

Chapter 2

A Preview of Particle Physics: The Experimentalist's Perspective

The upcoming chapters in this book, as well as the books to come later in this series, are very, very mathematical. The current structure of elementary particle theory is in many ways barely distinguishable from pure math. As one works through the intricate mathematical formalism, it can be easy to lose sight of how it all relates to what we observe in nature. Therefore, before going any farther, we'll take a few pages to summarize the current state of affairs, and how we got here, from an experimentalist's point of view.

The culmination of theoretical particle physics is called the Standard Model of Particle Physics, or usually just the **Standard Model**. For three and a half decades, the standard model has provided an excellent description of the inner workings of nature. Over the years, carefully designed experiments have probed numerous properties of the physical universe, and every experimental measurement has affirmed the standard model.

Starting in Chap. 3, we'll build up a mathematical foundation for the standard model in a way that will seem obvious – as if it simply has to be that way. Of course, we'll be providing this foundation based on our modern-day knowledge, with the benefit of many decades of hard work and tireless dedication. The twentieth century was marked by a continual exchange of ideas and measurements. New ideas stimulated the design and execution of clever new experiments, and experimental measurements either supported an idea or relegated it to the sidelines. No one dreamed up the $SU(3) \times SU(2) \times U(1)$ gauge theory¹ back in the 1950s when a slew of new particles started popping up in laboratories. It was the careful categorization of those particles, the baryons and mesons, that led to the quark model of nature. In this way, theory and experiment have worked hand in hand, each extending the reach of the other, to bring us where we are today.

Elementary particle physics in the twenty-first century continues to be a healthy and animated exchange between theorists and experimentalists. Bright young graduate students pursuing research in particle physics are usually channeled towards

¹We'll discuss what this means in plenty of detail later.

either theory or experiment. It is unfortunate, perhaps, that the skill set needed to succeed as an experimentalist has become quite different than the skill set needed to succeed as a theorist, and vice versa. While a theorist might study non-Cartan generators, local symmetry breaking, and Kähler geometry, an experimentalist gains expertise in things like data acquisition systems, field programmable gate arrays, ROOT, and the nuances of polymorphism and inheritance. The collective expertise needed to design, commission, test, and successfully run a modern-day particle physics experiment is immense. The day-to-day activities of an experimentalist rarely involve calculating matrix elements of Feynman diagrams.

So, what has happened is that theorists see the standard model as mathematical formalism, where “particles” aren’t so much real matter – instead they’re fields like Z_μ . At the other extreme, experimentalists think of Z^0 bosons as invisible particles that are “seen” only when they decay to something like a e^+e^- or $\mu^+\mu^-$ pair, mere remnants that we can identify in particle detectors. We authors have remarked to each other that theorists and experimentalists sometimes speak about the standard model in such different ways that it seems like we are not even talking about the same thing at all, and we are reminded of the well-known story of the blind men describing the elephant.

In the next few chapters, you’ll learn about a series of mathematical tricks for various types of “fields.” We’ll talk about “massless scalars with a $U(1)$ charge,” and about things “in a $j = \frac{1}{2}$ representation of $SU(2)$.” While the primary purpose of this text is indeed to provide the mathematical tools with which particle physics is performed, we are physicists, not mathematicians. It is therefore apt that we reunite theory and experiment and proceed with a “nature-based” preview of elementary particle physics.

2.1 The Ultimate “Atoms”

Since the time of the ancient Greeks, physicists have been progressing toward a simple, elegant, all-encompassing model that attempts to explain the workings of the universe. Humankind’s curiosity about the nature of nature can be traced back to the fifth century BC, when a Greek named Empedocles combined the ideas of several others before him to say that all structures of the world were made up of earth, air, fire, and water, and that there are two divine powers, Love and Strife, which govern the way these four elements combine and behave. More scientifically, he was saying that matter is made up of smaller substances that interact with each other through attraction and repulsion. Democritus, a contemporary of Empedocles, dared to propose that all matter is composed of invisible, indestructible particles called *atoms* from the Greek $\alpha\tau\omicron\mu\omega\sigma$, meaning “uncuttable” or “indivisible.”

Over the centuries, the yearning to identify the elementary constituents of matter has brought us from earth, air, fire, and water to a microworld over a million-billion times smaller than the book in your hands. The basic questions posed by Democritus over 2,400 years ago continue to drive the field of elementary particle physics today.

Are there fundamental, indivisible particles and if so, what are they? How do they behave? How do they group together to form the matter that we see? How do they interact with each other? Today, using the most sophisticated particle probes on earth, we think we might have finally discovered the ultimate $\alpha\tau\omega\mu\omega\sigma$. We call them *quarks* and *leptons*.

2.2 Quarks and Leptons

The twentieth century was a marvelous one for particle physics. It all began in 1897 when J.J. Thompson discovered the first truly elementary particle: the *electron*.² With this observation came the realization that the atoms of the nineteenth century – like hydrogen, oxygen, and lead – were not in fact the most basic building blocks of matter. In 1911, Ernest Rutherford and his associates bombarded thin gold foils with α -particles and found that some of them were deflected by huge angles, indicating the presence of a small yet massive kernel inside the atom: the atomic nucleus. The ensuing years revealed that the nucleus consisted of even smaller components, the *proton* and *neutron*, collectively referred to as *nucleons*. Physicists realized that every element in the periodic table could be constructed of a single atomic nucleus with a distinct number of protons and neutrons, surrounded by a cloud of electrons. And with that, modern elementary particle physics was born.

