

## Chapter 2

# Block Diagrams of Electromechanical Systems

The structure of an electromechanical drive system is given in Fig. 2.1. It consists of energy/power source, reference values for the quantities to be controlled, electronic controller, gating circuit for converter, electronic converter (e.g., rectifier, inverter, power electronic controller), current sensors (e.g., shunts, current transformer, Hall sensor), voltage sensors, (e.g., voltage dividers, potential transformer), speed sensors (e.g., tachometers), and displacement sensors (e.g., encoders), rotating three-phase machines, mechanical gear box, and the application-specific load (e.g., pump, fan, automobile). The latter component is given and one has to select all remaining components so that desirable operational steady-state and dynamic performances can be obtained: that is, the engineer has to match these electronic components with the given mechanical (or electronic) load. To be able to perform such a matching analysis, the performance of all components of a (drive) system must be understood. In Fig. 2.1 all but the mechanical gear are represented by transfer functions -that is, output variables  $X_{out}$  as a function of time  $t$ . The mechanical gear is represented by the transfer characteristic – that is  $X_{out}$  as a function of  $X_{in}$ .

Most constant-speed drives operate within the first quadrant I of the torque-angular velocity ( $T-\omega_m$ ) plane of Fig. 2.2. Whenever energy conservation via regeneration is required, a transition from either quadrant I to II or quadrant III to IV is required. Most variable-speed traction drives accelerating and decelerating in forward and backward directions operate within all four quadrants I–IV.

### Application Example 2.1: Motion of a Robot Arm

For accelerating a robot arm in forward direction the torque ( $T$ ) and angular velocity ( $\omega_m$ ) are positive, that means the power is  $P = T \cdot \omega_m > 0$ : the power required for the motion must be delivered from the source to the motor. If deceleration in forward direction is desired  $T < 0$  and  $\omega_m > 0$ , that is, the power required is  $P = T \cdot \omega_m < 0$ : the braking power to slow down the motion of the robot arm has to be generated by the motor and absorbed by the source of the drive.

For acceleration in reverse direction the torque  $T < 0$  and angular velocity  $\omega_m < 0$ , that is  $P = T \cdot \omega_m > 0$ , the power has to be delivered by the source of the drive. If deceleration in reverse direction is required, the braking power must be absorbed by the source of the drive (Fig. 2.3).

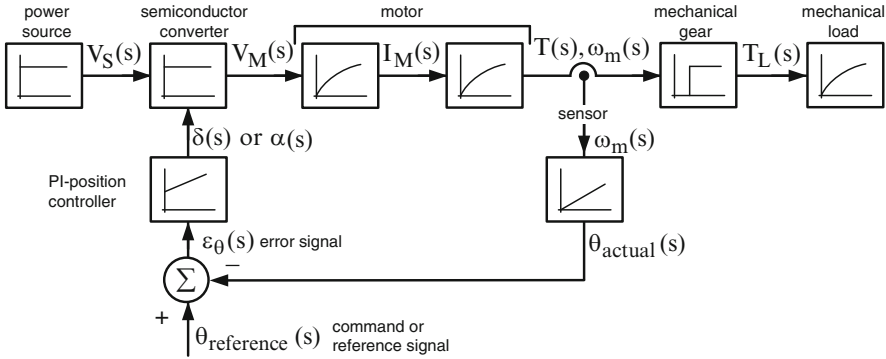


Fig. 2.1 Generic structure of an electromechanical drive system

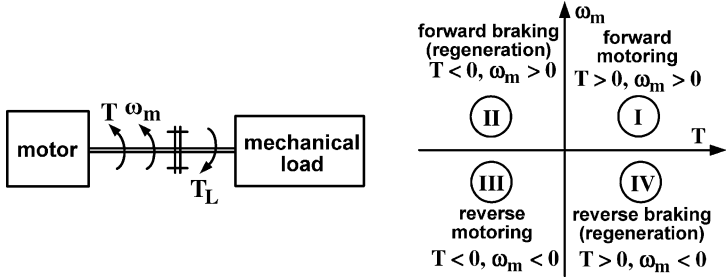


Fig. 2.2 Torque, angular velocity definitions and torque-angular velocity plane ( $T$ - $\omega_m$ )

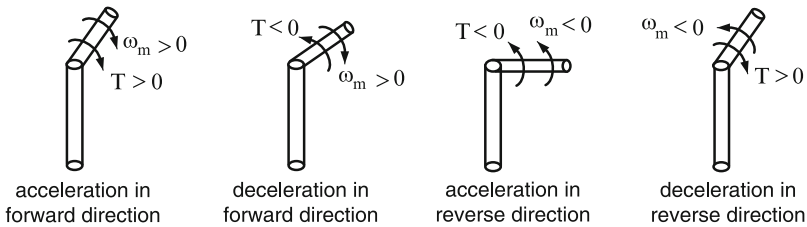


Fig. 2.3 Motion of a robot arm

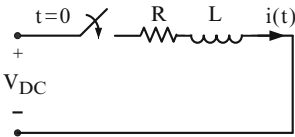


Fig. 2.4 R-L circuit with initial condition  $i_{init}(t=0)=0$

## 2.1 Relation Between Differential Equations of Motion and Transfer Functions

In general, the equations governing a motion are complicated differential equations with non-constant (time-dependent), nonlinear coefficients. Sometimes approximations permit us to represent these equations of motion by ordinary differential equations with constant coefficients. These can be solved either via Laplace solution techniques [1] or other mathematical methods. One of the latter is where the general solution of a differential equation with constant coefficients is obtained by the superposition of the (unforced, natural) solution of the homogeneous differential equation and a particular (forced) solution of the inhomogeneous differential equation.

### Application Example 2.2: Solution of Differential Equation with Constant Coefficients

The R-L circuit of Fig. 2.4 will be used to derive the solution of differential equation with constant coefficients and zero initial condition  $i(t=0) = i_{\text{init}}(t=0) = 0$ . Inhomogeneous (or forced) first-order differential equation is

$$Ri + L \frac{di}{dt} = V_{\text{DC}} \quad (2.1)$$

or

$$i + \tau \frac{di}{dt} = \frac{V_{\text{DC}}}{R}, \quad (2.2)$$

where  $\tau = L/R$  is the time constant.

Homogeneous (or unforced, natural) first-order differential equation is

$$i + \tau \frac{di}{dt} = 0. \quad (2.3)$$

1. Solution of the homogeneous (or unforced, natural response) differential equation: assume the solution to be of the form

$$i_{\text{hom}}(t) = Ae^{-(\alpha t)}, \quad (2.4)$$

$$\frac{di_{\text{hom}}(t)}{dt} = -A\alpha e^{-(\alpha t)}, \quad (2.5)$$

introducing these terms into the homogeneous differential equation, one obtains

$$Ae^{-(\alpha t)} - \tau A\alpha e^{-(\alpha t)} = 0, \quad (2.6)$$

with  $e^{-(\alpha t)} \neq 0$  and  $A \neq 0$  follows  $\tau = 1/\alpha$  or the homogeneous (unforced, natural) solution is

$$i_{\text{hom}}(t) = Ae^{-\left(\frac{t}{\tau}\right)}. \quad (2.7)$$

2. Particular solution of inhomogeneous differential equation: assume the solution to be of the form

$$i_{\text{particular}} = C, \quad (2.8a)$$

$$\frac{di_{\text{particular}}}{dt} = 0, \quad (2.8b)$$

with

$$i + \tau \frac{di}{dt} = \frac{V_{\text{DC}}}{R}, \quad (2.9)$$

follows the forced solution

$$i_{\text{particular}} = V_{\text{DC}}/R. \quad (2.10)$$

3. General solution consists of the superposition of homogeneous and particular solutions:

$$i_{\text{general}} = i_{\text{hom}} + i_{\text{particular}}, \quad (2.11)$$

$$i_{\text{general}}(t) = Ae^{-\left(\frac{t}{\tau}\right)} + \frac{V_{\text{DC}}}{R}. \quad (2.12)$$

The coefficient  $A$  will be found by introducing the initial condition at  $t=0$ , e.g.,  $i_{\text{init}}(t)=0$ , therefore

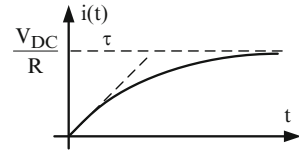
$$0 = Ae^{-0} + \frac{V_{\text{DC}}}{R} \quad (2.13a)$$

or

$$A = -\frac{V_{\text{DC}}}{R}. \quad (2.13b)$$

The solution for the given initial condition is now (Fig. 2.5)

$$i(t) = \frac{V_{\text{DC}}}{R} \left( 1 - e^{-\left(\frac{t}{\tau}\right)} \right). \quad (2.14)$$

**Fig. 2.5** Solution for  $i(t)$ 

The solution for the general initial condition  $i(t=0) = i_{\text{init}}(t=0)$  is

$$i(t) = \frac{V_{\text{DC}}}{R} \left( 1 - e^{-\left(\frac{t}{\tau}\right)} \right) + i_{\text{init}}(t=0) e^{-\left(\frac{t}{\tau}\right)}. \quad (2.15)$$

### 2.1.1 Circuits with a Constant or P (Proportional) Transfer Function

The following circuits have constant transfer functions.

(a) Voltage-divider circuit (Fig. 2.6)

$$\frac{v_{\text{out}}(s)}{v_{\text{in}}(s)} = \frac{R_1}{(R_1 + R_2)} = G_1 < 1. \quad (2.16)$$

The ratio  $G_1 = R_1/(R_1 + R_2)$  is called the gain of the circuit which is less than 1, where  $s$  is the Laplace operator.

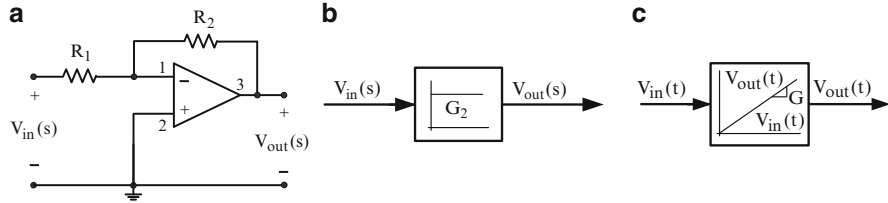
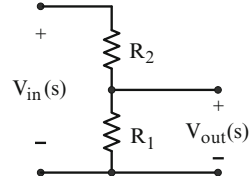
(b) Operational amplifier with negative (resistive) feedback: inverting [2] configuration (Fig. 2.7a).

$$\frac{v_{\text{out}}(s)}{v_{\text{in}}(s)} = -\frac{R_2}{R_1} = G_2. \quad (2.17)$$

As can be seen, the gain  $G_2$  is negative and can be larger or smaller than 1 (Fig. 2.7b).

The constant or proportional transfer function of (2.16) represents a gain  $G_1$  which is independent of time. For this reason the transfer function of Fig. 2.7b with gain  $G$  can be also represented as a transfer characteristic as illustrated in Fig. 2.7c. The slope of the linear input-output characteristic  $v_{\text{in}}(t)$  and  $v_{\text{out}}(t)$  is the gain  $G = G_1$  of the voltage divider (Fig. 2.6).

**Fig. 2.6** Voltage-divider circuit



**Fig. 2.7** (a) Inverting configuration of operational amplifier. (b) Representation of transfer function in a block diagram. (c) Representation of transfer characteristic in a block diagram

### 2.1.2 Circuits Acting as Integrators

The following circuits act as integrators, that is, the output signal is the integral of the input signal.

(a) R-C circuit (Fig. 2.8):

$i = C(dv_c/dt)$ , with  $v_c \ll v_R$  follows  $v_{in} = v_c + v_R$  or  $v_{in} \approx v_R$ ,  $C(dv_c/dt) = v_{in}/R$ ,  
 $v_c = v_{out} = (1/RC) \int v_{in} dt$ , with  $s = (d./dt)$

$$\frac{v_{out}(s)}{v_{in}(s)} = \frac{1}{s\tau}, \quad (2.18)$$

where  $\tau = RC$  is a time constant.

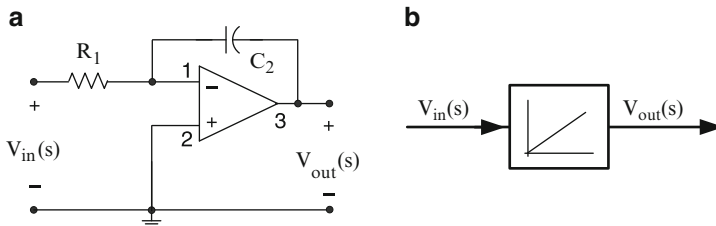
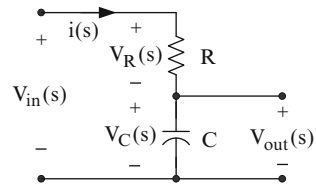
(b) Operational amplifier with negative (capacitive) feedback: inverting configuration (Fig. 2.9a).

$$\frac{v_{out}(s)}{v_{in}(s)} = -\frac{Z_2}{R_1} = -\left(\frac{1}{sC_2}\right) \quad (2.19a)$$

or

$$\frac{v_{out}(s)}{v_{in}(s)} = -\frac{1}{sR_1C_2}, \quad (2.19b)$$

**Fig. 2.8** R-C circuit as an integrator (I)



**Fig. 2.9** (a) Operational amplifier with capacitive feedback acting as integrator (I). (b) Representation of transfer function of an integrator in a block diagram

where  $\tau = R_1 C_2$  is a time constant, thus

$$\frac{v_{out}(s)}{v_{in}(s)} = -\frac{1}{s\tau}. \quad (2.20)$$

Figure 2.9b shows the block diagram representation of an integrator circuit.

### 2.1.3 First-Order Delay Transfer Function

The following circuits have first-order delay transfer functions.

(a) R-L circuit (Fig. 2.10)

First-order differential equation with constant coefficients in time domain reads  $i(t)R + L(di/dt) = v_{in}(t)$  or  $i(t) + (L/R)(di/dt) = v_{in}(t)/R$ , where  $\tau = L/R$  is a time constant.

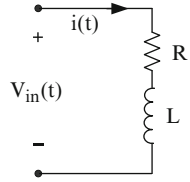
In Fig. 2.10 the voltage  $v_{in}(t)$  is the input of the network, and the current  $i(t)$  is its response (or output). That is, the R-L circuit can be represented by a transfer function within a block diagram (Fig. 2.11) as before:  $\frac{i(s)(1 + s\tau)}{(v_{in}(s)/R)} = 1$

or

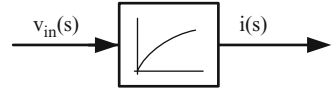
$$\frac{x_{out}(s)}{x_{in}(s)} = \frac{i(s)}{v_{in}(s)} = \frac{1}{R} \frac{1}{(1 + s\tau)} = \frac{G_3}{(1 + s\tau)}, \quad (2.21)$$

with  $\tau = L/R$  and gain  $G_3 = (1/R) < 1$ .

