

Chapter 2

Dimensions, Quantities and Units

Nomenclature

a	Acceleration
A	Area
d	Diameter
F	Force
g	Acceleration due to gravity
h	Height
m	Mass
P	Pressure
t	Time
u	Speed or velocity
u_x	Velocity in the x -direction
W	Work
x	Distance

Greek symbols

ρ	Density
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2.1 Dimensions and Units

The dimensions of all physical quantities can be expressed in terms of the four basic dimensions: mass, length, time and temperature. Thus velocity has the dimensions of length per unit time and density has the dimensions of mass per unit length cubed. A system of units is required so that the magnitudes of physical quantities may be determined and compared one with another. The internationally agreed system which is used for science and engineering is the Systeme International d'Unites, usually abbreviated to SI. Table 2.1 lists the SI units for the four basic dimensions together with those for electrical current and plane angle which, although strictly are derived quantities, are usually treated as basic quantities. Also included is the unit of molar mass which somewhat illogically is the gram molecular weight or gram mole and which is usually referred to simply as a 'mole'. However, it is often more convenient to use the kilogram molecular weight or kmol.

The SI system is based upon the general metric system of units which itself arose from the attempts during the French Revolution to impose a more rational order upon human affairs. Thus the metre was originally defined as one ten-millionth part of the distance from the North Pole to the equator

Table 2.1 Dimensions and SI units of the four basic quantities and some derived quantities

Dimension	Symbol	SI unit	Symbol
Mass	M	Kilogram	kg
Length	L	Metre	m
Time	T	Second	s
Temperature	Θ	Degree kelvin	K
Plane angle	–	Radian	rad
Electrical current	–	Ampere	A
Molar mass	–	Gram-molecular weight	mol

along the meridian which passes through Paris. It was subsequently defined as the length of a bar of platinum–iridium maintained at a given temperature and pressure at the Bureau International des Poids et Mesures (BIPM) in Paris, but is defined now by the wavelength of a particular spectral line emitted by a Krypton 86 atom.

The remaining units in Table 2.1 are defined as follows:

<i>kilogram:</i>	The mass of a cylinder of platinum–iridium kept under given conditions at BIPM, Paris.
<i>second:</i>	A particular fraction of a certain oscillation within a caesium 133 atom.
<i>degree kelvin:</i>	The temperature of the triple point of water, on an absolute scale, divided by 273.16. The degree kelvin is the unit of <i>temperature difference</i> as well as the unit of thermodynamic temperature.
<i>radian:</i>	The angle subtended at the centre of a circle by an arc equal in length to the radius.
<i>ampere:</i>	The electrical current which if maintained in two straight parallel conductors of infinite length and negligible cross-section, placed 1 m apart in a vacuum, produces a force between them of 2×10^{-7} N per metre length.
<i>mol:</i>	The amount of substance containing as many elementary units (atoms or molecules) as there are in 12 g of carbon 12.

The SI system is very logical and, in a scientific and industrial context, has a great many advantages over previous systems of units. However, it is usually criticised on two counts. First that the names given to certain derived units, such as the pascal for the unit of pressure, of themselves mean nothing and that it would be better to remain with, for example, the kilogram per square metre. This is erroneous; the definitions of newton, joule, watt and pascal are simple and straightforward if the underlying principles are understood. Derived units which have their own symbols, and which are encountered in this book, are listed in Table 2.2.

The second criticism concerns the magnitude of many units and the resulting numbers which are often inconveniently large or small. This problem would occur with any system of units and is not peculiar to SI. However, there are instances when strictly non-SI units may be preferred. For example, the wavelengths of certain kinds of electromagnetic radiation may be more conveniently written in

Table 2.2 Some derived SI units

Name	Symbol	Quantity represented	Basic units
newton	N	Force	kg m s^{-2}
joule	J	Energy or work	N m
watt	W	Power	J s^{-1}
pascal	Pa	Pressure	N m^{-2}
hertz	Hz	Frequency	s^{-1}
volt	V	Electrical potential	W A^{-1}

terms of the angstrom, 1 Å being equal to 10^{-10} m. Flow rates and production figures, when expressed in kg h^{-1} or even in t day^{-1} , may be more convenient than in kg s^{-1} . Pressures are still often quoted in bars or standard atmospheres rather than pascals simply because the pascal is a very small unit. Many of these latter disadvantages can be overcome by using prefixes. Thus a pressure of 10^5 Pa might better be expressed as 100 kPa. A list of prefixes is given in Appendix A.

