616518	0.0164	48.33
Mars		0.524060	0.1000	114.13
	1.5198	0.524032	0.0973	107.75
Jupiter		0.083122	0.0458	159.62
	5.2192	0.083091	0.0458	154.06
Saturn		0.033489	0.0569	231.63
	9.1743	0.033460	0.0570	225.00

[a] Ptolemaic values are in the top line and the Copernican on the lower for each planetary entry.

ascending node, Ω (and, below, its variation in arc-seconds per year), the argument of perihelion, ω (and, below, its variation in arc-seconds per year), the mean motion in degrees per day, n, the average sidereal period in mean solar days, and the average synodic period in mean solar days (and the number of integral Earth sidereal years, ⊕, and remainder in days). The mean motion is not independent of other elements, but it directly indicates the orbital motion of the planet; so we include it here. As we have noted, a combination of angles, the *longitude of perihelion* ($\tilde{\omega}$) is sometimes given in place of the *argument of perihelion* (ω): $\tilde{\omega} = \Omega + \omega$. The data for the telescopic planets Neptune and Pluto are included only for completeness. Uranus is marginally visible to the unaided eye. It is conceivable that the motion of Uranus could have been noticed during an *appulse* or close approach to a star, but its motion is so small, only 20 arc-minutes per month, that this is unlikely to have been noticed in antiquity. Whether it was or was not noticed by someone (see Hertzog 1988), to the present day, no evidence for early nontelescopic observations of Uranus has been found.

The data of Table 2.9 can be used to find the position of a planet in its orbit at subsequent times and its position in the ecliptic and equatorial systems. The mean longitude, ℓ, is related to the mean anomaly through the relation, $M = \ell - \tilde{\omega} = \ell - \omega - \Omega$ [(2.16) to (2.18) in §2.3.5]. A full discussion of the required procedures is beyond the scope of this book, but is provided by several sources.[41] Appendix A provides lists of published tables of planetary positions for the remote past, as well as some of the currently available commercial software packages for computing them.

Some of the elements of Table 2.9 may be compared with those of Table 2.10, which lists planetary parameters as reckoned by Ptolemy (2nd century) and by Copernicus (16th century), extracted from values provided by Gingerich (1993, p. 128, fn. *38*; p. 214, Table 4). The Ptolemaic values are on the top line, and the Copernican are on the lower, for each planetary entry. The solar distance parameter a is given in units of the average Earth-Sun distance and is tabulated only for the heliocentric model; n, e, and ω follow. The parameters that were used to characterize orbits in antiquity are not always the same as the modern elements. All orbits were circular, but a planet's orbit was not centered on the Earth (or, in the Copernican model, on the Sun), so that the Copernican "eccentricity," e, for instance, is the mean distance between the center of the orbit and the Sun and expressed in units of a. In Copernicus's model, this "eccentricity" varies with time, because the center of the orbit moves on a circle (the mean value is given in Table 2.10). As a consequence, the argument of perihelion also varies and adds to the perturbation-induced variation. Altogether, the model of Copernicus required at least six parameters to compute each planet's longitude and five additional parameters to include the effects of his (incorrect) theory of precession (see §§3.1.6, 7.7).

The periodicities that were most noticable and most noted by ancient astronomers were the synodic periods of the planets and those that were commensurate with the solar calendar or other calendars. The formulation of Kepler's Third Law, which relates the sidereal period to the semi-major axis, had to await understanding of the difference between the synodic and sidereal periods, correct planetary distances from the Sun, and, of course, the heliocentric perspective.