The notion that protons and neutrons were elementary particles was shattered in the late 1950s and 1960s by a population explosion of newly observed particles. With the construction of large particle accelerators, experiments produced hundreds of “elementary” particles, called *hadrons*, with properties very similar to the nucleons. Underlying symmetries in the masses, charges, and intrinsic angular momenta (spins) of the hadrons pointed to an even deeper order within the chaos. In 1963, Murray Gell-Mann and George Zweig independently proposed a scheme in which hadrons are composed of yet smaller particles, called *quarks*.³ Some hadrons, like the proton and neutron, consist of three quarks. Experimental evidence for the proton’s substructure was eventually established in 1968 by a team at the Stanford Linear Accelerator Center (SLAC). In an experiment not so different than Rutherford’s, a high-energy beam of electrons was aimed at a small vat of liquid hydrogen. The resulting scattering pattern revealed that the proton is not elementary at all.

²It is truly elementary, as far as we currently know.

³Although quark may sound inherently like a scientific term, its origin is surprisingly from literature. For the name of this type of particle, Murray Gell-Mann came up not with the word first, but with the sound (which he described as “kwork”, the sound a duck makes). Soon thereafter, Gell-Mann came across the phrase “Three quarks for Muster Mark” in *Finnegans Wake* by James Joyce. Gell-Mann immediately latched on to quark as the spelling – which seemed very appropo since he was theorizing that hadrons were composed of three different types of elementary particles. Zweig sought (unsuccessfully) to attach the name *aces* to the particles, in connection with his expectation of the discovery of a fourth such particle.

The original quark model of Gell-Mann and Zweig required only three *flavors* of quarks – the *up* (u), *down* (d), and *strange* (s) – to explain the proliferation of new hadrons. Nucleons are comprised of combinations of up and down quarks. Strange quarks explained the existence of odd, short-lived particles in cosmic rays. Each flavor of quark also has an associated *antiquark*, a corresponding particle with an identical mass but opposite electric charge.

Since the early 1970s, three more quarks have been discovered, bringing the total to six. For reasons we'll see shortly, they are often grouped in pairs, or *doublers*, as shown here:

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$

The *charm* (c) quark was discovered in 1974 in the form of the J/ψ (J/ψ) meson, a bound charm-anticharm pair, by two independent teams led by Samuel Ting at Brookhaven National Laboratory and Burton Richter at SLAC. In 1977, Leon Lederman and colleagues at the Fermi National Accelerator Laboratory (Fermilab) found the analogue of the J/ψ for *bottom* (b) quarks, which was named the *upsilon* (Υ). By this point, one more quark was obviously needed to pair with the b quark and fill the gaping hole in the third doublet. The last of the quarks, the *top* (t), was discovered in 1995 in high-energy proton-antiproton collisions by the Collider Detector at Fermilab (CDF) and DZero (DØ) collaborations. Far more massive than anyone expected – more than 186 times the protons mass! – the top quark's fleeting existence prevents it from joining with other quarks to create hadrons.

The arrangement of the six quark flavors in three *generations* of doublets, as shown above, reflects their intrinsic properties. The up and down quarks are the lightest of all and therefore the most stable. As a result, they make up ordinary matter. The proton, with a total electric charge of $+1$, contains two up quarks, each with charge $+2/3$, and a down quark with a charge $-1/3$. The *udd* configuration of the neutron gives it a net charge of zero. The second and third generations are just heavier duplicates of the first, with quarks that are produced only in high-energy interactions.

The theoretical and experimental advances that led to the quark model also predicted the existence of *leptons*, a second set of six elementary particles, together with their corresponding antiparticles. Like the quarks, the leptons can be arranged in three generations of doublets:

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$$

Of the three charged leptons, the lightest is the familiar electron. The *muon* (μ), a heavy replica of the electron, was first observed in 1938 in cosmic rays by Carl David Anderson. The heaviest known lepton, the *tau* (τ), was discovered decades later in 1975 by Martin Perl with colleagues at SLAC. Unlike the electron, the muon and tau are unstable and exist for only fractions of a second before decaying to less massive particles.

Each of the three charged leptons is complemented by a neutral partner, the *neutrino* (ν). Wolfgang Pauli originally proposed the idea of a neutrino in 1930 as the mysterious, unobserved particle that carried energy from nuclear β -decay. Neutrinos weren't actually "seen" until twenty-six years later, when Clyde Cowan and Fred Reines observed the interactions of electron antineutrinos with protons in a huge instrumented tank of water. Then, in 1961, a group led by Melvin Schwartz, Leon Lederman, and Jack Steinberger developed a neutrino beam at Brookhaven National Laboratory which resulted in the discovery of the second species of neutrino: the muon neutrino. The tau neutrino was ultimately discovered at Fermilab in 2000. Neutrinos, particles with a tiny mass, interact with matter only via the weak interaction. They interact so weakly, in fact, that a single neutrino can pass unscathed through millions of miles of solid steel!

The table below summarizes several details relating to the elementary quarks and leptons⁴:

Particle name	Symbol	Charge ($ e $)	Mass (MeV/ c^2)	Spin
<i>Quarks</i>				
Up	u	$+2/3$	1.7–3.3	1/2
Down	d	$-1/3$	4.1–5.8	1/2
Charm	c	$+2/3$	1180–1340	1/2
Strange	s	$-1/3$	80–130	1/2
Top	t	$+2/3$	≈ 172000	1/2
Bottom	b	$-1/3$	4130–4370	1/2
<i>Leptons</i>				
Electron	e	-1	0.51100	1/2
Electron neutrino	ν_e	0	≈ 0	1/2
Muon	μ	-1	105.66	1/2
Muon neutrino	ν_μ	0	< 0.19	1/2
Tau	τ	-1	1776.8	1/2
Tau neutrino	ν_τ	0	< 18.2	1/2

2.3 The Fundamental Interactions

At the most intuitive level, a *force* is any kind of push or pull on an object. You experience forces every day. To push open a door, for example, your hand exerts a contact force on the door. The force of friction ultimately stops a book that slides across a table. Every "Physics I" student has drawn a free-body diagram with the gravitational pull pointing down and the so-called normal force pointing up.