It must be stressed that whatever shorthand methods are used to present data, the strict SI unit must be used in calculations. Mistakes are made frequently by using, for example, $\text{kW m}^{-2} \text{K}^{-1}$ for the units of heat transfer coefficients in place of $\text{W m}^{-2} \text{K}^{-1}$. Although such errors ought to be obvious it is often the case that compound errors of this kind result in plausible values based upon erroneous calculations.

A further note on presentation is appropriate at this point. I believe firmly that the use of negative indices, as in $\text{W m}^{-2} \text{K}^{-1}$, avoids confusion and is to be preferred to the solidus as in $\text{W/m}^2 \text{K}$. The former method is used throughout this book. Appendix B gives a list of conversion factors between different units.

2.2 Definitions of Some Basic Physical Quantities

2.2.1 Velocity and Speed

Velocity and speed are both defined as the rate of change of distance with time. Thus, speed u is given by

$$u = \frac{dx}{dt} \quad (2.1)$$

where x is distance and t is time. Average speed is then the distance covered in unit elapsed time. Velocity and speed differ in that speed is a scalar quantity (its definition requires only a magnitude together with the relevant units) but velocity is a vector quantity and requires a direction to be specified. A process engineering example would be the velocity of a fluid flowing in a pipeline; the velocity must be specified as the velocity in the direction of flow as opposed to, for example, the velocity perpendicular to the direction of flow. Thus velocity in the x -direction might be designated u_x where

$$u_x = \frac{dx}{dt} \quad (2.2)$$

In practice the term velocity is used widely without specifying direction explicitly because the direction is obvious from the context. The SI unit of velocity is m s^{-1} .

2.2.2 Acceleration

Acceleration is the rate of change of velocity with time. Thus the acceleration at any instant, a , is given by

$$a = \frac{du}{dt} \quad (2.3)$$

It should be noted that the term acceleration does not necessarily indicate an increase in velocity with time. Acceleration may be positive or negative and this should be indicated along with the magnitude. The SI unit of acceleration is m s^{-2} .

2.2.3 Force and Momentum

The concept of force can only be understood by reference to Newton's laws of motion.

First law: A body will continue in its state of rest or uniform motion in a straight line unless acted upon by an impressed force.

Second law: The rate of change of momentum of the body with time is proportional to the impressed force and takes place in the direction of the force.

Third law: To each force there is an equal and opposite reaction.

These laws cannot be proved but they have never been disproved by any experimental observation. The momentum of an object is the product of its mass m and velocity u :

$$\text{momentum} = mu \quad (2.4)$$

From Newton's second law, the magnitude of a force F acting on a body may be expressed as

$$F \propto \frac{d(mu)}{dt} \quad (2.5)$$

If the mass is constant, then

$$F \propto m \frac{du}{dt} \quad (2.6)$$

or

$$F \propto ma \quad (2.7)$$

In other words, force is proportional to the product of mass and acceleration. A suitable definition of the unit of force will result in a constant of proportionality in Eq. (2.7) of unity. In the SI system the unit of force is the newton (N). One newton is that force, acting upon a body with a mass of 1 kg, which produces an acceleration of 1 m s^{-2} . Hence

$$F = ma \quad (2.8)$$

2.2.4 Weight

Weight is a term for the localised gravitational force acting upon a body. The unit of weight is therefore the newton and not the kilogram. The acceleration produced by gravitational force varies with the distance from the centre of the earth and at sea level the standard value is 9.80665 m s^{-2} which is usually approximated to 9.81 m s^{-2} . The acceleration due to gravity is normally accorded the symbol g .

The magnitude of a newton can be gauged by considering the apple falling from a tree which was observed supposedly by Isaac Newton before he formulated the theory of universal gravitation. The force acting upon an average-sized apple with a mass of, say, 0.10 kg, falling under gravity, would be, using Eq. (2.8)

$$F = 0.10 \times 9.81 \text{ N}$$

$$F = 0.981 \text{ N}$$

$$F \approx 1.0 \text{ N}$$

The fact that an average-sized apple falls with a force of about 1.0 N is nothing more than an interesting coincidence. However, this simple illustration serves to show that the newton is a very small unit.

2.2.5 Pressure

A force F acting over a specified surface area A gives rise to a pressure P . Thus

$$P = \frac{F}{A} \quad (2.9)$$

In the SI system the unit of pressure is the pascal (Pa). One pascal is that pressure generated by a force of 1 N acting over an area of 1 m². Note that standard atmospheric pressure is 1.01325×10^5 Pa. There are many non-SI units with which it is necessary to become familiar; the bar is equal to 10^5 Pa and finds use particularly for pressures exceeding atmospheric pressure. A pressure can be expressed in terms of the height of a column of liquid which it would support and this leads to many common pressure units, derived from the use of the simple barometer for pressure measurement. Thus atmospheric pressure is approximately 0.76 m of mercury or 10.34 m of water. Very small pressures are sometimes expressed as mm of water or 'mm water gauge'. Unfortunately it is still common to find imperial units of pressure, particularly pounds per square inch or psi.