Finally, we supply positions of a planet at a particular configuration. Table 2.11 (based on information provided by Jet Propulsion Laboratory astronomer E. Myles Standish) is a partial list[42] of dates of inferior conjunctions of Venus. The dates indicated are Julian Day Numbers and decimals thereof and Julian calendar (36,525 days in a century) dates and hours; the uncertainty is about 3 hours. There is a cycle of 251 tropical years for Venus conjunction events. Purely boldfaced dates indicate entries for one such series, and the bold-italicized dates those for another; the latter is carried forward into the 20th century at the end of the table. The 20th century dates, however, are given in the Gregorian calendar (see §4.2.3). Note that the difference in JDN (an accurate indication of the number of days between the conjunctions) is only about 0.03^d/cycle.[43] Although they certainly did not use the Gregorian or Julian calendars, Mayan astronomers were well aware of these sorts of periodicities of Venus, and of the tropical year, and tied some of them into their sacred calendar (see §12, where the repetitions of Venus phenomena are discussed in the context of the Mesoamerican calendar). Calendrical and iconographic evidence strongly suggests that the complicated series of motions of Venus in the sky over many years were observed carefully. The motion of the perihelion of a planet means

[41] For example, Brouwer and Clemence (1961), Danby (1962/1988), or for less-critical determinations, Schlosser et al. (1991/1994, pp. 70–76 and Appendix E).

[42] This is an updated version of part of a table from Spinden 1930, pp. 82–87.

[43] This can be seen as follows: 251 × 365.2422 = 91675.79, while 1955664.29 − 1863988.47 = 91675.82, for example.

TABLE 2.11. Venus inferior conjunctions.