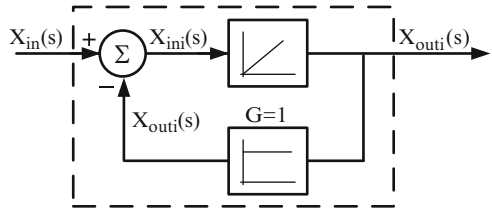
**Fig. 2.10** R-L circuit acting as a first-order delay network



**Fig. 2.11** Representation of transfer function of a first-order delay network in a block diagram



**Fig. 2.12** Integrator with negative feedback generates a first-order delay network



(b) Integrator with negative feedback (Fig. 2.12).

$$x_{outi}(s) = x_{ini}(s) \frac{1}{s\tau}, \quad x_{in}(s) - x_{outi}(s) = x_{ini}(s), \quad x_{outi}(s) = (x_{in}(s) - x_{outi}(s)) \frac{1}{s\tau}$$

or

$$x_{outi}(s) \left( 1 + \frac{1}{s\tau} \right) = \frac{x_{in}(s)}{s\tau}$$

$$\frac{x_{outi}(s)}{x_{in}(s)} = \frac{1}{s\tau(1 + \frac{1}{s\tau})} = \left( \frac{1}{s\tau + 1} \right), \quad (2.22)$$

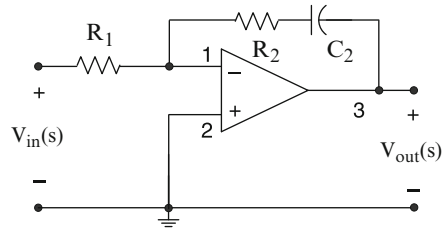
with gain  $G = 1$ .

### 2.1.4 Circuit with Combined Constant (P) and Integrating (I) Transfer Function

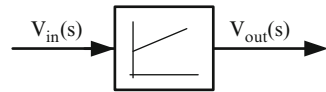
The circuit of Fig. 2.13 has a constant (P) transfer function combined with an integrating (I) transfer function.



**Fig. 2.13** Operational amplifier with resistive/capacitive feedback



**Fig. 2.14** Block diagram for proportional/integrator (PI) network



$$\frac{v_{out}(s)}{v_{in}(s)} = -\frac{Z_2}{R_1} = -\frac{\left(R_2 + \frac{1}{sC_2}\right)}{R_1} \quad (2.23a)$$

or

$$\frac{v_{out}(s)}{v_{in}(s)} = -\frac{R_2}{R_1} - \frac{1}{sR_1C_2}, \quad (2.23b)$$

where  $\tau = R_1C_2$  is a time constant, thus

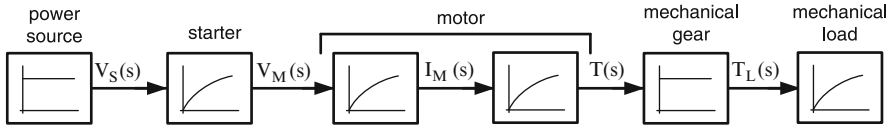
$$\frac{v_{out}(s)}{v_{in}(s)} = -\frac{R_2}{R_1} - \frac{1}{s\tau} = -\frac{\left(\frac{R_2}{R_1}\right)s\tau + 1}{s\tau} = G_4 \frac{(1 + s\tau_1)}{s}, \quad (2.24)$$

where  $G_4 = -(1/\tau) = (-1/R_1C_2)$ , and  $\tau_1 = (R_2/R_1)\tau = R_2C_2$ .

Figure 2.14 shows the block diagram representation of a proportional/integrator (PI) circuit.

## 2.2 Commonly Occurring (Drive) Systems

Some of the more commonly occurring drive systems are presented using linear transfer functions within each and every block of these diagrams. In real designs, nonlinear elements frequently occur. However, such nonlinear components cannot be approximated by linear differential equations with constant coefficients (e.g., Laplace solution technique) and, therefore, this introductory text limits the analysis to mostly linear systems.



**Fig. 2.15** Open-loop operation

### 2.2.1 Open-Loop Operation

The most commonly used electromechanical system is that of Fig. 2.15 operating at about constant speed without feedback control. The speed regulation is defined as [3]

$$\text{speed regulation} = \frac{(\text{no-load speed}) - (\text{full-load speed})}{(\text{full-load speed})}. \quad (2.25)$$

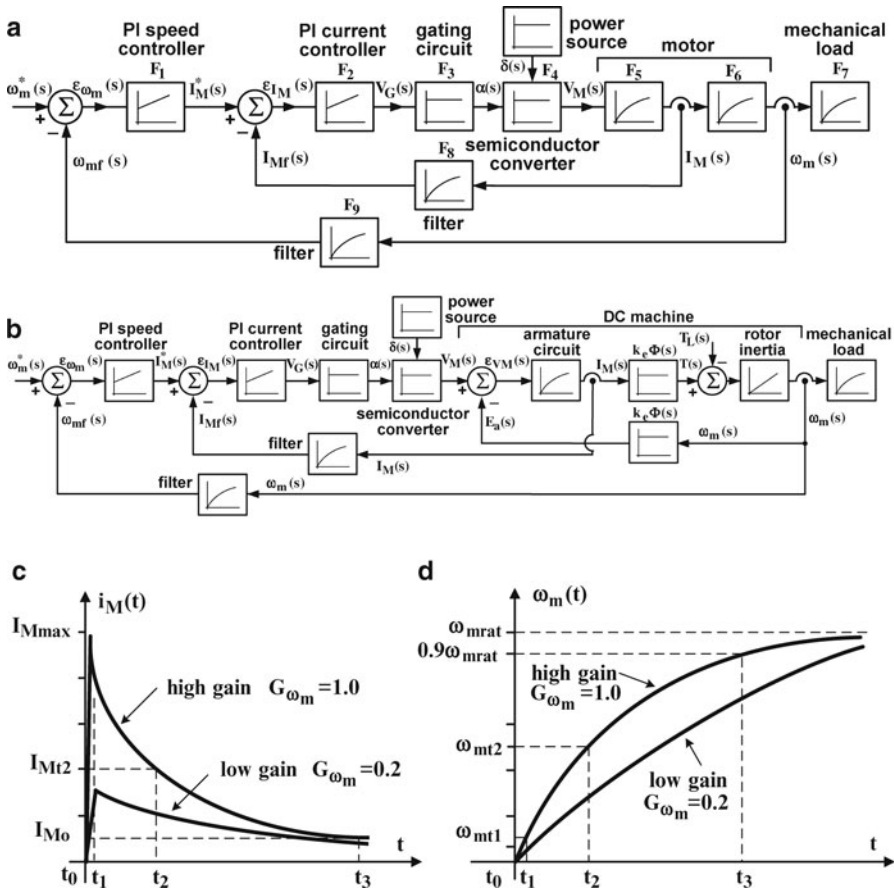
### 2.2.2 Closed-Loop Operation

If the speed regulation is less than a few percent ( $< 3\%$ ) and constant speed is sufficient, then open-loop operation is acceptable for the majority of drive applications. However, if the speed regulation must be small and variable-speed is required, a closed-loop speed control, with negative feedback, must be chosen. Such a system is represented in Fig. 2.16a.

In this system, the (inner) current control loop is desirable because it improves the dynamic behavior of the drive and the current can be limited to a value below a given maximum value if a nonlinear element (see Application Example 2.8) is included.

#### Application Example 2.3: Description of Start-Up Behavior of a Current- and Speed-Controlled Drive

The dynamic response during start-up of the drive of Fig. 2.16a with inner current feedback and outer speed feedback loop – where the internal feedback loop of the electric machine is not shown in Fig. 2.16a, but shown in Fig. 2.16b – is as follows. Provided the reference value of the mechanical angular velocity is  $\omega_m^* \cong \omega_{mrat}$  and the measured mechanical angular velocity at  $t = t_0 = 0$  is  $\omega_m \cong 0$  then the error angular velocity is  $\varepsilon_{\omega m} \cong \omega_{mrat}$ . If the gain of the speed controller is assumed to be  $G_{\omega m} = 1.0$ , then the reference signal of the current is  $I_M^* \cong I_{Mmax}$ . Because the filtered current is  $I_{Mf} \cong 0$  the error signal of the current controller is  $\varepsilon_{IM} \cong I_{Mmax}$ . Therefore, the gating circuit provides a gating signal  $V_G \cong V_{Gmin}$  for the thyristor rectifier resulting in  $\alpha \approx 0^\circ$  and a maximum motor current  $I_M \cong I_{Mmax}$  corresponding to the maximum torque  $T \cong T_{Mmax}$  at  $\omega_m \cong \omega_{mt1}$  (see Fig. 2.16c, d at time  $t_1$ ).



Due to this large torque the motor gains speed quickly, say  $\omega_m \cong \omega_{m2}$  at  $t=t_2$ : the speed error becomes  $\varepsilon_{\omega m} \cong \omega_{mmax} - \omega_{m2}$ , at the same time the motor current reduces to  $I_M \cong I_{M2}$  (see Fig. 2.16c, d at time  $t_2$ ). After some time the current error signal becomes  $\varepsilon_{IM} \cong 0.1 I_{Mmax}$ ; the gating voltage is increased to  $V_G \cong V_{Gmax}$  corresponding to  $\alpha = 175^\circ$  and the thyristor rectifier delivers to the motor the no-load current corresponding to  $I_M \cong I_{Mo}$  at  $t=t_3$ . In the meantime no-load speed has about been reached at  $\omega_m \cong 0.9 \omega_{mrat}$  (see Fig. 2.16c, d at time  $t_3$ ).

### 2.2.3 Multiple Closed-Loop Systems

The system of Fig. 2.16a can be augmented (see Fig. 2.17) with a position control feedback loop so that the rotor angular displacement can be controlled. Although there are different types of motors (induction, synchronous, reluctance, permanent-magnet, DC, etc.), all these motors can -in principle- be equipped with inner current control and outer speed and position feedback loops. It is up to the designer to select the best motor for a specific drive application with respect to size, costs, and safety issues related to explosive environments. The latter point disqualifies most DC motors because of their mechanical commutation and their associated arcs between brushes and commutator segments. For this reason DC motors will not be covered in this text in great detail, however, DC machines will be used to demonstrate control principles – indeed they serve as a role model for the control of AC machines.

Some of the most recently investigated systems relate to renewable energy, such as drive trains for electric (hybrid) automotive applications, the generation of electricity from photovoltaic sources, and from wind. Figure 2.18 depicts a closed-loop control system for an electric car drive consisting of either battery or fuel cell as

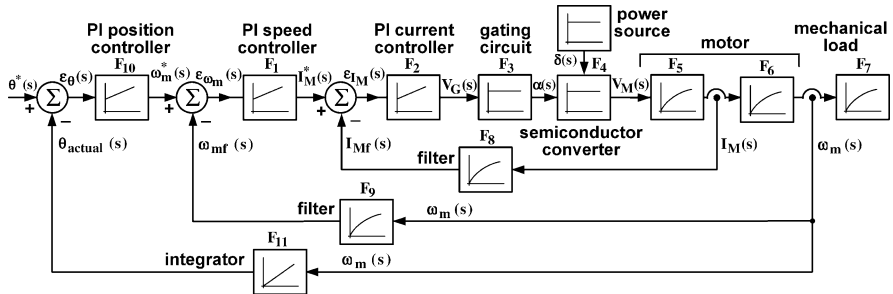


Fig. 2.17 Closed-loop current, speed, and position control

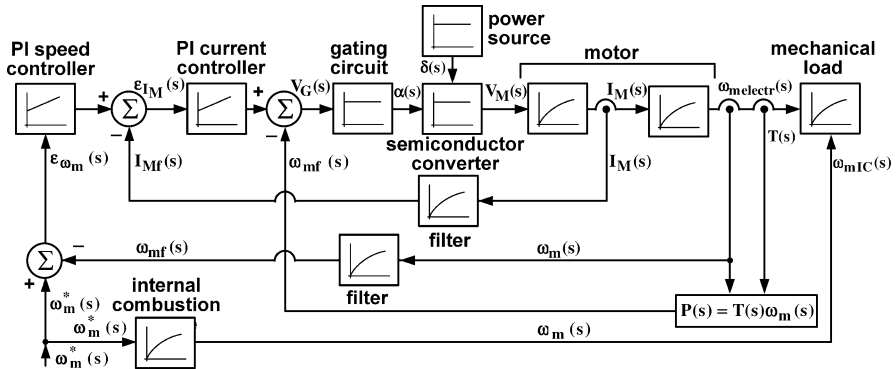
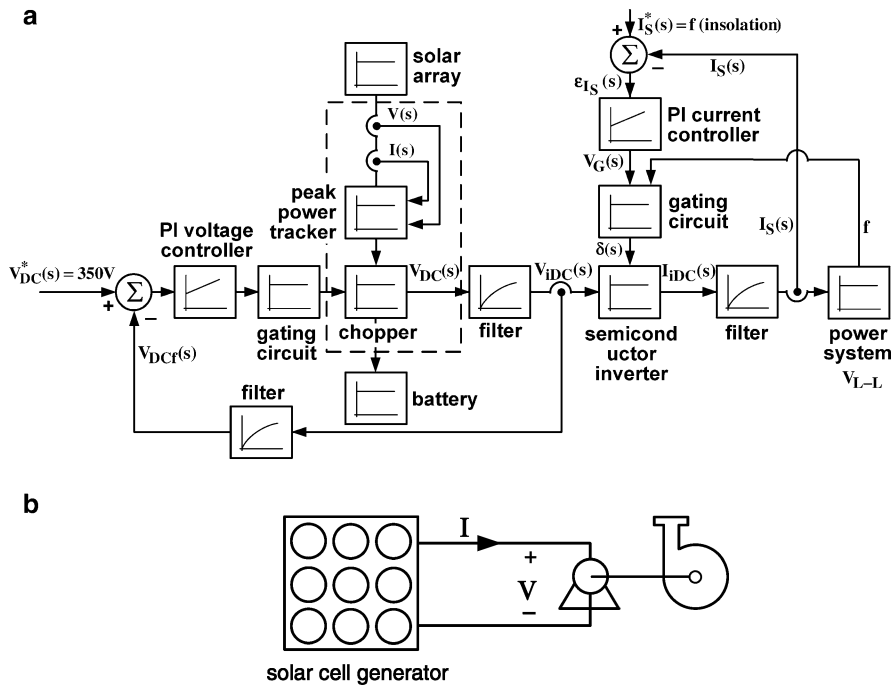


Fig. 2.18 Drive train for a parallel hybrid electric-internal combustion engine automobile drive



**Fig. 2.19** (a) Photovoltaic power generation with maximum power tracker and inverter. (b) Photovoltaic system supplying power to DC motor and pump

energy source, a six-step or pulse-width-modulated (PWM) inverter, and either a permanent-magnet motor (where the combination of an inverter and a permanent-magnet machine/motor/generator is called a brushless DC machine/motor/generator) or an induction motor as well as an internal combustion (IC) engine.

A commonly employed solar power generation system is illustrated in Fig. 2.19. It consists of an array of solar cells, a peak-power tracking component, a DC-to-DC converter which is able to operate either in a step-down or step-up mode, and an inverter feeding AC power into the utility system. Less sophisticated photovoltaic drive applications also exist, where a solar array directly feeds a DC motor driving a pump [4], as shown in Fig. 2.19b; in this case the costs of the entire drive are reduced. Figure 2.20 illustrates the block diagram of a variable-speed, direct-drive permanent magnet 20 kW wind power plant [5].