Consider a narrow tube held vertically so that the lower open end is below the surface of a liquid (Fig. 2.1). The pressure of the surrounding atmosphere P forces a column of liquid up the tube to a height h . The pressure at the base of the column (point B) is given by the weight of liquid in the column acting over the cross-sectional area of the tube:

$$\text{mass of liquid in tube} = \frac{\pi d^2}{4} h \rho \quad (2.10)$$

where ρ is the density of the liquid. The weight of liquid (force acting on the cross-section) F is

$$F = \frac{\pi d^2}{4} h \rho g \quad (2.11)$$

The pressure at B must equal the pressure at A, therefore

$$P = \frac{\pi d^2}{4} h \rho g \frac{4}{\pi d^2} \quad (2.12)$$

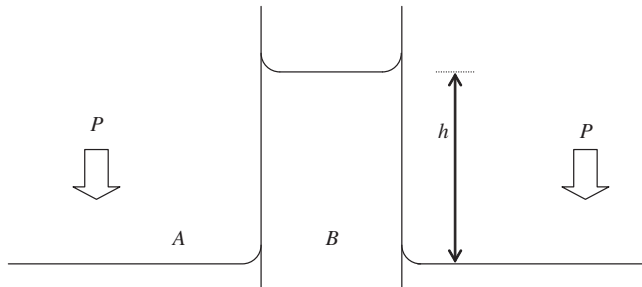


Fig. 2.1 Pressure at the base of a column of liquid

or

$$P = \rho gh \quad (2.13)$$

Example 2.1

What pressure will support a column of mercury (density = $13,600 \text{ kg m}^{-3}$) 80 cm high?

From Eq. (2.13) the pressure at the base of the column is

$$P = 13,600 \times 9.81 \times 0.80 \text{ Pa}$$

and therefore

$$P = 1.0673 \times 10^5 \text{ Pa}$$

2.2.6 Work and Energy

Work and energy are interchangeable quantities; work may be thought of as energy in transition. Thus, for example, in an internal combustion engine chemical energy in the fuel is changed into thermal energy which in turn produces expansion in a gas and then motion, first of a piston within a cylinder and then of a crankshaft and of a vehicle. The SI unit of all forms of energy and of mechanical work is the joule (J). A joule is defined as the work done when a force of 1 N moves through a distance of 1 m. For example, if an apple of weight 1 N (see Section 2.2.4) is lifted through a vertical distance of 1 m then the net work done on the apple is 1 J. This is the energy required simply to lift the apple against gravity and does not take into account any inefficiencies in the device – be it a mechanical device or the human arm. Clearly the joule is a small quantity. A further illustration of the magnitude of the joule is that approximately 4180 J of thermal energy is required to increase the temperature of 1 kg of water by 1 K (see Section 3.5).

Example 2.2

A mass of 250 kg is to be raised by 5 m against gravity. What energy input is required to achieve this?

From Eq. (2.8) the gravitational force acting on the mass is

$$F = 250 \times 9.81 \text{ N}$$

or

$$F = 2452.5 \text{ N}$$

The work done (or energy required) W in raising the mass against this force is

$$W = F \times \text{distance}$$

and therefore

$$W = 2452.5 \times 5 \text{ J}$$

or

$$W = 1.23 \times 10^4 \text{ J}$$

2.2.7 Power

Power is defined as the rate of working or the rate of usage or transfer of energy and thus it involves time. The apple may be lifted slowly or quickly. The faster it is lifted through a given distance the greater is the power of the mechanical device. Alternatively the greater the mass lifted (hence the greater the force overcome) within the same period, the greater will be the power. A rate of energy usage of 1 joule per second (1 J s^{-1}) is defined as 1 watt (W).

Example 2.3

The mass in Example 2.2 is lifted in 5 s. What power is required to do this? What power is required to lift it in 1 min?