JDN	Julian date	JDN	Julian date	JDN	Julian date
1863988.47	**0391^y APR 28^d 23^h**	1864572.51	0392^y DEC 03^d 00^h	1865154.11	0394^y JUL 07^d 14^h
1865741.85	0396 FEB 15 08	1866321.64	0397 SEP 17 03	1866908.16	0399 APR 26 15
1867492.01	0400 NOV 30 12	1868073.78	0402 JUL 05 06	1868661.46	0404 FEB 12 22
1869241.20	0405 SEP 14 16	1869827.85	0407 APR 24 08	1870411.51	0408 NOV 28 00
1870993.46	0410 JUL 02 22	1871581.07	0412 FEB 10 13	1872160.77	0413 SEP 12 06
1872747.53	0415 APR 22 00	1873331.02	0416 NOV 25 12	1873913.12	0418 JUN 30 14
1874500.68	0420 FEB 08 04	1875080.34	0421 SEP 09 20	1875667.21	0423 APR 19 17
1876250.53	0424 NOV 23 00	1876832.80	0426 JUN 28 07	1877420.28	0428 FEB 05 18
1877999.92	0429 SEP 07 10	1878586.90	0431 APR 17 09	1879170.03	0432 NOV 20 12
1879752.47	0434 JUN 25 23	1880339.87	0436 FEB 03 08	1880919.50	0437 SEP 05 00
1881506.59	0439 APR 15 02	1882089.53	0440 NOV 18 00	1882672.14	0442 JUN 23 15
1883259.46	0444 JAN 31 23	1883839.09	0445 SEP 02 14	1884426.26	0447 APR 12 18
1885009.03	0448 NOV 15 12	1885591.82	0450 JUN 21 07	1886179.05	0452 JAN 29 13
1886758.68	0453 AUG 31 04	1887345.95	0455 APR 10 10	1887928.53	0456 NOV 13 00
1888511.50	0458 JUN 18 23	1889098.63	0460 JAN 27 03	1889678.28	0461 AUG 28 18
1890265.63	0463 APR 08 03	1890848.03	0464 NOV 10 12	1891431.18	0466 JUN 16 16
1892018.21	0468 JAN 24 17	1892597.88	0469 AUG 26 09	1893185.30	0471 APR 05 19
1893767.53	0472 NOV 08 00	1894350.86	0474 JUN 14 08	1894937.79	0476 JAN 22 07
1895517.48	0477 AUG 23 23	1896104.98	0479 APR 03 11	1896687.04	0480 NOV 05 12
1897270.55	0482 JUN 12 01	1897857.36	0484 JAN 19 20	1898437.08	0485 AUG 21 13
1899024.65	0487 APR 01 03	1899606.54	0488 NOV 03 01	***1900190.24***	***0490 JUN 09 17***
1900776.92	0492 JAN 17 10	1901356.69	0493 AUG 19 04	1901944.31	0495 MAR 29 19
1902526.05	0496 OCT 31 13	1903109.91	0498 JUN 07 09	1903696.49	0500 JAN 14 23
1904276.30	0501 AUG 16 19	1904863.99	0503 MAR 27 11	1905445.56	0504 OCT 29 01
1906029.60	0506 JUN 05 02	1906616.03	0508 JAN 12 12	1907195.92	0509 AUG 14 10
1907783.65	0511 MAR 25 03	1908365.06	0512 OCT 26 13	1908949.28	0514 JUN 02 18
1909535.59	0516 JAN 10 02	1910115.55	0517 AUG 12 01	1910703.31	0519 MAR 22 19
1911284.58	0520 OCT 24 01	1911868.97	0522 MAY 31 11	1912455.15	0524 JAN 07 15
1913035.17	0525 AUG 09 16	1913622.97	0527 MAR 20 11	1914204.09	0528 OCT 21 14
1914788.65	0530 MAY 29 03	1915374.68	0532 JAN 05 04	1915954.79	0533 AUG 07 07