## 2.3 Principle of Operation of DC Machines

The principle of operation of a DC machine can be best explained based on the Gramme ring named for its Belgian inventor, Zénobe Gramme. It was the first generator to produce power on a commercial scale for industry. Gramme

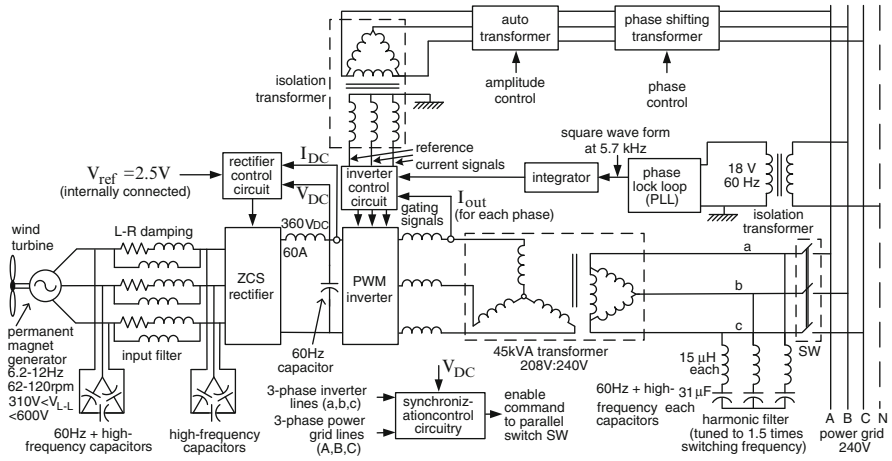


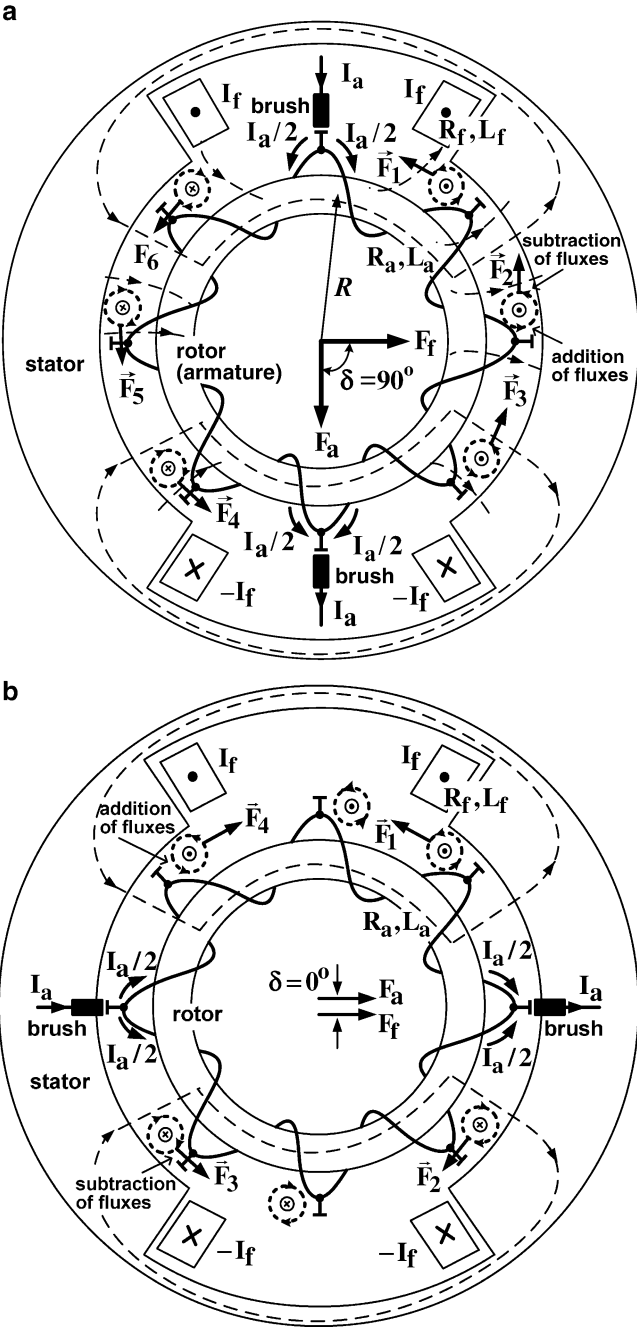
Fig. 2.20 Variable-speed, direct-drive wind power plant [5]

demonstrated this apparatus to the Academy of Sciences in Paris in 1871. The Gramme machine used a ring armature (or rotor), i.e., a series of thirty armature coils, wound around a revolving ring of soft iron. The coils are connected in series, and the junction between each pair is connected to a commutator on which two brushes run. The field excitation winding with resistance  $R_f$  and self inductance  $L_f$  or permanent magnets magnetize the stator core and the rotor (armature) iron ring, producing a magnetic field with the magnetomotive force (mmf) vector  $\mathbf{F}_f$  which is stationary with respect to the stator or field iron core. The rotor coils (having resistance  $R_a$  and self inductance  $L_a$ ) reside on the Gramme ring and rotate due the torque developed through the interaction of the armature current  $I_a$  with the field flux density  $\vec{B}_f$  – the armature current  $I_a$  sets up a rotor or armature mmf vector  $\mathbf{F}_a$  and interacts with the stator mmf  $\mathbf{F}_f$  –  $I_a$  is supplied to the commutator segments via the two brushes. That is, the stator magnetic field  $\vec{B}_f$  or mmf  $\mathbf{F}_f$  and the rotor magnetic field or mmf  $\mathbf{F}_a$  are stationary with respect to one another. The interaction of the stationary stator and rotor magnetic fields produce a torque  $\vec{T} = \mathbf{R} \times \vec{F}$  due to on the Lorentz force relation

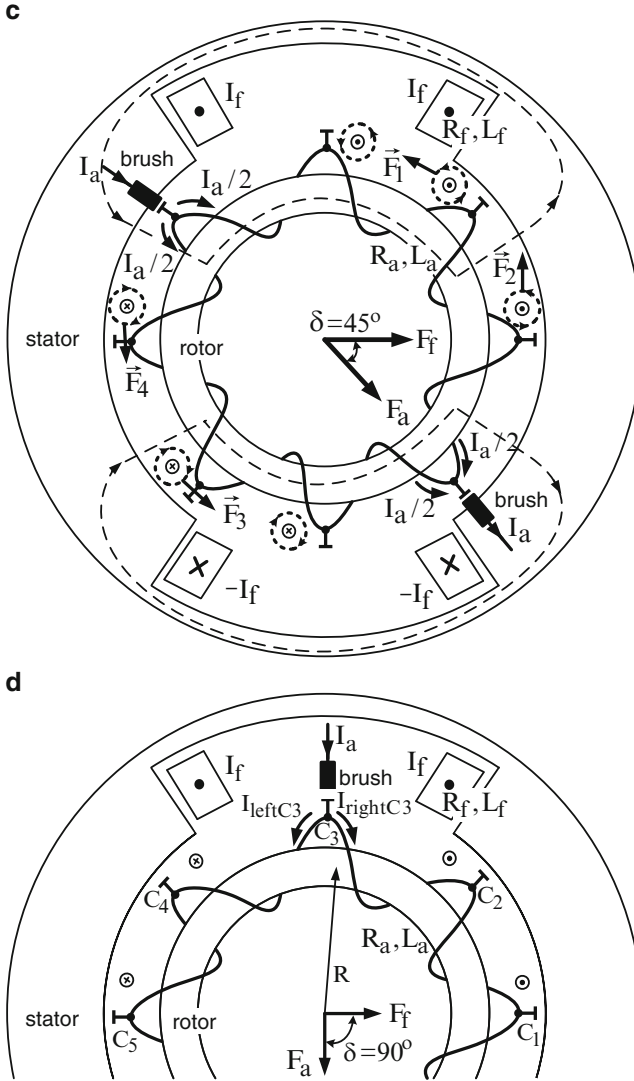
$$\vec{F} = (\vec{I}_a \times \vec{B}_f)\ell, \quad (2.26)$$

where  $\vec{I}_a$  is proportional to the mmf vector  $\mathbf{F}_a$  and  $\vec{B}_f$  is proportional to the mmf vector  $\mathbf{F}_f$ , and  $\ell$  is the length of the DC machine (depth of the Gramme ring/cylinder), and  $R$  is the radius of the Gramme ring as indicated in Fig. 2.21a.

In Fig. 2.21a the brushes reside in the quadrature (interpolar) axis. The field mmf vector  $\mathbf{F}_f$  and the armature mmf vector  $\mathbf{F}_a$  are orthogonal, that is, the angle between both vectors is  $\delta = 90^\circ$  which is called the torque angle. For the configuration of Fig. 2.21a the torque can be calculated from



**Fig. 2.21** (a) DC machine with brushes located in quadrature (interpolar) axis; maximum torque production,  $T = T_{\max}$ . (b) DC machine with brushes located in direct (polar) axis; torque production is zero,  $T = 0$ .



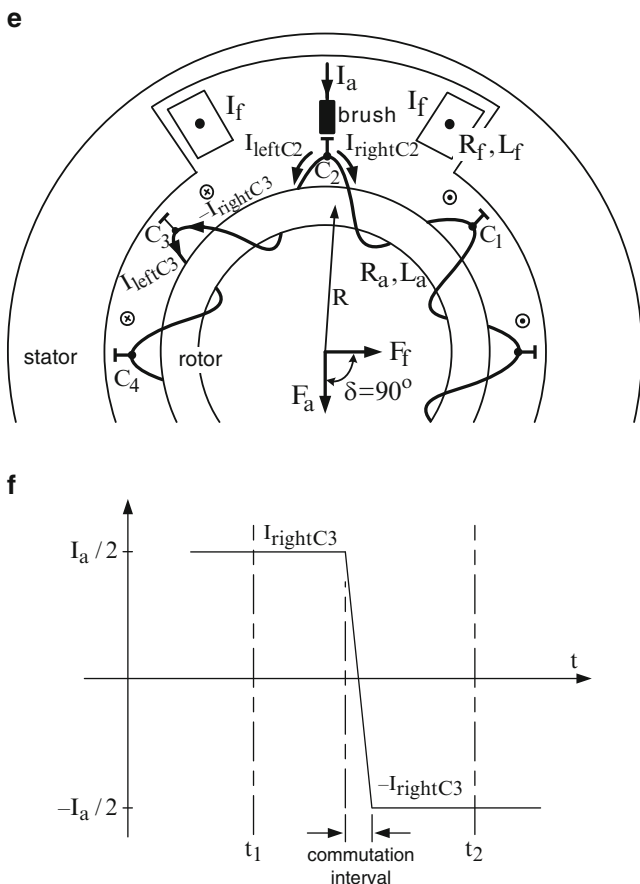
**Fig 2.21** (continued) (c) DC machine with brushes located between polar and interpolar axes; torque production larger than zero but less than  $T_{\max}$  or  $0 \leq T \leq T_{\max}$ . (d) Brush is connected to commutator segment  $C_3$  at time  $t_1$ .

$$|\vec{T}| = R \cdot \sum_{i=1}^6 |\vec{F}_i| \sin \delta, \quad (2.27)$$

for  $\delta = 90^\circ$  one obtains

$$|\vec{T}| = R \cdot \sum_{i=1}^6 |\vec{F}_i|. \quad (2.28)$$





**Fig 2.21** (continued) (e) Brush is connected to commutator segment  $C_2$  at time  $t_2$ . (f) Commutation (change from positive current  $I_{rightC3} = I_a/2$  at time  $t_1$  when brush is connected to segment  $C_3$  to negative current  $(-I_{rightC3}) = -I_a/2$  at time  $t_2$  when brush is connected to segment  $C_2$ ) of rotor current. The linear commutation interval is indicated

In Fig. 2.21b the brushes reside in the direct (polar) axis. The field mmf vector  $F_f$  and the armature mmf vector  $F_a$  are collinear or parallel, that is, the angle between both vectors is  $\delta=0^\circ$ . For the configuration of Fig. 2.21b the torque can be calculated from (2.27) and is  $|\vec{T}| = 0$ . In Fig. 2.21c the brushes are in between the direct (d) and quadrature (q) axes. The angle between the field mmf vector  $F_f$  and the armature mmf vector  $F_a$  is  $\delta$ , and the torque can be calculated from (2.27). This equation indicates that the torque  $\vec{T}$  can be controlled as a function of  $\delta$  by moving the brushes around the circumference of the machine. This concept will be exploited through electronic switching for the torque control of permanent-magnet machines and brushless DC machines.

When the rotor moves in counterclockwise direction, the currents in the turns to the right of the top brush of Fig. 2.21a can be defined as positive and the currents in the turns to the left of the top brush are flowing in opposite direction as compared to the currents in the turns to the right of the top brush – that is, they are negative. This demonstrates that the currents within the rotor winding are AC currents although the machine is called a DC machine. This change in current direction as the rotor moves is called commutation: the rotor current in the turns to the right of the top brush commutes or commutates from positive values to negative values as the turn moves to the left-hand side of the top brush. This commutation of the rotor current is depicted in Fig. 2.21d, e. The time functions of the commutating current  $I_{\text{rightC3}}$  is illustrated in Fig. 2.21f. The commutation can be linear or nonlinear. Figure 2.21f shows a linear commutation interval.

Today the design of the Gramme machine forms the basis of nearly all DC electric motors and DC generators [6–24]. Gramme’s use of multiple commutator contacts with multiple overlapped coils, and his innovation of using a ring armature (rotor), was an improvement on earlier dynamos and helped usher in development of large-scale electrical devices.

The transient (including steady-state) response of a separately excited DC machine (either motor or generator) is governed by the following equations:

$$\frac{d\omega_m}{dt} = \frac{1}{J} [T - T_L], \quad (2.29)$$

$$T = (k_e \Phi) i_a, \quad (2.30)$$

$$T_L = T_m + B\omega_m + T_c + c(\omega_m)^2 + T_s, \quad (2.31)$$

$$\frac{di_a}{dt} = \frac{1}{L_a} (V_a - R_a i_a - e_a), \quad (2.32)$$

$$e_a = (k_e \Phi) \omega_m. \quad (2.33)$$

In (2.29)  $\omega_m$  is the mechanical angular velocity (measured in rad/s) of the rotor (armature) of the DC machine,  $J$  is the polar moment of inertia of the rotor,  $T$  is the magnitude of the motor torque [see (2.27)], and  $T_L$  is the magnitude of the load torque.

Equation 2.30 defines the magnitude of the motor torque as a function of the resultant flux  $\Phi$  within the machine taking into account field flux and rotor flux (armature reaction),  $i_a$  is the instantaneous value of the rotor current whose steady-state value is  $I_a$ , and  $k_e$  is a constant. Equation 2.31 defines the load torque components:  $T_m$  is the mechanical torque doing useful work,  $B$  is a viscous damping coefficient,  $T_c$  is the coulomb frictional torque,  $c$  is a constant, and  $T_s$  is the standstill torque.