⁴For complete, up-to-date information, see <http://pdg.lbl.gov>.

If all matter can be described in terms of a few fundamental building blocks, can we also categorize the everyday forces in terms of a few fundamental forces? We believe the answer is yes. Physicists have identified four known *interactions* that appear to underlie all of the phenomena we observe in nature. They are *gravitation*, *electromagnetism*, the *weak interaction*, and the *strong interaction*. The term interactions has become a common way to describe them, for they are the ways that the simplest particles in the universe interact with each other. They are fundamental in that all other forces, even the everyday forces, can be described in terms of them.

2.3.1 Gravitation

Everyone is intimately familiar with gravity. One of Sir Isaac Newton's many discoveries was that the mysterious force that pulls common objects down toward the earth's center is the same force that holds the moon in place in its orbit. As the timeless story goes, Newton observed an apple fall from a tree, and with one brilliant revelation, he unified gravity on the earth and gravity in the heavens.

Newton's Law of Universal Gravitation states that every particle in the universe attracts every other particle with a force that is proportional to the product of the masses of the particles and inversely proportional to the square of the distance between them. The magnitude of the force can be written as

$$F = G \frac{m_1 m_2}{r^2}, \quad (2.1)$$

where G , the *gravitational constant*, has an accepted value equal to $6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$. Despite the way it feels, gravity is by far the weakest of the four interactions. To calculate the force of gravity that you experience on Earth's surface, we must first multiply your mass by the mass of the *entire* earth in the equation above. It is the huge mass of the earth that compensates for the tiny value of G .

Although gravitation is the oldest of the known interactions, it is in many respects the least well understood. The gravitational constant G is among the least well-measured physical constants, and scientists are not completely certain that the above equation remains accurate for distances less than a fraction of a millimeter.⁵ The general theory of relativity, published by Albert Einstein in 1915, beautifully predicts the behavior of objects in the celestial realm of stars and planets. Unfortunately, it completely breaks down in the tiny realm of atoms and nuclei. Einstein spent the last decades of his life in a desperate attempt to unify gravity and electromagnetism but came up empty-handed.

⁵The reader may be surprised to learn that the $1/r^2$ form of Newtonian gravity was not actually verified for distances below 1 cm until very recently, as discussed in Sect. 5.4.1.

Despite its broad success in describing physical phenomena in the microscopic realm, the standard model is an incomplete theory because it fails to describe gravitation. Physicists continue to work towards a theory that describes all four fundamental interactions, with *string theory* currently showing the most promise.

2.3.2 Electromagnetism

Most of the forces described at the beginning of this section – like contact forces, friction, and the normal force – are actually manifestations of the interaction called electromagnetism. The force we experience when we push or pull ordinary material objects, such as doorknobs, comes from the intermolecular forces between the individual molecules in our bodies and those in the objects. These result from the forces involved in interactions between atoms, which in turn can be traced to electromagnetism acting on the electrically charged protons and electrons inside the atoms. When you rub your hands together, the charged particles near the surface of your hands experience the electromagnetic interaction, giving rise to friction. Electromagnetism, acting between the soles of our shoes and the floor, is responsible for the upward normal force which keeps us all from falling through solid ground toward the center of the earth.

The term “electromagnetism” hints at the interesting history surrounding this interaction. Originally, electricity and magnetism were considered two separate forces. Then, while preparing for a lecture in 1820, Hans Christian Ørsted made a surprising discovery. As he was setting up his materials, he observed the deflection of a compass needle from magnetic north when the electric current from a nearby battery was switched on and off. This deflection convinced him that magnetic fields radiate from all sides of a wire carrying an electric current, and it confirmed a direct relationship between electricity and magnetism.

In 1873, Scottish theoretical physicist and mathematician James Clerk Maxwell published a set of equations that relate both electric and magnetic forces to their sources: charges and currents. *Maxwell's equations* not only brilliantly intertwined the two forces into one unified force, but they also explained the origin of electromagnetic radiation, which includes x-rays, radio waves, visible light, and more. His famous set of equations, reviewed in Sect. 1.6, demonstrated that electricity, magnetism, and light are all manifestations of the same physical phenomenon: the electromagnetic field. His electromagnetic theory successfully synthesized previously unrelated experimental observations and equations of electricity and magnetism into a consistent theory. Maxwell's groundbreaking work has been dubbed the “second great unification in physics” after the first one achieved by Sir Isaac Newton.

2.3.3 *The Strong Interaction*

In the 1920s, only two of the four fundamental interactions, gravitation and electromagnetism, were known. Gravity controls the motion of the heavenly bodies and keeps our feet on the ground. Electromagnetism dominates all atomic interactions and is ultimately responsible for all that we see and feel. Believing these to be the only two forces, Theodor Kaluza and Oskar Klein developed a theory of unified gravity and electromagnetism.⁶ Although it won the support of Albert Einstein, Kaluza-Klein theory faded from importance during the next decade, as the true nature of the nucleus became more and more clear. Studies of the atom during the 1930s led to the realization that gravity and electromagnetism were not the only two forces.

Imagine an atomic nucleus. What holds the constituent protons and neutrons together? Gravity is far too weak, and the electromagnetic interaction would push protons apart because of their like charge – not hold them together. In the 1930s, physicists had to admit that another force was needed to overcome the electromagnetic repulsion and allow nuclei to remain stable.