1916542.63	0535 MAR 18 03	1917123.60	0536 OCT 19 02	1917708.35	0538 MAY 26 20
1918294.22	0540 JAN 02 17	1918874.43	0541 AUG 04 22	1919462.28	0543 MAR 15 18
1920043.13	0544 OCT 16 15	1920628.03	0546 MAY 24 12	1921213.76	0547 DEC 31 06
1921794.06	0549 AUG 02 13	1922381.93	0551 MAR 13 10	1922962.65	0552 OCT 14 03
1923547.72	0554 MAY 22 05	1924133.29	0555 DEC 28 18	1924713.70	0557 JUL 31 04
1925301.58	0559 MAR 11 01	1925882.18	0560 OCT 11 16	1926467.41	0562 MAY 19 21
1927052.81	0563 DEC 26 07	1927633.34	0565 JUL 28 20	1928221.23	0567 MAR 08 17
1928801.71	0568 OCT 09 05	1929387.10	0570 MAY 17 14	1929972.35	0571 DEC 23 20
1930552.99	0573 JUL 26 11	1931140.88	0575 MAR 06 09	1931721.24	0576 OCT 06 17
1932306.78	0578 MAY 15 06	1932891.86	0579 DEC 21 08	1933472.63	0581 JUL 24 03
1934060.52	0583 MAR 04 00	1934640.77	0584 OCT 04 06	1935226.48	0586 MAY 12 23
1935811.39	0587 DEC 18 21	1936392.29	0589 JUL 21 18	1936980.15	0591 MAR 01 15
1937560.32	0592 OCT 01 19	1938146.17	0594 MAY 10 16	1938730.91	0595 DEC 16 09
1939311.93	0597 JUL 19 10	1939899.77	0599 FEB 27 06	1940479.87	0600 SEP 29 08
1941065.86	0602 MAY 08 08	1941650.42	0603 DEC 13 22	1942231.59	0605 JUL 17 02
1942819.40	0607 FEB 24 21	1943399.41	0608 SEP 26 21	1943985.55	0610 MAY 06 01
1944569.93	0611 DEC 11 10	1945151.25	0613 JUL 14 18	1945739.03	0615 FEB 22 12
1946318.97	0616 SEP 24 11	1946905.24	0618 MAY 03 17	1947489.44	0619 DEC 08 22
1948070.91	0621 JUL 12 09	1948658.65	0623 FEB 20 03	1949238.53	0624 SEP 22 00
1949824.92	0626 MAY 01 09	1950408.94	0627 DEC 06 10	1950990.58	0629 JUL 10 01
1951578.27	0631 FEB 17 18	1952158.08	0632 SEP 19 13	1952744.61	0634 APR 29 02
1953328.45	0635 DEC 03 22	1953910.25	0637 JUL 07 17	1954497.88	0639 FEB 15 09
1955077.65	0640 SEP 17 03	**1955664.29**	**0642 APR 26 18**	1956247.96	0643 DEC 01 10
1956829.91	0645 JUL 05 09	1957417.49	0647 FEB 12 23	1957997.22	0648 SEP 14 17
1958583.97	0650 APR 24 11	1959167.47	0651 NOV 28 23	1959749.58	0653 JUL 03 01
1960337.10	0655 FEB 10 14	1960916.79	0656 SEP 12 06	1961503.66	0658 APR 22 03
1962086.97	0659 NOV 26 11	1962669.25	0661 JUN 30 18	1963256.70	0663 FEB 08 04
1963836.37	0664 SEP 09 20	1964423.35	0666 APR 19 20	1965006.47	0667 NOV 23 23
1965588.92	0669 JUN 28 10	1966176.29	0671 FEB 05 18	1966755.96	0672 SEP 07 10
1967343.02	0674 APR 17 12	1967925.97	0675 NOV 21 11	1968508.60	0677 JUN 26 02