In (2.32)  $V_a$  is the terminal voltage, and  $e_a$  the induced voltage within the rotor winding.

**Application Example 2.4: Identify Transfer Functions of a Block Diagram**

In this example, define all transfer functions of the block diagram of Fig. 2.16a and find the transfer function  $\{\omega_m(s)/\omega_m^*(s)\}$ .

**Solution.**

Individual transfer functions

$$F_1 = \frac{G_1(1 + s\tau_1)}{s}, F_2 = \frac{G_2(1 + s\tau_2)}{s}, F_3 = G_3, F_4 = G_4, F_5 = \frac{G_5}{(1 + s\tau_5)},$$

$$F_6 = \frac{G_6}{(1 + s\tau_6)}, F_7 = \frac{G_7}{(1 + s\tau_7)}, F_8 = \frac{G_8}{(1 + s\tau_8)}, F_9 = \frac{G_9}{(1 + s\tau_9)}.$$

$$\begin{aligned} I_M(s) &= F_2 F_3 F_4 F_5 (I_M^*(s) - F_8 I_M(s)), I_M(s)(1 + F_2 F_3 F_4 F_5 F_8) \\ &= F_2 F_3 F_4 F_5 I_M^*(s), \text{ or } \frac{I_M(s)}{I_M^*(s)} = \frac{F_2 F_3 F_4 F_5}{(1 + F_2 F_3 F_4 F_5 F_8)}, \omega_m(s) = F_6 I_M(s), \omega_m(s) \\ &= \frac{F_2 F_3 F_4 F_5 F_6}{(1 + F_2 F_3 F_4 F_5 F_8)} I_M^*(s), I_M^*(s) = F_1 (\omega_m^*(s) - F_9 \omega_m(s)), \frac{\omega_m(s)}{\omega_m^*(s)} \\ &= \frac{F_1 F_2 F_3 F_4 F_5 F_6}{(1 + F_2 F_3 F_4 F_5 F_8 + F_1 F_2 F_3 F_4 F_5 F_6 F_9)} = F_{12}. \end{aligned}$$

**2.4 Central Air-Conditioning System for Residence**

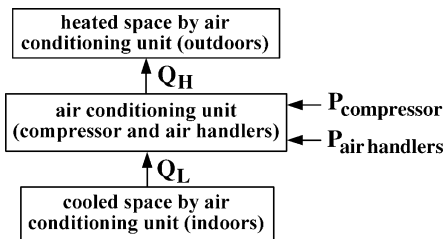
The central air-conditioning unit of a residence is shown in the block diagram of Fig. 2.22.

Air conditioning systems are used for space cooling (e.g., rooms, residences, etc.). They are similar to the cooling unit of a refrigerator.

The indoor heat (power)  $Q_L$  will be transported by an air handling ( $P_{\text{air handlers}}$ ) and a compressor ( $P_{\text{compressor}}$ ) system from the indoors to the outdoors.

- The constant-speed compressor(s) of the air conditioning unit is (are) located outdoors.
- The coil with the warm cooling medium is located outdoors, where the coil is cooled by a variable-speed outdoor fan (air handler), which dissipates the heat energy to the surrounding air, or there is a coil embedded in the ground and dissipates the heat to the soil or ground water. No fan will be required for the outside unit in this latter case.
- There is also a variable-speed indoor fan (air handler), which circulates the cool air originating from the coil with the cool medium within the residence.

**Fig. 2.22** Block diagram of a central air conditioning unit for a residence, where L and H means low and high temperature, respectively



It is the advantageous feature of an air conditioning system that for the given input electrical power  $P_{in} = (P_{compressor} + P_{air\ handlers})$  one can provide cooling power  $Q_L$  for the indoors, where  $P_{in}$  is significantly smaller than  $Q_L$ . The coefficient of performance of an air conditioning unit is defined as

$$COP_L = Q_L / (P_{compressor} + P_{air\ handlers}).$$

Air conditioning systems are classified by the Seasonal Energy Efficiency Ratio (SEER). The relation between SEER and  $COP_L$  is:

$$SEER = 3.413 \cdot COP_L.$$

According to federal law [25], SEER=13 is the minimum seasonal energy efficiency ratio which is commercially available for residential air conditioning systems. Central air-conditioning systems are most efficient if they operate all the time at either high or low speed, and on-off operation (that is the start-up phase) should be avoided for high-efficient central air-conditioning systems. This is also good for extending the lifetime of compressors. One can classify air conditioning systems according to Table 2.1.

The size of an air conditioning unit is specified in terms of “tons”, where 1 ton  $\equiv$  12,000 BTU/h.

Additional useful units are: 1 BTU  $\equiv$  1.055 kJ, 1 J  $\equiv$  1 Ws, 1 kWh  $\equiv$  3.6  $\cdot$  10<sup>3</sup> kJ, and 1 quad  $\equiv$  10<sup>15</sup> BTU.

A new evaporative cooling technology [26, 27] can deliver cooler supply air temperatures than either direct or indirect evaporative cooling systems, without increasing humidity. The technology, known as the Coolerado Cooler™, has been described as an “ultra cooler” because of its performance capabilities relative to other evaporative cooling products. The Coolerado Cooler evaporates water in a secondary (or working) air stream, which is discharged in multiple stages. No water or humidity is added to the primary (or product) air stream in the process. This approach takes advantage of the thermodynamic properties of air, and it applies to both direct and indirect cooling technologies in an innovative cooling system that is drier than direct evaporative cooling and cooler than indirect cooling. The technology also uses much

**Table 2.1** Classification of air conditioning units

Low efficiency	SEER = 13–14	Two (outdoor and indoor) constant-speed fans (air handlers) with either one or two speed ranges (high and low speed), one constant-speed compressor
Medium efficiency	SEER = 15–16	Two variable-speed (outdoor and indoor) fans (air handlers), two (one small rating and one large rating) constant-speed compressors. The difference between SEER = 15–16 and SEER = 17–19 are the cooling surfaces of the outdoor and indoor coils of the air handlers: the larger the cooling surface the higher the efficiency
High efficiency	SEER = 17–19	

less energy than conventional vapor compression air-conditioning systems. Performance tests have shown that the efficiency of the Coolerado Cooler is 1.5–4 times higher than that of conventional vapor compression cooling systems, while it provides the same amount of cooling. It is suitable for climates having low to average humidity, as is the case in much of the Western half of the United States. This technology can also be used to pre-cool air in conventional heating, ventilating, and air-conditioning systems in more humid climates because it can lower incoming air temperatures without adding moisture.

**Application Example 2.5: A Central Air-Conditioning Unit with SEER = 13**

Central air-conditioning unit of rated capacity of  $Q_L = 3$  tons (average residence of 2,500 (ft)<sup>2</sup>), SEER = 13, one constant-speed compressor, and two constant-speed air handlers. Input power of two (outdoor and indoor fans) air handlers at 50% of rated cfm (cubic feet per minute):  $P_{\text{air handlers}} = 2 \cdot I_{\text{rms}} \cdot \cos\Phi \cdot V_{\text{rms}} = 2 \cdot 7\text{A} \cdot 0.9 \cdot 120\text{V} = 0.864$  kW. Coefficient of performance is a function of SEER:  $\text{COP}_L = \text{SEER}/3.413 = 3.81$ . Cooling power provided to the cooled space (indoors) corresponding to 3 tons:  $Q_L = 10.55$  kW; for given  $Q_L$ ,  $\text{COP}_L$  and  $P_{\text{air handlers}}$  values:  $P_{\text{compressor}} = 1.91$  kW;  $P_{\text{in}} = (P_{\text{compressor}} + P_{\text{air handlers}}) = 2.77$  kW

**Application Example 2.6: A Central Air-Conditioning Unit with SEER = 19**

Central air-conditioning unit of rated capacity of  $Q_L = 3$  tons, SEER = 19, two constant-speed compressors (only one is on at any time, but not both) and two variable-speed air handlers:  $P_{\text{air handlers}} = 2 \cdot 1.03\text{A} \cdot 120\text{V} = 0.247$  kW at 50% of rated cfm;  $\text{COP}_L = \text{SEER}/3.413 = 5.57$ ;  $Q_L = 10.55$  kW;  $P_{\text{compressor}} = 1.65$  kW;  $P_{\text{in}} = (P_{\text{compressor}} + P_{\text{air handlers}}) = 1.897$  kW. For a high-efficient air conditioner, the fan and coil of the outdoor air handler is rather large to increase the cooling surface.

The heat pump is a similar system with a reverse heat (power) flow  $Q_H$  from the outdoors to the indoors, where the heat is being released. A similar coefficient of performance can be defined for heat pumps:  $\text{COP}_H = Q_H/(P_{\text{compressor}} + P_{\text{air handlers}})$ .

### Application Example 2.7: Emergency Standby Generating Set for Commercial Building

An emergency standby generating set for a commercial building is to be sized. The building has an air conditioning unit (10 tons, SEER=14) and two air handlers, two high-efficient refrigerators/freezers (18 (ft)<sup>3</sup>, each), 100 light bulbs, three fans, two microwave ovens, three drip coffee makers, one clothes washer (not including energy to heat water), one clothes dryer, one electric range (with oven), TV and VCR, three computers, and one 3 hp motor for a water pump.

- Find the wattage of the required standby emergency power equipment.
- What is the estimated price for such standby equipment (without installation)?

#### Solution.

Typical wattages of appliances are listed in Table 2.2.

**Table 2.2** Typical wattage of appliances

---

Clothes dryer:	4,800 W
Clothes washer (not including energy to heat hot water):	500 W
Drip coffee maker:	1,500 W
Fan (ceiling):	100 W
Fan(box/window):	200 W
Microwave oven:	1,450 W
Dishwasher:	1,200 W
Vacuum cleaner:	700 W
Range (with conventional oven):	3,200 W
Range (with self-cleaning oven):	4,000W
Refrigerator/freezer 18 (ft) <sup>3</sup> :	150 W (new high-efficient) to 700 W (old)
VCR (including operation of TV):	120 W
Television:	250 W
Toaster:	1,100 W
Hair dryer:	1,250 W
Compact fluorescent light bulb (cfl) and incandescent light bulbs:	13 W (cfl) to 200 W (incandescent)
Computer:	360 W
Air conditioning unit, compressor (8 tons, SEER = 14):	6,000 W.
Two air handlers of air conditioning unit:	3,000 W.

---

- Required wattage

10 ton, SEER=14 air conditioning unit:  $6,000 \cdot (10/8) = 7,500$  W  
 air handlers for 10 ton, SEER=14 air conditioner:  $3,000 \cdot (10/8) = 3,750$  W  
 2 refrigerators/freezers (high-efficient):  $2 \cdot 150 = 300$  W  
 100 light bulbs (cfl):  $100 \cdot 65 = 6,500$  W  
 3 fans (box/window):  $3 \cdot 200 = 600$  W  
 2 microwave ovens:  $2 \cdot 1,450 = 2,900$  W  
 3 drip coffee makers:  $3 \cdot 1,500 = 4,500$  W  
 1 clothes washer (not including energy to heat hot water): 500 W  
 1 clothes dryer: 4,800 W  
 1 range (with oven): 3,200 W

TV and VCR: 370 W

2 computers:  $3 \cdot 360 = 720$  W

3 one-horse power motors (note 1 hp=746 W):  $3 \cdot 746 = 2,238$  W

Total wattage required: 37,878 W

(b) Price for a 40 kW standby generator according to [28] is about \$10,210.

### Application Example 2.8: Open-and Closed-Loop Operation of DC Machine Drive

*Open-loop operation:* Compute the transient response of a separately excited DC machine for changes of the armature voltage ( $V_a$ ), of the load torque ( $T_L$ ) doing the mechanical work ( $T_m$ ), and of the flux ( $k_e\Phi$ ) (see Fig. 2.23).

The transient response of a separately excited DC machine is governed by the following two first-order differential equations:

$$\frac{di_a}{dt} = \frac{1}{L_a} [V_a - R_a i_a - (k_e\Phi)\omega_m], \quad (2.34)$$

where the induced voltage is  $e_a = (k_e\Phi)\omega_m$ .

$$\frac{d\omega_m}{dt} = \frac{1}{J} [(k_e\Phi)i_a - T_m - B\omega_m - T_c - c(\omega_m)^2 - T_s], \quad (2.35)$$

where the mechanical load torque (including frictional torques  $B\omega_m$ ,  $T_c$ ,  $c(\omega_m)^2$ ,  $T_s$ ) is  $T_L = T_m + B\omega_m + T_c + c(\omega_m)^2 + T_s$ , and  $\omega_m$  is the angular velocity related to the speed of the motor.

The electrical DC machine torque is

$$T = (k_e\Phi)i_a, \quad (2.36)$$

and the DC machine output power (neglecting windage and frictional losses) is

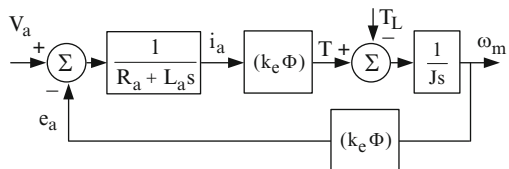
$$P = T\omega_m. \quad (2.37)$$

The constant machine parameters are defined as follows:

$L_a = 3.6$  mH,  $R_a = 0.153$   $\Omega$ ,  $(k_e\Phi) = 2.314$  Vs/rad,  $J = 0.56$  kgm<sup>2</sup>,  $T_c = 5$  Nm,

$B = 0.1$  Nm/(rad/s),  $c = 0.002$  Nm/(rad/s<sup>2</sup>), and  $T_s = 0$  Nm.

$L_a$  is the armature inductance,  $R_a$  the armature resistance,  $(k_e\Phi)$  corresponds to the exciting flux of the machine,  $J$  is the polar moment of inertia,  $T_c$  is the Coulomb frictional torque,  $B$  is the damping coefficient of the viscous torque,  $c$  is a



**Fig. 2.23** Equivalent circuit of separately excited DC machine in time domain

**Table 2.3** Variable input parameters for start-up based on armature voltage control, rated operation at rated speed, increased speed via flux weakening, and reversing the speed under reduced flux conditions

Time $t$ [s]	0	2	4	6	8	10	12
Terminal voltage of DC motor $V_a$ [V]	110	220	220	220	220	−220	
Field flux of DC motor ( $k_e\Phi$ ) [Vs/rad]	2.314	2.314	2.314	2.314	1.067	1.067	
Mechanical torque doing the work $T_m$ [Nm]	0	0	150	190	75	−75	
Coulomb frictional torque $T_c$ [Nm]	5	5	5	5	5	5	
Coefficient of windage torque $c$ [Nm/(rad/s) <sup>2</sup> ]	0.002	0.002	0.002	0.002	0.002	−0.002	

constant for the windage torque,  $T_s$  is the torque at standstill, and in Fig. 2.23 “ $s$ ” is the Laplace operator.

The input parameters  $V_a$ ,  $T_m$ , ( $k_e\Phi$ ) can be variable (see Table 2.3).