Enter the strong interaction. By far the strongest of the four fundamental interactions, the strong interaction not only binds protons and neutrons into atomic nuclei, it also unites quarks into composite particles – the hadrons. The range of the strong interaction is extremely small; it acts over a mere 10^{-15} m, a millionth of a billionth of a meter, the size of a proton. As we'll see shortly, the nature of the strong interaction is completely unlike the other interactions.

2.3.4 *The Weak Interaction*

The weak interaction is rather unfamiliar in our day-to-day existence. It is responsible for certain types of radioactive decay; for example, it permits a proton to turn into a neutron and vice versa. Aptly named, the strength of the weak interaction is 100 billion times less than the strength of the electromagnetic interaction! Like the strong force, it only acts over very short distances – about 10^{-18} m, a billionth of a billionth of a meter.

The weak interaction is the only one that can cause a quark to change its flavor. For instance, it can transform an up quark into a down quark. That is, in fact, *precisely* what happens in the case of β -decay. If one of the two up quarks in a proton changes to a down quark, we're left with the *udd* structure of the neutron. In the process, electric charge is conserved by the emission of a positively charged *positron* (the antimatter partner of the electron), and lepton number is conserved by the emission of a neutrino.

⁶Kaluza-Klein theory, of which an extended version is a natural result of string theory, is reviewed in Sect. 5.4.1 of Chap. 5 and will be discussed in depth in a later text in this series.

2.3.5 Summary

A table summarizing all four fundamental interactions is shown here. The approximate relative strengths have been normalized to unity for the strong interaction. We say “approximate” because we will learn later⁷ that the strength of a force depends on the length scale being considered.

Interactions	Acts On	Strength	Range
Strong	Hadrons	1	10^{-15} m
Electromagnetism	Electric Charges	10^{-2}	$\infty (1/r^2)$
Weak	Leptons and Hadrons	10^{-5}	10^{-18} m
Gravitation	Mass	10^{-39}	$\infty (1/r^2)$

2.4 Categorizing Particles

In the middle of the twentieth century, before quarks were discovered, elementary particle physicists were shocked by the sudden population explosion of new particles discovered in the laboratory. Things seemed far too disorganized. How could all of these particles be elementary? Over time, the properties of these particles were measured and eventually it became apparent that they were not elementary at all, but *composite*: comprised of two or more other particles. Just as early biologists sorted living organisms by their appearance and defining features, physicists classified particles based on their measured properties such as mass, electric charge, and intrinsic angular momentum. The identification of common characteristics within the “zoo” of new particles ultimately led to the quark model of nature. Moreover, the names of the categories of particles have become a part of the daily vocabulary of experimental particle physics.

2.4.1 Fermions and Bosons

Every type of particle, elementary or composite, has an intrinsic angular momentum, or quantum mechanical spin. A particle with a half-integer spin ($1/2, 3/2, 5/2, \dots$), in units of Planck’s constant \hbar , is a *fermion*. A particle with an integer spin ($0, 1, 2, \dots$) is a *boson*. The spin, in addition to being the particle’s intrinsic angular momentum, governs the statistics of a set of such particles, so fermions and bosons may also be

⁷Much later – in a later book, in fact.

defined according to the statistics they obey. Fermions obey Fermi-Dirac statistics, and also the Pauli exclusion principle, which says that no two identical fermions can be found in the same quantum state at the same time. Bosons, on the other hand, obey Bose-Einstein statistics, which means that any number of the same type of particle can be in the same state at the same time.

The elementary particles that make up matter, the quarks and leptons, all have spin $1/2$ and are thus fermions. As we will see, there are also elementary particles that govern the fundamental interactions of the standard model – the *photon*, *W* and *Z* bosons, and the *gluons* – which have spin 1, and are thus bosons. The Higgs boson, as yet undiscovered, is predicted to have spin 0.

2.4.2 *Baryons and Mesons*

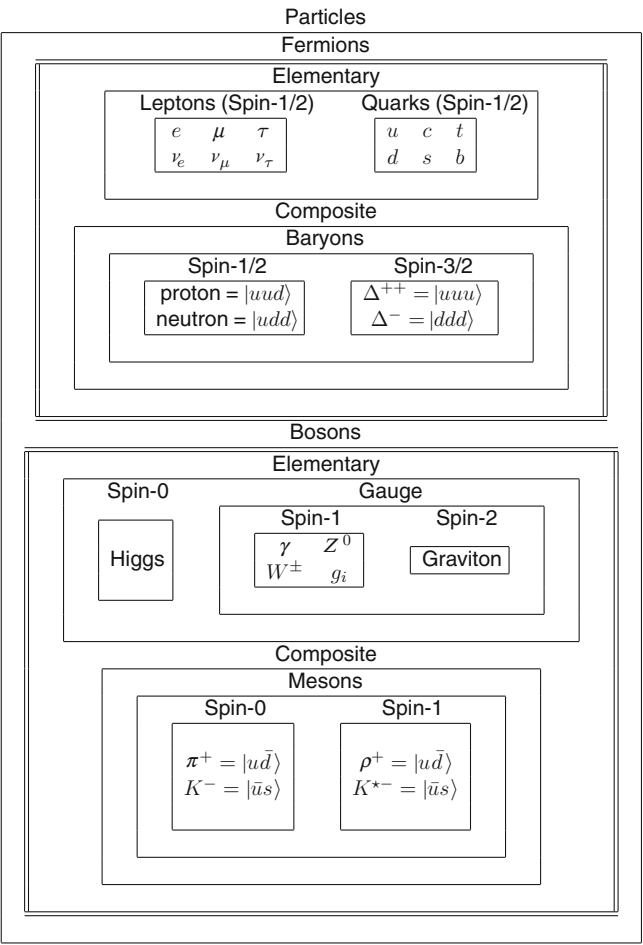
One way to distinguish the various elementary fermions is by whether or not they interact via the strong interaction: quarks interact via the strong interaction, while leptons do not. Hadrons are composite particles constructed of quarks bound together by the strong interaction. They can be either fermions or bosons, depending on the number of quarks that comprise them. Three bound quarks (or three bound antiquarks) form spin- $1/2$ or spin- $3/2$ hadrons, which are called *baryons*. The baryons are made of “normal” matter quarks and their antimatter counterparts are made of the corresponding antiquarks. The most well known examples of baryons are protons and neutrons. *Mesons* are spin-0 or spin-1 hadrons consisting of a quark and antiquark, though not necessarily of the same flavor. Examples include the π^+ ($u\bar{d}$) (a positively-charged *pion*) and the K^- ($\bar{u}s$) (a negatively-charged *kaon*). Because of their values of spin, all baryons are fermions and all mesons are bosons.