The power required is equal to the energy used (or work done) divided by the time over which the energy is expended. Therefore

$$\begin{aligned}\text{power} &= \frac{1.23 \times 10^4}{5} \text{ W} \\ &= 2452.5 \text{ W or } 2.45 \text{ kW}.\end{aligned}$$

If now the mass is lifted over a period of 1 min,

$$\begin{aligned}\text{power} &= \frac{1.23 \times 10^4}{60} \text{ W} \\ &= 204.4 \text{ W}\end{aligned}$$

Whilst most examples in this book are concerned with rates of thermal energy transfer, it is important to understand (and may be easier to visualise) the definitions of work and power in mechanical terms. In addition, students of food technology and food engineering do require some basic knowledge of electrical power supply and usage, although that is outside the scope of this book. Suffice it to say that the electrical power consumed (in watts) when a current flows in a wire is given by the product of the current (in amperes) and the electrical potential (in volts).

2.3 Dimensional Analysis

2.3.1 Dimensional Consistency

All mathematical relationships which are used to describe physical phenomena should be dimensionally consistent. That is, the dimensions (and hence the units) should be the same on each side of the equality. Take Eq. (2.13) as an example.

$$P = \rho gh \quad (2.13)$$

Using square brackets to denote dimensions or units, the dimensions of the terms on the *right*-hand side are as follows:

$$[\rho] = \text{ML}^{-3}$$

$$[g] = \text{LT}^{-2}$$

$$[h] = \text{L}$$

Thus the dimensions of the *right*-hand side of Eq. (2.13) are

$$[\rho gh] = (\text{ML}^{-3})(\text{LT}^{-2})(\text{L})$$

or

$$[\rho gh] = \text{ML}^{-1}\text{T}^{-2}$$

Pressure is a force per unit area, force is given by the product of mass and acceleration and therefore the dimensions of the *left*-hand side of Eq. (2.13) are

$$[P] = \frac{[\text{mass}] [\text{acceleration}]}{[\text{area}]}$$

or

$$[P] = \frac{(\text{M})(\text{LT}^{-2})}{\text{L}^2}$$

or

$$[P] = \text{ML}^{-1}\text{T}^{-2}$$

Similarly the units must be the same on each side of the equation. This is simply a warning not to mix SI and non-SI units and to be aware of prefixes. The units of the various quantities in Eq. (2.13) are

$$[P] = \text{Pa}$$

$$[\rho] = \text{kg m}^{-3}$$

$$[g] = \text{m s}^{-2}$$

$$[h] = \text{m}$$

and therefore the right-hand side of Eq. (2.13) becomes

$$[\rho gh] = (\text{kgm}^{-3}) \times (\text{ms}^{-2}) \times (\text{m})$$

or

$$[\rho gh] = \text{kg} \times \text{ms}^{-2} \times \text{m}^{-2}$$

As a force of 1 N, acting upon a body of mass 1 kg, produces an acceleration of 1 m s^{-2} , the units of the *right-hand* side of Eq. (2.13) are now

$$[\rho gh] = \text{Nm}^{-2}$$

and thus

$$[\rho gh] = \text{Pa}$$

2.3.2 Dimensional Analysis

The technique of dimensional analysis is used to rearrange the variables which represent a physical relation so as to give a relationship that can more easily be determined by experimentation. For example, in the case of the transfer of heat to a food fluid in a pipeline, the rate of heat transfer might depend upon the density and viscosity of the food, the velocity in the pipeline, the pipe diameter and other variables. Dimensional analysis, by using the principle of dimensional consistency, suggests a more detailed relationship which is then tested experimentally to give a working equation for predictive or design purposes. It is important to stress that this procedure must always consist of dimensional analysis followed by detailed experimentation. The theoretical basis of dimensional analysis is too involved for this text but an example, related to heat transfer (Chapter 7), is set out in Appendix C.

Problems

- 2.1 What is the pressure at the base of a column of water 20 m high? The density of water is 1000 kg m^{-3} .
- 2.2 What density of liquid is required to give standard atmospheric pressure (101.325 kPa) at the base of a 12 m high column of that liquid?
- 2.3 A person of mass 75 kg climbs a vertical distance of 25 m in 30 s. Ignoring inefficiencies and friction (a) how much energy does the person expend and (b) how much power must the person develop?
- 2.4 Determine the dimensions of the following:
 - (a) linear momentum (mass \times velocity)
 - (b) work
 - (c) power
 - (d) pressure
 - (e) weight
 - (f) moment of a force (force \times distance)
 - (g) angular momentum (linear momentum \times distance)
 - (h) pressure gradient
 - (i) stress
 - (j) velocity gradient



<http://www.springer.com/978-1-4419-7661-1>

Introduction to Food Process Engineering

Smith, P.G.

2011, XVII, 510 p., Hardcover

ISBN: 978-1-4419-7661-1