TABLE 2.11. *Continued.*

JDN	Julian date	JDN	Julian date	JDN	Julian date
1969095.89	0679 FEB 03 09	1969675.54	0680 SEP 05 01	1970262.71	0682 APR 15 04
1970845.47	0683 NOV 18 23	1971428.28	0685 JUN 23 18	1972015.47	0687 JAN 31 23
1972595.13	0688 SEP 02 15	1973182.39	0690 APR 12 21	1973764.97	0691 NOV 16 11
1974347.96	0693 JUN 21 11	1974935.06	0695 JAN 29 13	1975514.73	0696 AUG 31 05
1976102.06	0698 APR 10 13	1976684.48	0699 NOV 13 23	1977267.64	0701 JUN 19 03
1977854.65	0703 JAN 27 03	1978434.32	0704 AUG 28 19	1979021.74	0706 APR 08 05
1979603.98	0707 NOV 11 11	1980187.32	0709 JUN 16 19	1980774.21	0712 AUG 26 10
1981941.41	0714 APR 05 21	1982523.49	0715 NOV 08 23	1983107.01	0717 JUN 14 12
1983693.78	0719 JAN 22 06	1984273.53	0720 AUG 24 00	1984861.08	0722 APR 03 14
1985442.99	0723 NOV 06 11	1986026.69	0725 JUN 12 04	1986613.35	0727 JAN 19 20
1987193.15	0728 AUG 21 15	1987780.75	0730 APR 01 06	1988362.49	0731 NOV 03 23
1988946.36	0733 JUN 09 20	1989532.90	0735 JAN 17 09	1990112.76	0736 AUG 19 06
1990700.42	0738 MAR 29 22	1991282.00	0739 NOV 01 11	***1991866.06***	***0741 JUN 07 13***
1992452.47	0743 JAN 14 23	1993032.38	0744 AUG 16 21	1993620.08	0746 MAR 27 13
1994201.51	0747 OCT 30 00	1994785.74	0749 JUN 05 05	1995372.03	0751 JAN 12 12
1995952.00	0752 AUG 14 11	1996539.74	0754 MAR 25 05	1997121.02	0755 OCT 27 12
1997705.43	0757 JUN 02 22	1998291.57	0759 JAN 10 01	1998871.62	0760 AUG 12 02
1999459.40	0762 MAR 22 21	2000040.54	0763 OCT 25 00	2000625.12	0765 MAY 31 14
2001211.12	0767 JAN 07 14	2001791.25	0768 AUG 09 17	2002379.05	0770 MAR 20 13
2002960.06	0771 OCT 22 13	2003544.81	0773 MAY 29 07	2004130.65	0775 JAN 05 03
2004710.88	0776 AUG 07 09	2005298.71	0778 MAR 18 05	2005879.58	0779 OCT 20 02
2006464.49	0781 MAY 26 23	2007050.19	0783 JAN 02 16	2007630.51	0784 AUG 05 00
2008218.37	0786 MAR 15 20	2008799.10	0787 OCT 17 14	2009384.18	0789 MAY 24 16
2009969.72	0790 DEC 31 05	2010550.16	0792 AUG 02 15	2011138.01	0794 MAR 13 12
2011718.63	0795 OCT 15 03	2012303.86	0797 MAY 22 08	2012889.25	0798 DEC 28 18
2013469.80	0800 JUL 31 07	2014057.65	0802 MAR 11 03	2014638.15	0803 OCT 12 15
2015223.55	0805 MAY 20 01	2015808.78	0806 DEC 26 06	2016389.44	0808 JUL 28 22
2016977.30	0810 MAR 08 19	2017557.69	0811 OCT 10 04	2018143.24	0813 MAY 17 17
2018728.31	0814 DEC 23 19	2019309.09	0816 JUL 26 14	2019896.93	0818 MAR 06 10
2020477.23	0819 OCT 07 17	2021062.93	0821 MAY 15 10	2021647.82	0822 DEC 21 07
2022228.74	0824 JUL 24 05	2022816.57	0826 MAR 04 01	2023396.77	0827 OCT 05 06
2023982.62	0829 MAY 13 02	2024567.34	0830 DEC 18 20	2025148.39	0832 JUL 21 21
2025736.20	0834 MAR 01 16	2026316.32	0835 OCT 02 19	2026902.31	0837 MAY 10 19
2027486.85	0838 DEC 16 08	2028068.05	0840 JUL 19 13	2028655.83	0842 FEB 27 07
2029235.86	0843 SEP 30 08	2029821.99	0845 MAY 08 11	2030406.36	0846 DEC 13 20
2030987.71	0848 JUL 17 04	2031575.45	0850 FEB 24 22	2032155.41	0851 SEP 27 21
2032741.68	0853 MAY 06 04	2033325.88	0854 DEC 11 09	2033907.37	0856 JUL 14 20
2034495.08	0858 FEB 22 13	2035074.97	0859 SEP 25 11	2035661.37	0861 MAY 03 20
2036245.39	0862 DEC 08 21	2036827.03	0864 JUL 12 12	2037414.69	0866 FEB 20 04
2037994.53	0867 SEP 23 00	2038581.06	0869 MAY 01 13	2039164.89	0870 DEC 06 09
2039746.70	0872 JUL 10 04	2040334.30	0874 FEB 17 19	2040914.10	0875 SEP 20 14
2041500.74	0877 APR 29 05	2042084.41	0878 DEC 03 21	2042666.36	0880 JUL 07 20
2043253.91	0882 FEB 15 09	2043833.67	0883 SEP 18 04	2044420.42	0885 APR 26 22
2045003.91	0886 DEC 01 09	2045586.04	0888 JUL 05 12	2046173.52	0890 FEB 13 00
2046753.24	0891 SEP 15 17	**2047340.11**	**0893 APR 24 14**	2047923.40	0894 NOV 28 21
2048505.71	0896 JUL 03 04	2049093.12	0898 FEB 10 14	2049672.83	0899 SEP 13 07
2050259.79	0901 APR 22 06	2050842.91	0902 NOV 26 09	2051425.39	0904 JUN 30 21
2052012.72	0906 FEB 08 05	2052592.40	0907 SEP 10 21	2053179.46	0909 APR 19 23
2053762.41	0910 NOV 23 21	2054345.06	0912 JUN 28 13	2054932.31	0914 FEB 05 19
2055511.99	0915 SEP 08 11	2056099.15	0917 APR 17 15	2056681.92	0918 NOV 21 09
2057264.74	0920 JUN 26 05	2057851.90	0922 FEB 03 09	2058431.58	0923 SEP 06 01
2059018.82	0925 APR 15 07	2059601.42	0926 NOV 18 22	2060184.42	0928 JUN 23 22
2060771.48	0930 JAN 31 23	2061351.17	0931 SEP 03 16	2061938.50	0933 APR 13 00
2062520.92	0934 NOV 16 10	2063104.10	0936 JUN 21 14	2063691.06	0938 JAN 29 13
2064270.77	0939 SEP 01 06	2064858.18	0941 APR 10 16	2065440.42	0942 NOV 13 22
2066023.78	0944 JUN 19 06	2066610.63	0946 JAN 27 03	2067190.38	0947 AUG 29 21
2067777.85	0949 APR 08 08	2068359.93	0950 NOV 11 10	2068943.46	0952 JUN 16 22
2069530.20	0954 JAN 24 16	2070109.99	0955 AUG 27 11	2070697.52	0957 APR 06 00
2071279.43	0958 NOV 08 22	2071863.14	0960 JUN 14 15	2072449.77	0962 JAN 22 06
2073029.60	0963 AUG 25 02	2073617.19	0965 APR 03 16	2074198.94	0966 NOV 06 10