One of the reasons for the plethora of particles discovered in the past century is the numerous possible combinations of different quark flavors one can put into a three-quark baryon or two-quark meson. Additionally, each of these combinations can be in one of multiple quantum mechanical states. For example, a ρ^+ meson has the same combination of quarks as a π^+ , but the ρ^+ is a spin-1 particle whereas the π^+ is a spin-0 particle.

2.4.3 *Visualizing the Particle Hierarchy*

Newcomers to elementary particle physics quickly notice that particle names typically end in “-on,” like “proton” and “pion.” However, as we have just seen, even categories of particles have names that end in “-on,” like “fermion” and “lepton.” A muon is a lepton and a fermion. A pion is a meson, hadron, and boson. This tangled taxonomy is enough to bewilder the brightest minds. Names have even been assigned to particles that have yet to be observed, such as the *preon*: the hypothetical subcomponents of quarks and leptons.

To help make sense of it all, it is useful to visualize the various categories of particles in a Venn diagram, as shown here. A few of the specific particles mentioned throughout this chapter are included as examples.



2.5 Relativistic Quantum Field Theories of the Standard Model

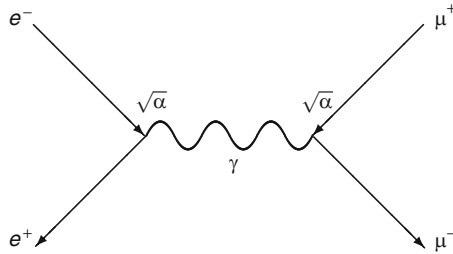
Most theories in modern particle physics, including the Standard Model of Particle Physics, are formulated as *relativistic quantum field theories*. Quantum field theory is widely considered to be the only correct approach for combining quantum mechanics and special relativity. In perturbative quantum field theory, the forces

between particles are mediated by other particles, the *gauge bosons*. In the following sections, we'll see how the gauge bosons are intricately connected to the three fundamental interactions of the standard model: electromagnetism, the weak interaction, and the strong interaction.⁸

2.5.1 Quantum Electrodynamics (QED)

Quantum electrodynamics (QED) is a precise, quantitative description of electromagnetic interactions. Arguably one of the most successful theoretical achievements of the twentieth century, QED is the quantum field theory that connects the modern formalism of quantum mechanics with the classical principles of electromagnetism. One of its many noteworthy achievements is the precise calculation of the electron's magnetic moment, which agrees with experimental measurements to at least 10 decimal places. For their contributions to the development of QED, Sinitiro Tomonaga, Julian Schwinger, and Richard Feynman shared the Nobel Prize in Physics in 1965.

In QED, the force between two charged particles is characterized by the exchange of a field quantum, the photon. By virtue of the gauge invariance of QED, electric charge is conserved in all electromagnetic interactions. Below is a graphical representation of this interaction:



This diagram of $e^+e^- \rightarrow \mu^+\mu^-$ scattering is an example of a Feynman diagram. Feynman diagrams play a crucial role in calculating measurable quantities such as *cross-sections*, which tell us about the probability for particular interactions to occur, and *decay rates*, which tell us how quickly particular particles decay. Every line and vertex of the diagram is associated with a mathematical term in the QED calculation. For example, each vertex contributes a factor proportional to $\sqrt{\alpha}$ to the amplitude \mathcal{M} , where $\alpha = e^2/4\pi$ represents the strength of the electromagnetic

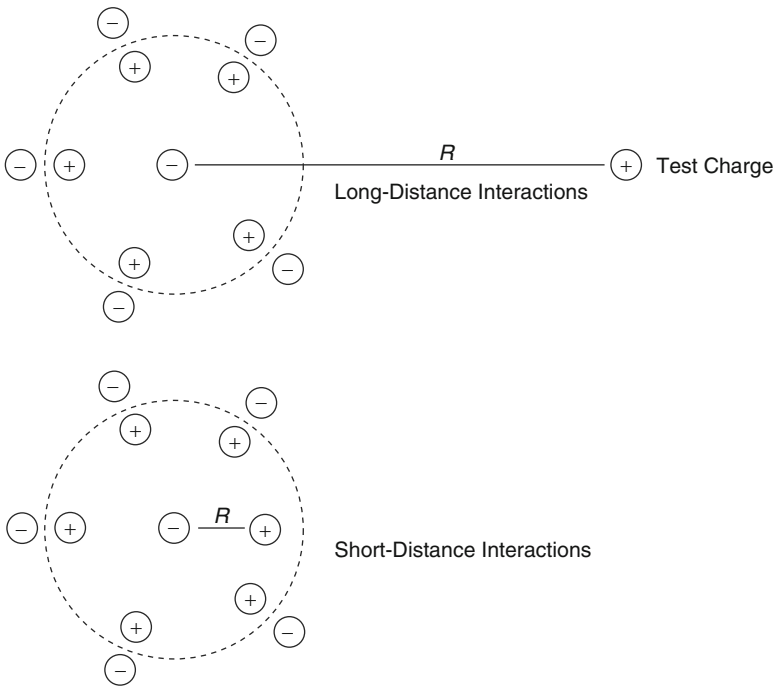
⁸There is currently no complete quantum theory of the remaining fundamental interaction, gravitation, but many of the proposed theories postulate the existence of a spin-2 particle that mediates gravity, the *graviton*.