- (a) Compute and plot ( $0 \leq t \leq 12$  s) using either Matlab or Mathematica the armature current  $i_a(t)$ , the angular velocity  $\omega_m(t)$ , the electrical machine torque  $T(t)$ , and the machine output power  $P(t)$  as a function of time  $t$  for the initial conditions :  $i_a(t=0)=0$ ,  $\omega_m(t=0)=0$ , and for the variations of the input parameters as detailed in Table 2.3. Print either the Matlab or Mathematica program. The start-up of the machine from 0 to 4 s is achieved by armature-voltage control, and the speed increase above rated speed from 8 to 10 s is accomplished based on flux reduction (below rated flux) or flux-weakening control. Reversing of speed is performed from 10 to 12 s under reduced-flux conditions.

### Solution

Remove[] “removes symbols completely, so that their names are no longer recognized by Mathematica” rather than merely setting variables to zero, and “all of its properties and definitions are removed as well”. More common than removing variables one at a time as in the remove statement is Remove[“Global\*”], which removes everything except explicitly protected symbols in the Global (default) context. As an even more effective alternative, the Quit [] command can be used, which quits the kernel from the command line. All definitions are lost.

A way is to set at the very beginning of each and every program run all variables to zero push the button “Evaluation” and then choose “Quit Kernel” resulting in the response “local” and the question: “Do you really want to quit the kernel?”. Then select “Quit”.

*How do we run the software Mathematica?:*

1. In Windows (it depends how the software is installed): Start → Programs → Mathematica → Mathematica 6.

In Mac OS: Inside the Applications folder, there should be a Mathematica folder



2. All Mathematica files have the extension .nb or notebook.
3. When entering a line or equation, a single equal sign (=) sets the value of a parameter, whereas a double equal sign (==) means “equal” and relates to equations in Mathematica.
4. All functions of t are defined by  $x[t\_]:=$  where the two important parts are t\_ and := these are required whenever defining a function.
5. To run a program, one can execute the program one instruction at a time or have the computer go through all instructions for you.
  - To execute instructions one at a time, put the cursor on the line and press the return key while holding down the shift key. Or go to Kernel → Evaluation → Evaluate Cells.
  - To run the whole program at once, go to kernel → Evaluation → Evaluate Notebook.
6. Any blue messages from Mathematica signify an error in the notebook somewhere.

Remove statement

Remove[T, TL, Va, Wm, sol, eqn1, eqn2, ic1, ic2, kphi, Tm, Tic, icky, P, Ia]

Definition of constant input parameters:

La=0.0036;  
 Ra=0.153;  
 J=0.56;  
 B=0.1;  
 Ts=0;

Definition of time-dependent input parameters (see Table 2.3):

Va[t\_] := If[t<10.0, If[t<2, 110, 220], -220];  
 kphi[t\_] := If[t<8.0, 2.314, 1.067];  
 Tm[t\_] := If[t<10.0, If[t<8.0, If[t<6.0, If[t<4.0, 0, 150], 190], 75], -75];  
 Tic[t\_] := If[t<10.0, 5, -5];  
 icky[t\_] := If[t<10.0, 0.002, -0.002];

Initial conditions at time t=0:

ic1=Ia[0]= 0;  
 ic2=Wm[0]= 0;

Parameters as a function of the independent variables (e.g., Ia[t], T[t], Wm[t]):

T[t\_] := kphi[t]\*Ia[t];  
 P[t\_] := T[t]\*Wm[t];

Definition of the set of differential equations:

$$\text{eqn1} = \text{Ia}'[t] == (1/\text{La}) * (\text{Va}[t] - \text{Ra} * \text{Ia}[t] - \text{kphi}[t] * \text{Wm}[t]);$$

$$\text{eqn2} = \text{Wm}'[t] =$$

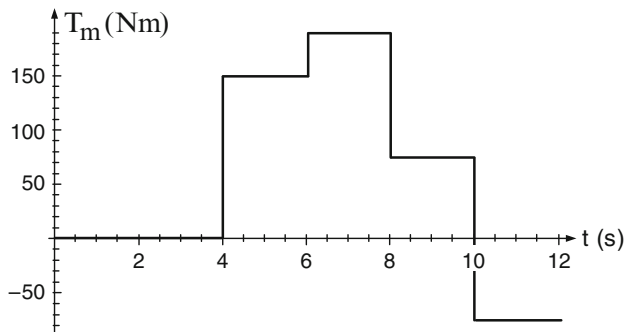
$$= (1/J) * (\text{kphi}[t] * \text{Ia}[t] - \text{Tm}[t] - \text{B} * \text{Wm}[t] - \text{Tic}[t] - \text{icky}[t] * (\text{Wm}[t])^2 - \text{Ts});$$

Numerical solution of differential equation system:

```
sol= NDSolve[{eqn1,eqn2, ic1, ic2}, {Ia[t], Wm[t]}, {t, 0, 12}, MaxSteps -> 10,000];
```

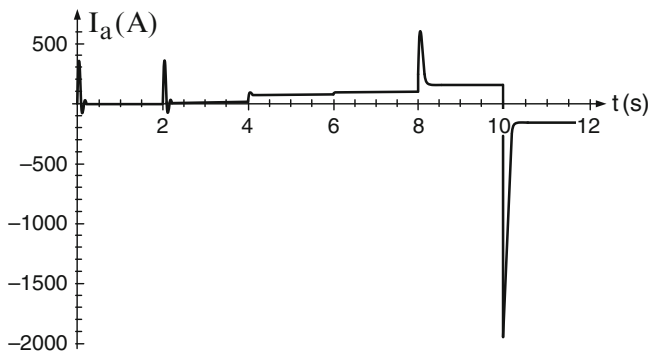
Plotting statements: (Figs. 2.24–2.28)

```
Plot[Evaluate[Tm[t]], {t, 0, 12}, PlotRange -> All, AxesLabel -> {"t (s)", "Tm (Nm)"}]
```



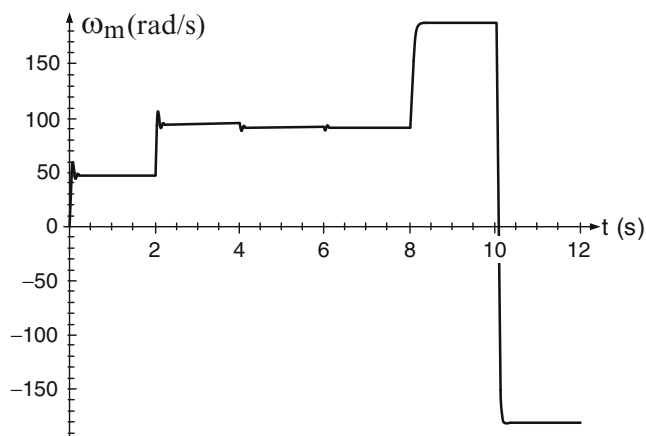
**Fig. 2.24** Mechanical shaft torque in Nm

```
Plot [Ia[t]/. sol, {t, 0, 12}, PlotRange -> All, AxesLabel -> {"t (s)", "Ia (A)"}]
```



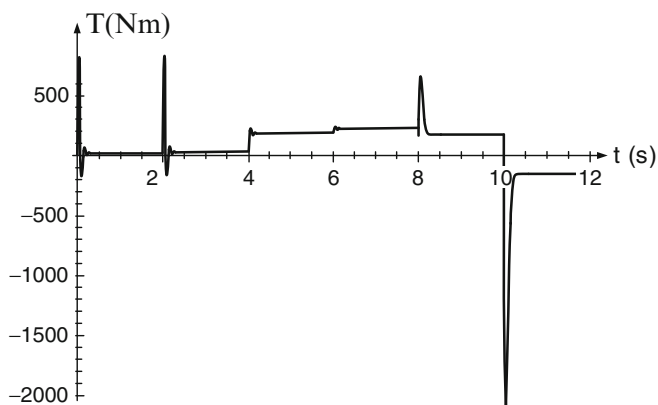
**Fig. 2.25** DC machine armature current in A without current limiter

Plot [Wm[t]/. sol, {t, 0, 12}, PlotRange  $\rightarrow$  All, AxesLabel  $\rightarrow$  {"t (s)", "Wm (rad/s)"}]



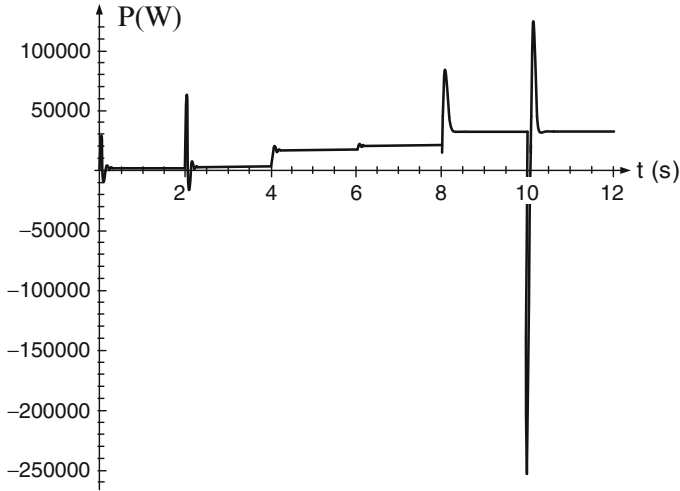
**Fig. 2.26** Mechanical angular velocity in rad/s without current limiter

Plot[T[t]/.sol, {t, 0, 12}, PlotRange  $\rightarrow$  All, AxesLabel  $\rightarrow$  {"t (s)", "T (Nm)"}]



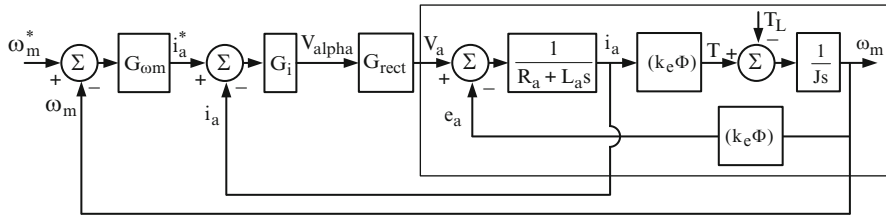
**Fig. 2.27** DC machine torque in Nm without current limiter

Plot[P[t]/.sol, {t, 0, 12}, PlotRange → All, AxesLabel → {"t (s)", "P (W)"}]



**Fig. 2.28** DC machine output power in W without current limiter

*Closed-loop operation:* Figure 2.29 illustrates the overall circuit of the DC machine with angular velocity  $\omega_m$  control (also called speed control).



**Fig. 2.29** Speed (angular velocity) control via armature voltage variation (reduction) with P-speed controller and without current limiter in time domain

The transient response of a separately excited DC machine with speed control, using armature–voltage variation, is governed by the following equations:

$$\frac{d\omega_m}{dt} = \frac{1}{J} [T - T_L], \quad (2.38)$$

$$T = (k_e \Phi) i_a, \quad (2.39)$$

$$T_L = T_m + B\omega_m + T_C + c\omega_m^2 + T_S, \quad (2.40)$$

$$\frac{di_a}{dt} = \frac{1}{L_a} (V_a - R_a i_a - e_a), \quad (2.41)$$

$$e_a = (k_e \Phi) \omega_m, \quad (2.42)$$

$$V_a = G_{\text{rect}} V_{\alpha}, \quad (2.43)$$

$$V_{\alpha} = G_i(i_a^* - i_a), \quad (2.44)$$

$$i_a^* = G_{\omega m}(\omega_m^* - \omega_m). \quad (2.45)$$

The constant machine parameters are defined as follows:

$L_a = 3.6$  mH,  $R_a = 0.153$   $\Omega$ ,  $(k_c \Phi) = 2.314$  Vs/rad,  $J = 0.56$  kgm<sup>2</sup>,  $T_c = 5$  Nm,  
 $B = 0.1$  Nm/(rad/s),  
 $c = 0.002$  Nm/(rad/s<sup>2</sup>),  $T_s = 0$  Nm,  $G_{\text{rect}} = 20$ ,  $G_i = 1.0$ ,  $G_{\omega m} = 10.0$ .

Note all above values are maintained constant throughout closed-loop control and Table 2.3 does not apply.  $G_{\text{rect}}$  is the gain of the rectifier,  $G_i$  is the gain of the P-current controller, and  $G_{\omega m}$  is the gain of the P-speed controller

- (b) Using either Matlab or Mathematica, compute for  $T_m = 190$  Nm and plot ( $0 \leq t \leq 2$  s) the armature current  $i_a(t)$ , the angular velocity  $\omega_m(t)$ , the machine torque  $T(t)$ , and the output power  $P(t)$  as a function of time for the initial conditions  $i_a[0] = 0$ ,  $\omega_m[0] = 0$ , where at time  $t = t_0 = 0$   $\omega_m^*$  changes from 0 to 100 rad/s.
- (c) Replace at time  $t_1 = 2$  s the P-speed controller by a PI-controller (where the final values of part b) serve as initial conditions). This means replace equation (2.45) by

$$\frac{di_a^*}{dt} = \frac{(\omega_m^* - \omega_m)}{0.2} - \frac{d\omega_m}{dt} \quad (2.46)$$

*Mathematica program for the time interval  $t = 0$  s to 2 s, and the speed P-controller gain of  $G_{wm} = 10.0$ :*

A “remove” statement can be used to set variables to zero:

Remove[T, TL, Ea, Va, Iastar, Wmstar, Valpha, Wm, sol, eqn1, eqn2, ic1, ic2, kphi, Tm, Tic, icky, P, Ia]

Definition of constant input parameters:

```
La=0.0036;
Ra=0.153;
J=0.56;
B=0.1;
Ts=0;
kphi=2.314;
Tc=5;
c=0.002;
Grect=20;
Gi=1;
Gwm=10;
Tm=190;
Wmstar=100;
```

Definition of time-dependent input parameters:

There are no time-dependent input parameters.