Table 2.11. *Continued.*

JDN	Julian date	JDN	Julian date	JDN	Julian date
2074782.83	0968 JUN 12 07	2075369.34	0970 JAN 19 20	2075949.21	0971 AUG 22 16
2076536.85	0973 APR 01 08	2077118.45	0974 NOV 03 22	2077702.52	0976 JUN 10 00
2078288.89	0978 JAN 17 09	2078868.82	0979 AUG 20 07	2079456.51	0981 MAR 30 00
2080037.96	0982 NOV 01 11	2080622.20	0984 JUN 07 16	2081208.45	0986 JAN 14 22
2081788.44	0987 AUG 17 22	2082376.18	0989 MAR 27 16	2082957.48	0990 OCT 29 23
2083541.89	***0992 JUN 05 09***	2084127.99	0994 JAN 12 11	2084708.08	0995 AUG 15 13
2085295.83	0997 MAR 25 07	2085876.99	0998 OCT 27 1	. . .	
. . .					
2415795.46	1902 FEB 14 22	2416375.38	1903 SEP 17 21	2416962.91	1905 APR 27 09
2417544.72	1906 NOV 30 05	2418128.65	1908 JUL 06 03	2418715.01	1910 FEB 12 12
2419295.00	1911 SEP 15 11	2419882.57	1913 APR 25 01	2420464.23	1914 NOV 27 17
2421048.33	1916 JUL 03 19	2421634.57	1918 FEB 10 01	2422214.62	1919 SEP 13 02
2422802.23	1921 APR 22 17	2423383.75	1922 NOV 25 06	2423968.02	1924 JUL 01 12
2424554.13	1926 FEB 07 15	2425134.25	1927 SEP 10 17	2425721.89	1929 APR 20 09
2426303.26	1930 NOV 22 18	2426887.69	1932 JUN 29 04	2427473.69	1934 FEB 05 04
2428053.87	1935 SEP 08 08	2428641.55	1937 APR 18 01	2429222.77	1938 NOV 20 06
2429807.38	1940 JUN 26 21	2430393.23	1942 FEB 02 17	2430973.50	1943 SEP 06 00
2431561.20	1945 APR 15 16	2432142.30	1946 NOV 17 19	2432727.07	1948 JUN 24 13
2433312.78	1950 JAN 31 06	2433893.13	1951 SEP 03 15	2434480.85	1953 APR 13 08
2435061.81	1954 NOV 15 07	2435646.76	1956 JUN 22 06	2436232.32	1958 JAN 28 19
2436812.77	1959 SEP 01 06	2437400.49	1961 APR 10 23	2437981.34	1962 NOV 12 20
2438566.45	1964 JUN 19 22	2439151.86	1966 JAN 26 08	2439732.40	1967 AUG 29 21
2440320.13	1969 APR 08 15	2440900.86	1970 NOV 10 08	2441486.13	1972 JUN 17 15
2442071.39	1974 JAN 23 21	2442652.05	1975 AUG 27 13	2443239.77	1977 APR 06 06
2443820.40	1978 NOV 07 21	2444405.81	1980 JUN 15 07	2444990.92	1982 JAN 21 10
2445571.69	1983 AUG 25 04	2446159.42	1985 APR 03 21	2446739.93	1986 NOV 05 10
2447325.50	1988 JUN 12 23	2447910.45	1990 JAN 18 22	2448491.35	1991 AUG 22 20
2449079.05	1993 APR 01 13	2449659.46	1994 NOV 02 23	***2450245.18***	***1996 JUN 10 16***
2450829.97	1998 JAN 16 11	2451411.00	1999 AUG 20 11		

that its anomalistic period will be different from that of its sidereal period in the same way that the Moon's anomalistic period differs from its sidereal period. Similarly, planets can be said to have "nodal" periods. When any of these are multiples of the synodic periods, cyclic similarity in sky movement patterns can be expected.

This concludes our discussion of the basic movements of the sky and of the Sun, Moon, and planets. We now move to the problems associated with the observation of these objects and touch on such topics as the discernment and measurement of their positions, motions, and brightnesses.

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