coupling between photons and charged particles. (Here, e is the magnitude of the electron's charge.) The mathematical evaluation of the above diagram yields a cross-section proportional to $|\mathcal{M}|^2$ (and therefore α^2):

$$\sigma(e^+e^- \longrightarrow \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}, \quad (2.2)$$

where \sqrt{s} is the center of mass energy of the e^+e^- collision.⁹

An interesting physical ramification of QED is the spontaneous production of *virtual* electron-positron pairs due to the uncertainty inherent in quantum mechanics. Because of this uncertainty, energy conservation can be violated for a very short time period, $\Delta t < \hbar/\Delta E$, where ΔE is the “borrowed” energy. This has important implications for the nature of the electromagnetic interaction. An electron in QED can spontaneously emit a virtual photon, which in turn can produce a virtual e^+e^- pair, and so on, until a single “bare” electron is surrounded by a cloud of virtual electrons and positrons. The diagram¹⁰ below illustrates this.



⁹As usual, we've chosen units where $\hbar = c = 1$.

¹⁰The diagram was adapted from Fig. 1.6 in *Quarks and Leptons* by F. Halzen and A.D. Martin, 1984.

Because opposite charges attract, the positrons will be located preferentially closer to the electron. If one measures the charge of the electron from a location outside of the e^+e^- cloud, the bare charge is reduced by the intervening positrons. This is referred to as *charge screening*. As one moves closer to the electron, penetrating the cloud of nearby positrons, the observed charge of the electron increases.

Since the strength of the electromagnetic coupling α is proportional to the square of the electric charge (e^2), the effect of charge screening is to reduce the coupling strength for long distance (low energy) interactions. Thus, α depends on the energy scale associated with the interaction. The value of α decreases asymptotically with energy to a constant value of $\approx 1/137$. Historically, this quantity is known as the *fine structure constant*.

Equation (2.2) gives the leading-order approximation to the exact $e^+e^- \rightarrow \mu^+\mu^-$ scattering cross-section. A full QED calculation requires summing an infinite series of diagrams with additional vertices and internal loops! Generally, as more photons are added to the diagrams, the number of vertices (and hence the order of α) increases and the calculations become quite cumbersome. Fortunately, the small value ($\alpha \approx 1/137 \ll 1$) makes it possible to ignore the contributions from higher-order diagrams. This is the basis of *perturbation theory*, and it greatly enhances the predictive power of QED. In most cases, very precise QED predictions of physical observables can be obtained using only a few simple (low order) diagrams.

2.5.2 The Unified Electroweak Theory

In 1954, C.N. (Frank) Yang and Robert Mills formulated a generalized principle of gauge invariance that eventually led to a new type of quantum field theory. Unlike QED, with a single force-mediating photon, the theory proposed by Yang and Mills required three massless gauge bosons: one with positive electric charge, one with negative electric charge, and one electrically neutral. The introduction of additional gauge bosons implied the existence of a force that is capable of transforming particles from one type to another. At the time, this seemed to describe the characteristics of the weak force, which, among other things, converted protons to neutrons (and vice versa) in nuclear β decay.

The mathematical groundwork of Yang and Mills led to substantial theoretical developments in the 1960s. In 1961, Sheldon Glashow irreversibly linked the weak interaction to QED by formulating a gauge field theory with three massless vector bosons in addition to the photon. There was only one problem: no massless charged field-mediating particles had ever been observed in nature. The conundrum was solved by the identification of *spontaneous symmetry breaking* by Jeffrey Goldstone and Peter Higgs. The *Higgs mechanism* was applied to Glashow's theory by Steven Weinberg in 1967 and Abdus Salam in 1968, thereby giving the gauge bosons mass, described further in Sect. 2.6. The result was a self-consistent unified electroweak theory that predicted one massless particle (the photon) and three new massive

particles: the W^+ , W^- , and Z^0 bosons.¹¹ The discovery of the W and Z bosons at the European Center for Nuclear Research (CERN) sixteen years later confirmed the theoretical predictions and marked a tremendous advance for the standard model. At sufficiently high energies, the difference between the electromagnetic and weak interactions becomes negligible and the two act together as a single, unified electroweak interaction.

The first measurements of the W and Z boson masses by the UA1 and UA2 collaborations in 1983 were based on a handful of events from $p\bar{p}$ collisions at the CERN SPS collider. To the surprise of many, these mediators of the electroweak force turned out to be over 85 times more massive than the proton! The masses of the W and Z bosons were about $80 \text{ GeV } c^{-2}$ and $91 \text{ GeV } c^{-2}$, respectively. These huge masses of the W and Z bosons mean that they are extremely short-lived, which explains the relatively small interaction strength of the weak interaction.