Initial conditions:

$$\begin{aligned} \text{ic1} &= \text{Ia}[0] = 0; \\ \text{ic2} &= \text{Wm}[0] = 0; \end{aligned}$$

Parameters as a function of the independent variables:

$$\begin{aligned} T[t\_ ] &:= k\phi i * \text{Ia}[t]; \\ P[t\_ ] &:= T[t] * \text{Wm}[t]; \\ TL[t\_ ] &:= T_m + B * \text{Wm}[t] + T_c + c * (\text{Wm}[t] * \text{Wm}[t]) + T_s; \\ Ea[t\_ ] &:= k\phi i * \text{Wm}[t]; \\ Va[t\_ ] &:= G_{rect} * V_{\alpha}[t]; \\ V_{\alpha}[t\_ ] &:= G_i * (\text{Ia}_{star}[t] - \text{Ia}[t]); \\ \text{Ia}_{star}[t\_ ] &:= G_{wm} * (\text{Wm}_{star} - \text{Wm}[t]); \end{aligned}$$

Definition of the set of differential equations:

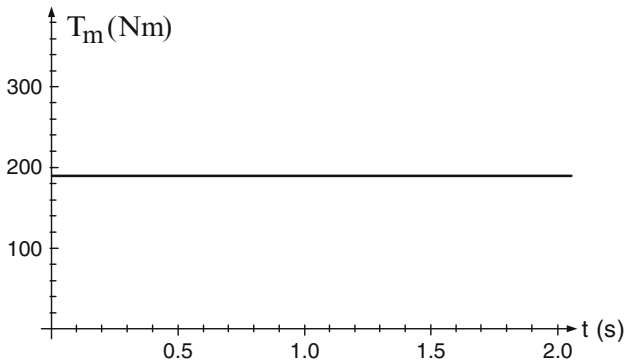
$$\begin{aligned} \text{eqn1} &= \text{Ia}'[t] == (1/L_a) * (V_a[t] - R_a * \text{Ia}[t] - E_a[t]); \\ \text{eqn2} &= \text{Wm}'[t] == (1/J) * (T[t] - TL[t]); \end{aligned}$$

Numerical solution of differential equation system:

$$\text{sol} = \text{NDSolve}[\{\text{eqn1}, \text{eqn2}, \text{ic1}, \text{ic2}\}, \{\text{Ia}[t], \text{Wm}[t]\}, \{t, 0, 2\}, \text{MaxSteps} \rightarrow 10,000];$$

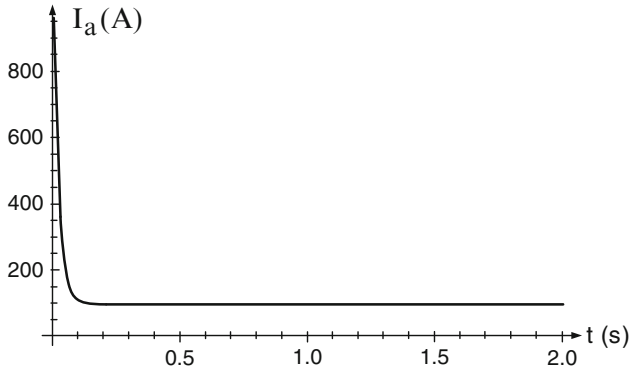
Plotting statements (Figs. 2.30–2.34):

$$\text{Plot}[\text{Evaluate}[T_m], \{t, 0, 2\}, \text{PlotRange} \rightarrow \text{All}, \text{AxesLabel} \rightarrow \{\text{"t (s)"}, \text{"T}_m \text{ (Nm)}\}]$$



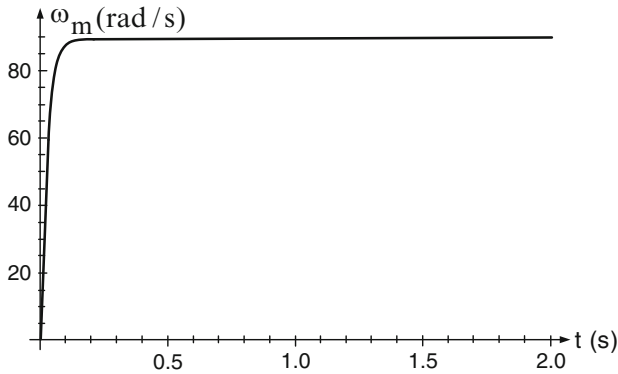
**Fig. 2.30** Mechanical shaft torque in Nm

$$\text{Plot}[\text{Ia}[t] /. \text{sol}[[1]], \{t, 0, 2\}, \text{PlotRange} \rightarrow \text{All}, \text{AxesLabel} \rightarrow \{\text{"t (s)"}, \text{"Ia (A)}\}]$$



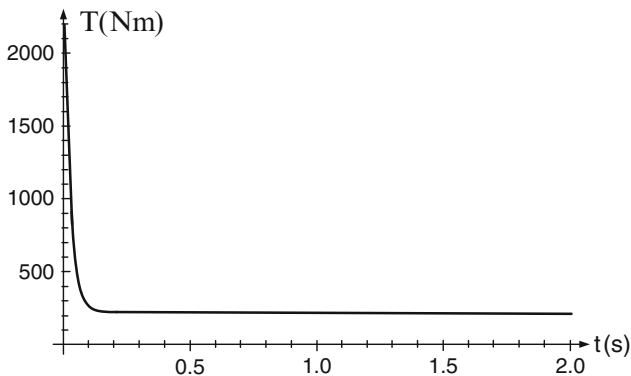
**Fig. 2.31** DC machine armature current in A without current limiter

`Plot[Wm[t]/.sol[[1]], {t, 0, 2}, PlotRange → All, AxesLabel → {"t (s)", "Wm (rad/s)"}]`



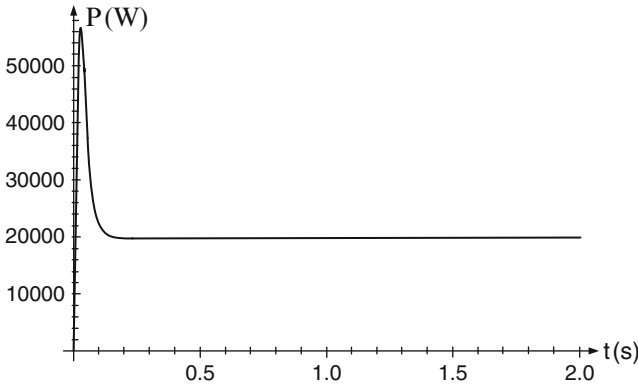
**Fig. 2.32** Mechanical angular velocity in rad/s without current limiter

`Plot[Evaluate[T[t]]/.sol[[1]], {t, 0, 2}, PlotRange → All, AxesLabel → {"t (s)", "T (Nm)"}]`



**Fig. 2.33** DC machine torque in Nm without current limiter

Plot[Evaluate[P[t]]/.sol[[1]], {t, 0, 2}, PlotRange → All, AxesLabel → {"t (s)", "P (W)"}]



**Fig. 2.34** DC machine output power in W without current limiter

*Continuation of the Mathematica program for the time interval from  $t=2$  s to 5 s replacing P-controller (2.45) by PI-speed controller (2.46):*

Storing the final conditions at time  $t = 2$ s:

```
Wmb=(Wm[t]/.sol[[1]])/.t → 2;
Iab=(Ia[t]/.sol[[1]])/.t → 2;
Iastarb=(Iastar[t]/.sol[[1]])/.t → 2;
```

Initial conditions for time interval  $t = (2 \text{ to } 5)$ s:

```
ic3=Wm[2] = Wmb;
ic4=Ia[2] = Iab;
ic5=Iastar[2] = Iastarb;
```

Parameters as a function of the independent variables:

```
T[t_] := kphi*Ia[t];
TL[t_] := Tm + B*Wm[t] + Tc + c*(Wm[t])^2 + Ts;
Ea[t_] := kphi*Wm[t];
Va[t_] := Grect*Valpha[t];
Valpha[t_] := Gi*(Iastar[t] - Ia[t]);
P[t_] := T[t]*Wm[t];
```

Definition of the set of differential equations:

```
eqn3 = Wm'[t] == (1/J) * (T[t] - TL[t]);
eqn4 = Ia'[t] == (1/La) * (Va[t] - Ra * Ia[t] - Ea[t]);
eqn5 = Iastar'[t] == (5 * Wmstar - 5 * Wm[t]) - (1/J) * (T[t] - TL[t]);
```

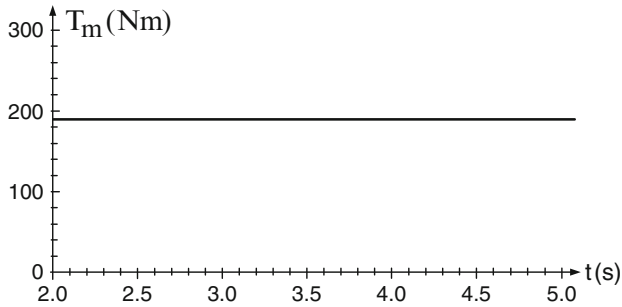


Numerical solution of differential equation system:

```
sol2= NDSolve[{eqn3, eqn4, eqn5, ic3, ic4, ic5}, Wm[t], Ianeu[t], Iastarnew[t], {t, 2, 5}, MaxSteps -> 10,000];
```

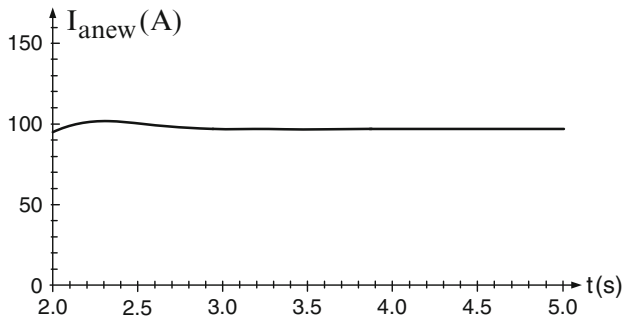
Plotting statements (Figs. 2.35–2.40):

```
Plot[Evaluate[Tm], {t, 2, 5}, PlotRange -> All, AxesLabel -> {"t (s)", "Tm (Nm)"}]
```



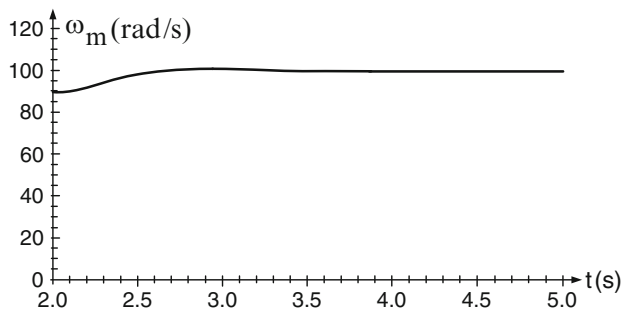
**Fig. 2.35** Mechanical shaft torque in Nm

```
Plot [Ianeu[t]/. sol2[[1]], {t, 2, 5}, PlotRange->{0, 200}, AxesLabel -> {"t (s)", "Ianeu (A)"}]
```



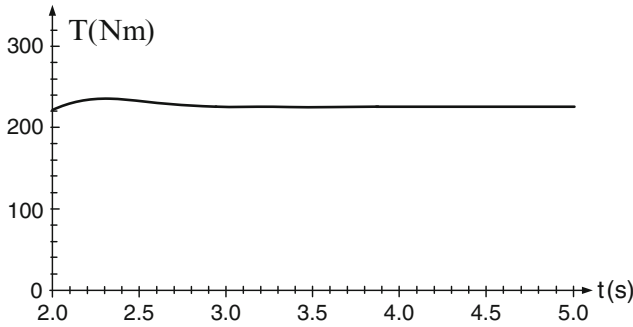
**Fig. 2.36** DC machine armature current in A without current limiter

```
Plot [Wm[t]/. sol2[[1]], {t, 2, 5}, PlotRange->{0,150} , AxesLabel -> {"t (s)", "Wm (rad/s)"}]
```



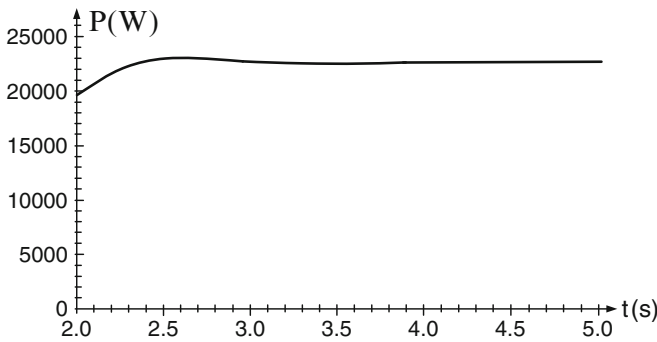
**Fig. 2.37** Mechanical angular velocity in rad/s without current limiter

Plot[Evaluate[T[t]]/.sol2[[1]], {t, 2, 5}, PlotRange->{0, 400}, AxesLabel -> {"t (s)", "T (Nm)"}]



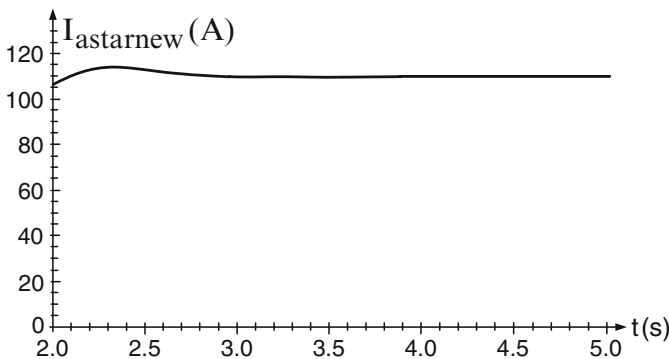
**Fig. 2.38** DC machine torque in Nm without current limiter

Plot[Evaluate[P[t]]/.sol2[[1]], {t, 2, 5}, PlotRange->{0, 30,000}, AxesLabel -> {"t (s)", "P (W)"}]



**Fig. 2.39** DC machine output power in W without current limiter

Plot [Iastarnew[t]/.sol2[[1]], {t, 2, 5}, PlotRange->{0, 150}, AxesLabel -> {"t (s)", "Iastarnew (A)"}]



**Fig. 2.40** DC machine reference current in A without current limiter

*Computation of the steady-state error  $\varepsilon_{om}$  of the angular velocity based on final-value theorem:*

observe that there is a steady-state error due to the P-speed controller (see Problem 2.5), which is why the final value of the angular velocity  $\omega_m$  is not 100 rad/s!