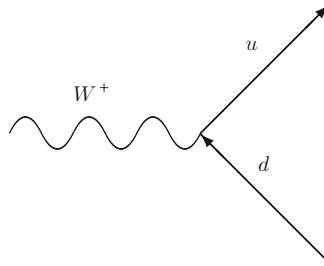
In the electroweak theory, the masses of the W and Z bosons are intricately connected with two gauge coupling constants, g and g' , via the Weinberg angle θ_W :

$$\tan \theta_W = \frac{g'}{g}, \quad \cos \theta_W = \frac{M_W}{M_Z}. \quad (2.3)$$

Also known as the *weak mixing angle*, θ_W is a parameter that relates the relative strengths of the weak and electromagnetic couplings. A fit to a variety of experimental measurements yields a value of

$$\sin^2 \theta_W = 0.2230 \pm 0.0004. \quad (2.4)$$

As a mediator of the weak force, the charged W bosons couple to fermion pairs that differ in charge by ± 1 . Unlike all of the other gauge bosons, the W^+ or W^- possesses the unique ability to change the flavor of fermions with which it interacts. The flavor-changing property of the W^+ boson is illustrated in the diagram below.



¹¹The W^+ and W^- bosons are often written as W bosons, where the superscripts representing electric charge are dropped. Likewise, the Z^0 boson is written as the Z boson.

Since the interaction requires a transfer of electric charge at the vertex, the W boson coupling is said to be associated with a weak *flavor-changing charged current*. This property is of great importance for the production of W bosons in $p\bar{p}$ collisions, where they are an integral part of many ongoing analyses at experiments like CDF and DØ at Fermilab, and have recently been “rediscovered” at CERN in proton-proton collisions produced by the Large Hadron Collider.

2.5.3 Quantum Chromodynamics (QCD)

In 1965, Moo-Young Han, Yoichiro Nambu, and Oscar Greenberg laid the foundation for *quantum chromodynamics* (QCD), the quantum gauge theory that describes the strongest of the four fundamental interactions. The strong interaction is mediated by massless gauge bosons called *gluons* (g). Gluons are the field quanta that carry a unique kind of charge, called *color*, for which the theory is named. Just as electric charge is conserved within the framework of QED, the color charge of QCD is conserved in all interactions between quarks and gluons. There are three distinct values of the color charge – commonly denoted red, green, and blue – together with their corresponding anticolors.

Although both QED and QCD are gauge-invariant field theories, QCD is *non-Abelian*. Physically, this implies a qualitative difference from QED: whereas photons couple only to electrically charged particles, gluons themselves carry the color charge and interact among themselves. In fact, there are actually eight distinct varieties of gluons composed of various permutations of color and anticolor charge. This has important ramifications. Unlike the charge screening of QED, in which virtual electron-positron pairs pop out of the vacuum and align themselves to shield a bare charge, a bare QCD color charge (e.g., a quark) is quickly surrounded by a “sea” of virtual quarks and gluons with the same color. As one probes the bare color charge at shorter and shorter distances, corresponding to higher and higher energies, the observed charge lessens until only the bare charge is seen. This is referred to as *asymptotic freedom*. Farther from the bare color charge, the intervening sea of color increases the observed charge, resulting in a strong attractive force between two distant color charges. The potential energy grows roughly linearly with the separation of charges, according to

$$V(r) = -\frac{4}{3} \frac{\alpha_s}{r} + kr. \quad (2.5)$$

At large distances, the potential energy between two quarks is sufficient to create a real (as opposed to virtual) quark-antiquark pair from the vacuum, thereby breaking the long-distance force and reducing the overall potential energy. This process is known as *fragmentation*. Since fragmentation will always occur as two quarks separate, solitary quarks cannot exist. Instead, quarks must eventually join with other quarks or antiquarks to form colorless bound states. This property of QCD, called *color confinement*, offers an explanation of why no free quarks or gluons have ever been observed in nature.

In Equation 2.5, the quantity α_s is the QCD coupling strength, which describes how the effective charge between two quarks depends on the distance between them. The lowest-order expression for α_s , also known as the *running coupling constant*, is given by

$$\alpha_s(Q) = \frac{6\pi}{(33 - 2n_f) \ln(Q/\Lambda_{\text{QCD}})}. \quad (2.6)$$

Here, Q denotes the square root of the momentum transfer (i.e. the energy of the probe), n_f is the allowed number of quark flavors at that energy, and Λ_{QCD} corresponds roughly to the energy boundary between asymptotically free quarks and hadrons. Measurements of Λ_{QCD} yield a value between 100 and 500 MeV, a scale that coincides well with the masses of the lightest hadrons.

Unlike the QED coupling α , which increases with energy, α_s falls off gradually and approaches an asymptotic value. For $Q \approx \Lambda_{\text{QCD}}$, quarks and gluons interact strongly and arrange themselves into hadrons. As Q becomes much larger than Λ_{QCD} , the effective coupling becomes small and quarks and gluons interact with each other only weakly. The value of α_s is about 0.1 for interactions with a momentum transfer in the 100 GeV to 1 TeV range.

Besides addressing the question of why quarks always appear in bound systems, the notion of color also solved a nagging dilemma in the quark model of hadrons. Baryons were thought to contain either three quarks or three antiquarks, and this recipe successfully described the huge spectrum of newly discovered hadrons in the late 1950s and early 1960s. The Δ^{++} baryon was a peculiar exception. With an electric charge of $+2|e|$, the Δ^{++} could only exist as a combination of three up quarks (uuu) in the lowest orbital momentum state ($l = 0$) with fully aligned spins ($J = S = 3/2$). This configuration violates the Pauli exclusion principle. If, however, each quark carried a different value of the color charge, the fermions would no longer be identical and the exclusion principle would not be violated.

The existence of three unique quark colors is experimentally validated by the measurement of the cross-section ratio:

$$R = \frac{\sigma(e^+e^- \rightarrow q\bar{q} \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \quad (2.7)$$

For a period of time before 1964, a serious discrepancy between the predicted and measured values of the ratio puzzled theorists and experimentalists alike. The experimental value was three times larger than the predicted value. However, when the numerator was summed over all of the quark colors, the theoretical cross-section ratio reduced to the simple expression

$$R = N_c \sum_i q_i^2, \quad (2.8)$$

where N_c is the number of colors and q_i is the charge of each quark flavor. The sum includes the quark flavors that are kinematically accessible ($2m_i < \sqrt{s}$). A value of $N_c = 3$ brought theory and experiment into excellent agreement.

2.6 The Higgs Boson

As discussed in Sect. 2.5.2, the work of Yang and Mills produced a generalized principle of gauge invariance that led to a new form of quantum field theory. Glashow then developed a technique to link QED with the weak interaction by formulating a gauge field theory with three massless vector bosons in addition to a photon. The problem was that no massless charged field-mediating particles had ever been observed in nature. By 1964, three independent groups (Guralnik, Hagen, and Kibble; Higgs; Brout and Englert) solved this conundrum by combining the principle of gauge invariance with spontaneous symmetry breaking.

Spontaneous symmetry breaking occurs when the true symmetry of a system is hidden by the choice of a particular ground state. As an example, consider a thin, flexible ruler held vertically by its ends, with its edge facing you. Pushing on the ends of the ruler causes it to bend, and the middle shifts either to the left or to the right, breaking the left-right symmetry of the ruler system. This example illustrates a way to generate a discrete set of ground states because there are only two ways for it to bend. However, you could generate a continuous set of ground states if you were to push on the ends of an object like a thin, cylindrical plastic rod rather than a ruler. The rod can bend in the middle, but in any direction. In a field theory, ground states are determined by the minima of a potential (the vacuum states) and the fields are treated as fluctuations about a chosen state. The choice of a single ground state seemingly breaks the symmetry of the system and is considered “spontaneous” because there is no external means by which this occurs. For example, 3-dimensional symmetry on the surface of the earth is broken because we cannot describe up and down in the same way that we can left and right. This is, however, due to the presence of gravity, an external agency.

By 1968, Weinburg and Salam applied it to the electroweak theory. In the simplest application of the general gauge theory, an extra Higgs field is added. Through spontaneous symmetry breaking – choice of a ground state – this field is caused to interact with the weak field to produce masses for the W and Z gauge bosons. The Higgs mechanism can also be applied to the other field theories to demonstrate how quarks and leptons gain mass.

A more intuitive way to think of the Higgs field is to imagine water that fills a pool. If you were to walk through a pool, you would feel heavier because of the water pushing against you. Your inertia is larger because the water makes it harder for you to move, as though you have gained mass. The Higgs field permeates space in the same way. As some particles travel through the universe, they interact with this field and acquire mass as a result. Some particles also interact more strongly with the Higgs field than others and so have a larger mass. Why this is true is still unknown.

Of the scientists that theorized the Higgs mechanism, Peter Higgs mentioned the possibility of a new elementary scalar (spin-0) boson being produced, now called the Higgs boson. This particle is the only standard model particle that has not been observed in nature. Evidence of this particle would verify the existence of the Higgs mechanism and confirm the manner in which particles gain mass.

The formalism of the Higgs mechanism does not provide the actual mass of the Higgs boson, so we experimentalists must search the entire mass range. The Large Electron-Positron Collider at CERN excluded¹² the mass below $114 \text{ GeV}/c^2$ and indirect electroweak measurements constrain the Higgs mass to be below $185 \text{ GeV}/c^2$. Efforts to reduce the range between these masses is ongoing for the CDF and DØ collaborations at Fermilab who have together, as of July 2010, additionally excluded the region between 158 and $175 \text{ GeV}/c^2$. One of the main goals of the Large Hadron Collider (LHC), located at CERN in Switzerland, is to provide evidence for the Higgs. The LHC produced proton-proton collisions at 3.5 TeV for the first time in November 2009, making it the highest energy particle collider in the world. As the CDF and DØ experiments wind down in 2012, it is expected that the LHC will fill in the final gaps of the mass range and, therefore, provide confirmation for or against the existence of the Higgs boson.

2.7 References and Further Reading

The primary sources for the material in this section are [20, 33–35, 37, 40, 56, 89, 93, 97, 102]. For further reading, we also recommend [25].

¹²All mass exclusions quoted here are with a 95% confidence level.

	Leptons		Hadrons			Higgs
	$(1, 2, -1/2)$	$(1, 1, 1)$	$(3, 2, 1/6)$	$(3, 1, -2/3)$	$(3, 1, 1/3)$	$(1, 2, -1/2)$
Generation 1	$\begin{pmatrix} \text{Electron neutrino} \\ \text{Electron} \end{pmatrix}$	Electron	$\begin{pmatrix} \text{Up} \\ \text{Down} \end{pmatrix}$	Up	Down	1 Generation only
Generation 2	$\begin{pmatrix} \text{Muon neutrino} \\ \text{Muon} \end{pmatrix}$	Muon	$\begin{pmatrix} \text{Charm} \\ \text{Strange} \end{pmatrix}$	Charm	Strange	
Generation 3	$\begin{pmatrix} \text{Tau neutrino} \\ \text{Tau} \end{pmatrix}$	Tau	$\begin{pmatrix} \text{Top} \\ \text{Bottom} \end{pmatrix}$	Top	Bottom	



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