We can compute this steady-state error of the angular velocity from:  $\varepsilon_{om}(s) = (\omega_m^*(s) - \omega_m(s))/\omega_m^*(s)$ : Considering Fig. 2.29, we know that

$$\omega_m(s) = \frac{1}{Js} \left[ \frac{[(\omega_m^*(s) - \omega_m(s))G_{om} - i_a]G_i G_{rect} - e_a}{R_a + L_a s} k_e \Phi - T_L \right], \quad (2.47)$$

$$\omega_m(s) = \frac{k_e \Phi i_a(s) - T_L}{Js}, \quad (2.48)$$

and

$$i_a(s) = \frac{Js\omega_m(s) + T_L}{k_e \Phi} \quad (2.49)$$

by substitution in the first equation, we obtain:

$$Js \omega_m(s) = \frac{(G_{om}\omega_m^*(s) - G_{om}\omega_m(s) - \frac{Js\omega_m(s) + T_L}{k_e \Phi})G_i G_{rect} - k_e \Phi \omega_m(s)}{R_a + L_a s} k_e \Phi - T_L \quad (2.50)$$

or

$$\begin{aligned} (R_a + L_a s)Js \omega_m(s) &= \left[ G_i G_{rect} G_{om} \omega_m^*(s) - G_i G_{rect} G_{om} \omega_m(s) \right. \\ &\quad \left. - \frac{G_i G_{rect}}{k_e \Phi} (Js \omega_m(s) + T_L) - k_e \Phi \omega_m(s) \right] k_e \Phi \\ &\quad - T_L(R_a + L_a s) \end{aligned} \quad (2.51)$$

or

$$\begin{aligned} \omega_m(s) &\left( R_a Js + J L_a s^2 + G_i G_{rect} G_{om} k_e \Phi + G_i G_{rect} Js + (k_e \Phi)^2 \right) \\ &= k_e \Phi G_i G_{rect} G_{om} \omega_m^*(s) - T_L (G_i G_{rect} + R_a + L_a s) \end{aligned} \quad (2.52)$$

resulting in the angular velocity as a function of the Laplace operator  $s$

$$\omega_m(s) = \frac{G_i G_{rect} G_{om} \omega_m^*(s) k_e \Phi - T_L (G_i G_{rect} + R_a + L_a s)}{R_a Js + J L_a s^2 + G_i G_{rect} G_{om} k_e \Phi + G_i G_{rect} Js + (k_e \Phi)^2}. \quad (2.53)$$

The final (steady-state) value of  $\omega_{mss} = \{\lim_{s \rightarrow 0} s \omega_m(s)\}$  is now for a  $\omega_m(s)$  step function at the input:

$$\lim_{t \rightarrow \infty} \omega_m(t) = \omega_{mss} = \frac{\omega_m^* G_i G_{rect} G_{om} k_e \Phi - T_L (R_a + G_i G_{rect})}{G_i G_{rect} G_{om} k_e \Phi + (k_e \Phi)^2}. \quad (2.54)$$



Plotting statements (Figs. 2.43–2.50):

```

Remove[T, Wmstar, eqn1, eqn2, eqn3, eqn4, eqn5, ic1, ic2, ic3, ic4, ic5];
La=0.0036;
Ra=0.153;
kphi=2.314;
J=0.56;
Tc=5;
B=0.1;
c=0.002;
Ts=0;
Grect=20;
Gi=1.0;
Gwm=10.0;
Tm=190;
Wmstar=100;
ic1=Ia[0]=0;
ic2=Wm[0]=0;
Valpha[t_]:=Gi*(Iastar[t]-Ia[t]);
Iastar[t_]:=Clip[Gwm*(Wmstar-Wm[t]),{-200,200},{-200,200}];
Va[t_]:=Grect*Valpha[t];
T[t_]:=kphi*Ia[t];
P[t_]:=T[t]*Wm[t];
eqn1=Ia'[t]==(1/La)*(Va[t]-Ra*Ia[t]-kphi*Wm[t]);
eqn2=Wm'[t]==(1/J)*(kphi*Ia[t]-Tm-B*Wm[t]-Tc-c*(Wm[t]^2)-Ts);
sol1=NDSolve[{eqn1, eqn2, ic1, ic2}, {Ia[t], Wm[t]}, {t, 0, 2}, MaxSteps ->
10,000];
Plot[Ia[t]/.sol1[[1]],{t, 0, 2}, PlotRange -> All, AxesLabel -> {"t (s)", "Ia (A)"}]

```

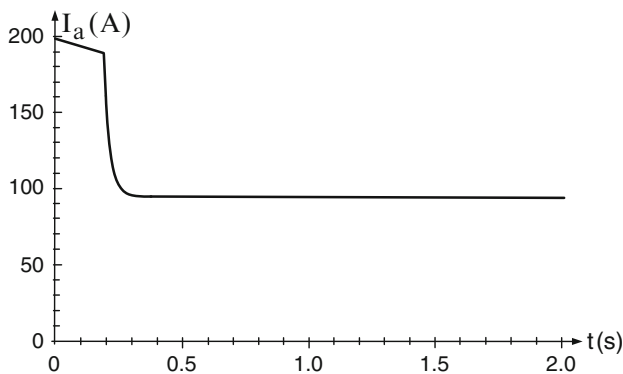
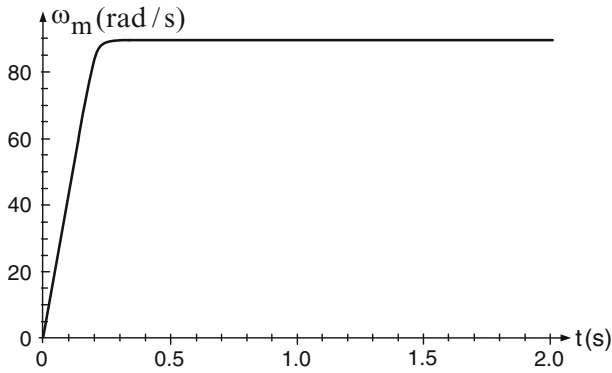


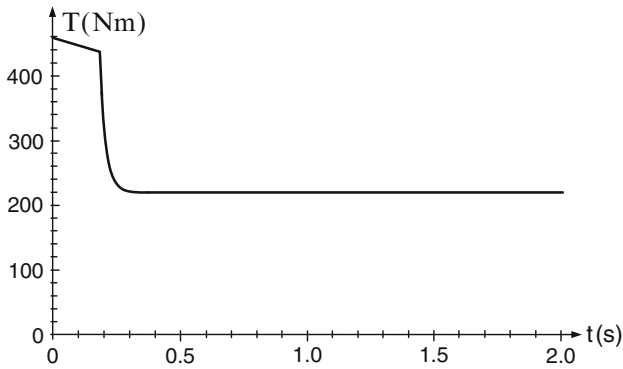
Fig. 2.43 DC machine armature current in A with current limiter

`Plot[Wm[t]/.sol1[[1]],{t, 0, 2}, PlotRange → All, AxesLabel → {"t (s)", "Wm (rad/s)"}]`



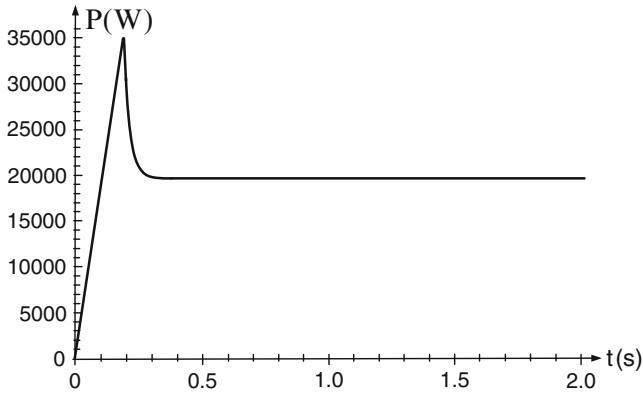
**Fig. 2.44** Mechanical angular velocity in rad/s with current limiter

`Plot[Evaluate[T[t]/.sol1[[1]]],{t, 0, 2}, PlotRange → All, AxesLabel→ {"t (s)", "T (Nm)"}]`



**Fig. 2.45** DC machine torque in Nm with current limiter

`Plot[Evaluate[T[t]/.sol1[[1]]],{t, 0, 2}, PlotRange → All, AxesLabel→ {"t (s)", "P (W)"}]`

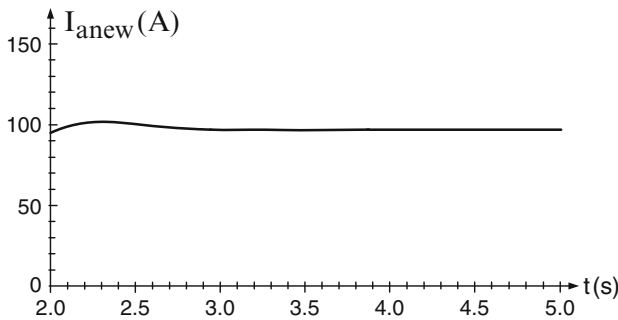


**Fig. 2.46** DC machine output power in W with current limiter

```

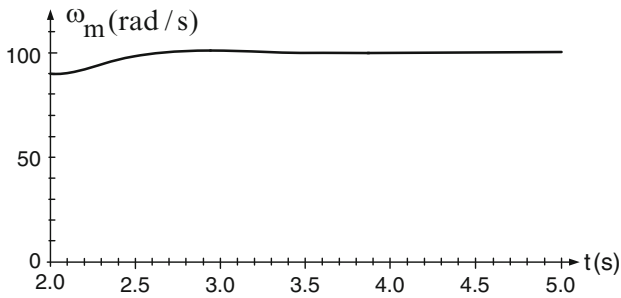
Wmb=(Wm[t]/.sol1[[1]])/.t → 2;
Iab=(Ia[t]/.sol1[[1]])/.t → 2;
Iastarb=(Iastar[t]/.sol1[[1]])/.t → 2;
ic3=Wm[2]=Wmb;
ic4=Ianew[2]=Iab;
ic5=Iastarnew[2]=Iastarb;
Va[t_]:=Grect*Valpha[t];
Valpha[t_]:=Gi*(Iastarnew[t]-Ianew[t]);
T[t_]:=kphi*Ianew[t];
P[t_]:=T[t]*Wm[t];
eqn3=Wm'[t]= (1/J)*(kphi*Ianew[t]-Tm-B*Wm[t]-Tc-c*(Wm[t]^2)-Ts);
eqn4=Ianew'[t]= (1/La)*(Va[t]-Ra*Ianew[t]-kphi*Wm[t]);
eqn5=Iastarnew'[t]= (Wmstar-Wm[t])/0.2-Wm'[t];
sol2=NDSolve[{eqn3, eqn4, eqn5, ic3, ic4, ic5}, {Wm[t], Ianew[t], Iastarnew[t]}, {t, 2, 5}, MaxSteps→ 10,000];
Plot[Ianew[t]/.sol2[[1]], {t, 2, 5}, PlotRange→ {2, 5} {0, 150}, AxesLabel → {"t (s)", "Ianew (A)"}]

```



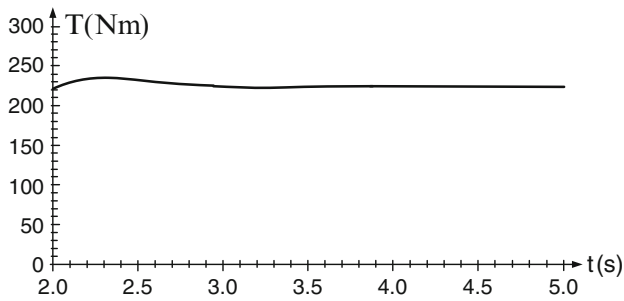
**Fig. 2.47** DC machine armature current in A with current limiter

`Plot[Wm[t]/.sol2[[1]], {t, 2, 5}, PlotRange → {2, 5} {0, 150}, AxesLabel → {"t (s)", "Wm (rad/s)"}]`



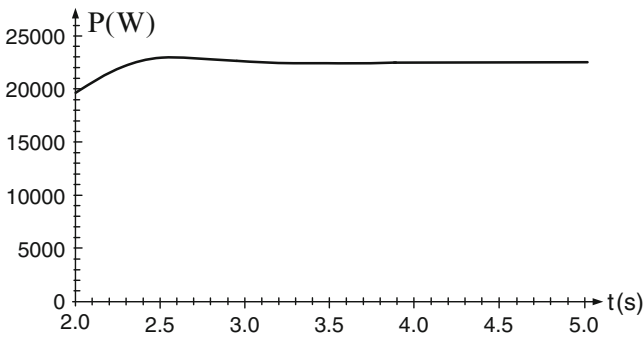
**Fig. 2.48** Mechanical angular velocity in rad/s with current limiter

`Plot[Evaluate[T[t]/.sol2[[1]]], {t, 2, 5}, PlotRange → {2, 5} {0, 300}, AxesLabel → {"t (s)", "T (Nm)"}]`



**Fig. 2.49** DC machine torque in Nm with current limiter

`Plot[Evaluate[P[t]/.sol2[[1]]], {t, 2, 5}, PlotRange → {2, 5} {0, 25,000}, AxesLabel → {"t (s)", "P (W)"}]`



**Fig. 2.50** DC machine output power in W with current limiter



- (h) The Matlab program is not listed, however, the results are identical with those obtained with Mathematica software.
- (i) Without the current limiter, the DC machine starts with a maximum current of about 900 A (Fig. 2.31), which permits a large starting torque above 2,000 Nm (Fig. 2.33). The mechanical angular velocity reaches its steady-state value of 89.4 rad/s within about 0.1 s, Fig. 2.32. When the current limiter is added, the maximum current becomes limited to 200 A at start up (Fig. 2.43) and limits the maximum starting torque to about 460 Nm, Fig. 2.45. This in turn delays the start up, and the angular velocity reaches its steady-state value of approximately 89.4 rad/s after about 0.25 s, Fig. 2.44. Thus there is a tradeoff when implementing a current limiter: the acceleration of the DC machine is reduced through the current limiter and the circuit is now protected against dangerously high levels of current

References [6–24, 29] provide a basic understanding of rotating machines and must be consulted if detailed operation and design parameters must be obtained.

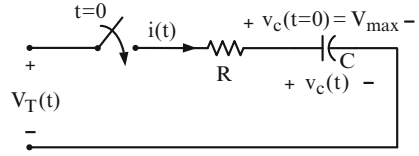
## 2.5 Summary

Block diagrams illustrate wind power plants, air conditioning systems and drives for electric/hybrid automobiles. Block diagrams are used to analyze the response of electromechanical systems based on either Mathematica or Matlab. From an electrical viewpoint, the DC machine is relatively simple, however, it is complicated from a mechanical point of view due to the commutator. The DC machine principle is explained based on the Gramme ring. The field winding of a DC machine is excited by DC while the rotor winding is excited by AC. The mechanical commutator makes the conversion from DC to AC in motoring mode and from AC to DC in generating mode. Indeed in motoring mode the Gramme ring works like a DC to AC inverter and in the generating mode the Gramme ring works like an AC to DC rectifier. The start-up, rated operation, flux weakening operation, and the speed reversal of a DC machine has been demonstrated. Nonlinear current limiters are discussed.

## 2.6 Problems

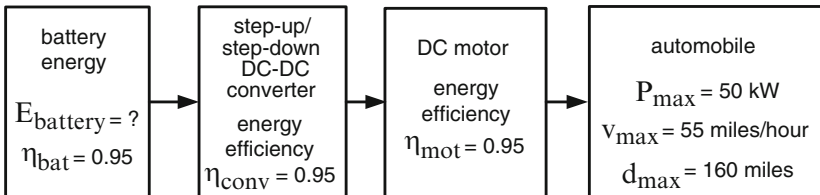
### **Problem 2.1:** *Solution of differential equation with constant coefficients*

Find the solution of the differential equation ( $v_c(t)$ ,  $i(t)$ ) as described by the circuit of Fig. 2.51 provided  $v_c(t=0) = V_{\max}$ , and the terminal voltage applied at time  $t=0$  is  $v_T(t) = V_{\max} \cos \omega t$ . Hint: Use the Laplace transformation or solve the differential equation (see Application Example 2.2) for the state-variable  $v_c(t)$ , use the initial condition, and then find  $i(t)$ .

**Fig. 2.51** R-C circuit**Problem 2.2:** *Design of an electric drive for an automobile*

An electric drive for a 1.4 (tons-force) automobile consists of battery, step-up/step-down DC-DC converter, and a DC motor, as shown in Fig. 2.52. The available maximum power at the wheels of the car is (output power)  $P_{\max\_rated} = 50$  kW, the maximum velocity of the car is  $v_{\max} = 55$  miles/h, and the operating range is  $d_{\max} = 160$  miles.

- Express  $P_{\max\_rated}$  in hp,  $v_{\max}$  in m/s, and  $d_{\max}$  in m.
- How many hours  $t_h$  does one trip last (55 miles/h, 160 miles)?
- What is the total maximum energy utilized at the wheels ( $E_{\text{wheels}}$ ) if the car has an output power of  $P_{\text{out}} = 30$  kW during the entire time it travels 160 miles at the constant velocity of  $v_{\max} = 55$  miles/h?
- If the energy efficiencies of the battery ( $\eta_{\text{bat}}$ ) DC-DC chopper ( $\eta_{\text{con}}$ ) and the DC motor ( $\eta_{\text{mot}}$ ) are 95% each, how much energy must be provided by the battery at the wheels ( $E_{\text{wheels}}$ ) during one trip ( $P_{\text{out}} = 30$  kW,  $v_{\max} = 55$  miles/h,  $d_{\max} = 160$  miles)?
- What is the figure of merit  $\left( FM = \frac{[(\text{tons} - \text{force}) \cdot \text{miles}]}{[\text{Energy used at the wheels } E_{\text{wheels}}]} \right)$  for this case [30]?
- What is  $E_{\text{battery}}$ ? How much does the nickel-metal hydride battery [30, 31] weigh, if its specific energy is 100 Wh/(kg-force) and its depth of discharge is DoD=0.5 ?
- How much does the energy supplied to the wheels ( $E_{\text{wheel}}$ ) cost if the price for 1 kWh is \$0.15?
- What is the inherent disadvantage of an electric automobile where the energy is supplied by a battery?

**Fig. 2.52** Block diagram of a drive for an electric car at rated conditions

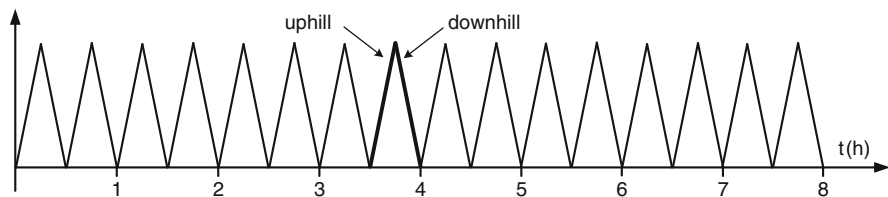
**Problem 2.3: Parallel hybrid drive vehicle**

A parallel hybrid vehicle has a  $P_{IC\_rated} = 100$  hp internal combustion (IC) engine and a  $P_{electric\_rated} = 15$  kW electric motor variable-speed drive consisting of brushless DC machine, solid-state converter (rectifier, inverter) and a 4 kWh nickel-metal hydride battery. You may assume that the motor/generator, converter and battery have energy efficiencies of  $\eta_{bat} = \eta_{conv} = \eta_{mot} = 95\%$  each.

The mileage of the hybrid vehicle (without electric drive) is 40 miles/gallon and during braking 30% of the energy can be recovered by regeneration. Note that the energy efficiency of the IC engine is about 20% due to its constant-speed operation.

- How much gasoline (1 gallon = 3.8 ℓ) must be in the tank provided one drives 400 miles with the IC engine and without use of the electric drive?
- What is the energy content ( $E_{provided}$ ) of the gasoline residing in the tank?
- What is the energy content ( $E_{used}$ ) used by the IC engine (energy efficiency of 20%)?
- Provided the car travels at an average speed of 50 miles/h how long will the trip take to travel 400 miles?
- How many (equivalent) kWh are used during 1 h of travel ( $E_{used}$  per hour)?
- Let us assume that during 1 h 30% of the energy provided by the IC engine is recycled (see Fig. 2.53) by regeneration, how much energy is stored in the battery ( $E_{battery\ stored}$ )?
- The energy stored ( $E_{battery\ stored}$ ) is then reused: how much of this energy ( $E_{battery\ reused}$ ) can be reused at the wheels?
- How much gasoline ( $Gal_{gas}$ ) must be in the tank provided one drives 400 miles with the IC engine and with use of the electric drive?
- How many percent does the electric drive increase the energy efficiency ( $\Delta\eta$ ) of this hybrid vehicle?
- What is the weight of the gasoline (tank is full), and that of the nickel-metal hydride battery? For the battery you may assume a specific energy density of 100 Wh/(kg-force) and a depth of discharge DoD  $\approx 0.4$ .

*Note: During the uphill climb the electric drive contributes power to vehicle, and during downhill travel the 4 kWh battery is being recharged due to braking/regeneration.*



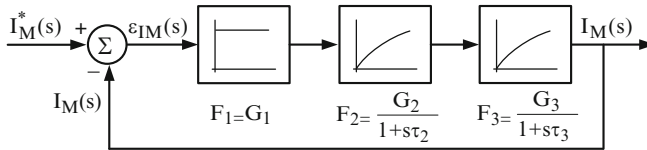
**Fig. 2.53** Uphill-downhill diagram as a function of distance

**Problem 2.4:** Find transfer function

Identify all transfer functions of the block diagram of Fig. 2.17 and find  $\{\theta_{\text{actual}}(s)/\theta^*(s)\}$ . Hint: see Application Example 2.4.

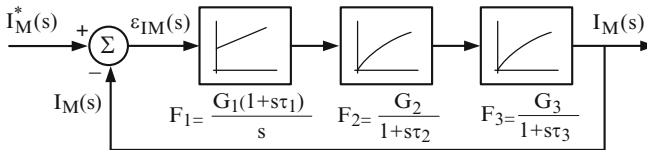
**Problem 2.5:** Steady-state errors using the final-value theorem

- (a) Compute for the block diagram of Fig. 2.54 the steady-state error  $\varepsilon_{\text{IM}}(s)$  if  $I_M^*(s)$  is a unit-step function. You may assume  $G_1=5$ ,  $G_2=2$ ,  $\tau_2=0.2$  s,  $G_3=10$ , and  $\tau_3=0.1$  s.



**Fig. 2.54** Current-control loop of an electric drive with P-controller

- (b) Show that for the block diagram of Fig. 2.55 the steady-state error  $\varepsilon_{\text{IM}}(s)$  is zero if  $I_M^*(s)$  is a unit-step function.



**Fig. 2.55** Current-control loop of an electric drive with PI-controller

**Problem 2.6:** Rated operation, rated efficiency, flux weakening, and reversal of DC machine

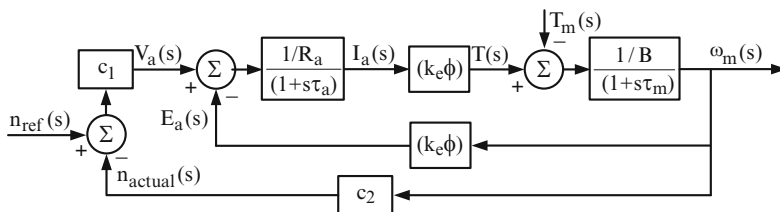
- (a) Repeat the Mathematica/Matlab analysis of Application Example 2.8 for a  $P_{\text{out\_rated}}=100$  hp,  $V_a=230$  V DC machine. The machine parameters are [16]  $L_a=1.1$  mH,  $R_a=0.0144$   $\Omega$ ,  $I_{a\_\text{rated}}=349$  A,  $n_{m\_\text{rated}}=1,750$  rpm,  $(k_e\Phi)=1.2277$  Vs/rad,  $J=1.82$  kgm<sup>2</sup>,  $T_c=21$  Nm,  $B=0.5$  Nm/(rad/s),  $c=0.02$  Nm/(rad/s<sup>2</sup>),  $T_s=10$  Nm,  $G_{\text{rect}}=10$ ,  $G_i=1.0$ , and  $G_{\text{om}}=5.0$ . The rated losses of the field winding are  $P_f=325$  W. The variation of the input variables for start-up, rated operation, flux weakening and reverse operation are given in Table 2.4
- (b) What is the rated efficiency of this machine in motoring mode?

**Table 2.4** Variable input parameters

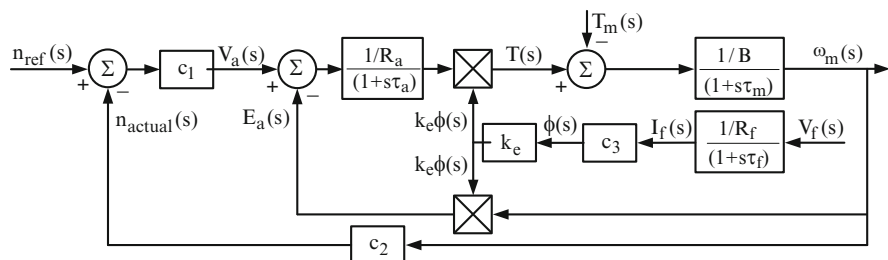
Time $t$ [s]	0	2	4	6	8	10	12
Terminal voltage of DC motor $V_a$ [V]	115	230	230	230	230	−230	
Field flux of DC motor $(k_e\phi)$ [Vs/rad]	1.2277	1.2277	1.2277	1.2277	0.614	0.614	
Mechanical torque doing the work $T_m$ [Nm]	0	0	214	428	214	−214	
Coulomb frictional torque $T_c$ [Nm]	21	21	21	21	21	−21	
Standstill frictional torque $T_s$ [Nm]	10	10	10	10	10	−10	
Coefficient of windage torque $c$ [Nm/(rad/s) <sup>2</sup> ]	0.02	0.02	0.02	0.02	0.02	−0.02	

**Problem 2.7:** *Speed control via flux weakening of a DC Motor*

A  $P_{\text{rated}} = 40$  kW,  $V_{a,\text{rated}} = 240$  V,  $n_{\text{rated}} = 1,100$  rpm separately excited DC motor is to be used in a speed control system which may be represented by the block diagram of Fig. 2.56. For  $k_e\Phi = 2.0$  Vs/rad,  $R_a = 0.1$   $\Omega$ ,  $B = 0.3$  Nms/rad,  $c_1 = 0.2$  V/rpm, and  $c_2 = 0.10$  rpm/rad/s:

**Fig. 2.56** Block diagram of separately excited DC motor with speed/angular velocity control via armature voltage

- Determine the steady-state angular velocity at no load  $\omega_{mo}$  if  $V_a = 240$  V. What is the no-load speed  $n_{mo}$ ?
- What is the reference speed  $n_{ref}$  for the condition of (a) ?
- What is  $(k_e\Phi)_{\text{new}}$  if the steady-state no-load angular velocity  $(\omega_{mo})_{\text{new}}$  must be two times the rated angular velocity (see Fig. 2.57)?
- For  $R_f = 1$   $\Omega$ ,  $V_f = 240$  V, and  $k_e\Phi = 2.0$  Vs/rad determine  $c_3$ . What is the value of  $V_f$  for part c)? You may assume linear conditions (Fig. 2.57).

**Fig. 2.57** Block diagram of separately excited DC motor with speed/angular velocity control via flux weakening

**Problem 2.8:** *Steady-state performance of shunt-connected DC motor*

A  $P = 8.8 \text{ kW}$ ,  $V_a = 220 \text{ V}$ ,  $n_m = 1,200 \text{ rpm}$  shunt-connected DC motor has armature resistance of  $R_a = 0.1 \Omega$  and field resistance of  $R_f = 100 \Omega$ . The rotational (friction, windage) loss is  $P_{fr\_wi} = 2,200 \text{ W}$ . Neglect the iron-core loss. Compute the rated motor output torque  $T$ , the armature current  $I_a$ , and the efficiency  $\eta$ .

**Problem 2.9:** *Steady-state performance of series-connected DC motor*

When operated from a  $V_a = 230 \text{ V}$  DC supply, a series-connected DC motor operates at  $n_m = 975 \text{ rpm}$  with an armature current of  $I_a = 90 \text{ A}$ . The armature and the series field resistances are  $R_a = 0.12 \Omega$ , and  $R_{sf} = 0.09 \Omega$ , respectively.

- Compute the induced voltage  $E_a$
- Compute the new value of the induced voltage  $E_{a\_new}$  when the armature current is reduced to  $I_{a\_new} = 30 \text{ A}$ .
- Find the new motor speed  $n_{m\_new}$  when the armature voltage is  $V_a = 230 \text{ V}$  and armature current is  $I_{a\_new} = 30 \text{ A}$ . Assume that due to saturation, the flux produced by an armature current of  $30 \text{ A}$  is 48% of that at an armature current of  $90 \text{ A}$ .

**Problem 2.10:** *Steady-state torques of shunt-connected DC motor*

A  $V_a = 240 \text{ V}$  shunt-connected DC motor operates at full load at  $n_m = 2,400 \text{ rpm}$ , and requires  $I_t = 24 \text{ A}$ . The armature and field circuit resistances are  $R_a = 0.4 \Omega$  and  $R_f = 160 \Omega$ , respectively. Rotational (friction, windage) losses at full load are  $P_{rotational} = 479 \text{ W}$ . Neglect the iron-core losses. Determine:

- Field current  $I_f$ , armature current  $I_a$  and the induced voltage  $E_a$ .
- Copper losses  $P_{cu}$ , developed internal motor power  $P_d = E_a \cdot I_a$ , output power  $P$ , and efficiency  $\eta$ .
- Developed internal motor torque  $T_d = P_d / \omega_m$ , and output load torque  $T$ .

**Problem 2.11:** *Speed control of shunt-connected DC motor*

A  $V_a = 120 \text{ V}$ ,  $n_m = 2,400 \text{ rpm}$  shunt-connected DC motor requires  $I_{t\_no-load} = 2 \text{ A}$  at no load, and  $I_{t\_load} = 14.75 \text{ A}$  at full load. The armature circuit resistance is  $R_a = 0.4 \Omega$  and the field winding resistance is  $R_f = 160 \Omega$ .

- Determine the no-load speed  $n_{no-load}$ , the rotational and core loss  $P_{r+fe}$ , and the full-load efficiency  $\eta_{full-load}$ .
- If an external resistance of  $R_{external} = 3.6 \Omega$  is connected in series with the armature circuit, calculate the motor speed  $n_{m\_new}$  and the efficiency of the motor  $\eta_{new}$  for  $I_a = 14 \text{ A}$ .  
Hint:  $(E_{a\_no-load} / E_{a\_full-load}) = (n_{m\_no-load} / n_{m\_full-load})$ .

**Problem 2.12:** *Influence of armature reaction on performance of shunt-connected DC motor*

A  $V_a = 220 \text{ V}$  shunt-connected DC motor has an armature resistance of  $R_a = 0.2 \Omega$  and field-circuit resistance of  $R_f = 110 \Omega$ . At no-load, the motor runs at  $n_{m\_no-load} = 1,000 \text{ rpm}$  and it draws a line current of  $I_{t\_no-load} = 7 \text{ A}$ . At full load, the input to

the motor is  $P_{in\_full-load} = 11 \text{ kW}$ . Consider that the air-gap flux remains constant at its no-load value; that is, neglect armature reaction.

- (a) Find the speed, speed regulation, and developed internal torque at full load  $T_{d\_full-load}$ .
- (b) Find the starting torque if the starting armature current is limited to 150% of the full load current.
- (c) Consider that the armature reaction reduces the air-gap flux by 5 % when full-load current flows in the armature. Repeat part (a).

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