

Preface

Motivation for This Series

First of all, we want to point out that this book is by no means meant to compete with or take the place of any of the standard quantum field theory, particle physics, or mathematics texts currently available. There are too many outstanding choices to try to add yet another to the list. Our goal is, simply put, to teach physicists the math that is used in particle physics.

The origin of this goal is the plight of upper-level undergraduate and first/second year graduate students in physics, especially those in theory. Generally, after four years of standard undergraduate coursework and two years of standard graduate coursework, the road to understanding a modern research paper in particle theory is a long, hard hike. And as the physics becomes more and more advanced, the necessary math becomes more sophisticated at an overwhelming rate. At least, it is overwhelming for those of us who don't understand everything immediately.

To make matters worse, the way physicists and mathematicians think about nearly everything in math and physics can be (and usually is) vastly different. The way a mathematician approaches differential equations, Lie groups, or fiber bundles is typically unlike the way a physicist approaches them. The language used is often very different, and the things that are important are almost always different. When physics students realize that they need a better understanding of how one does analysis on manifolds (e.g. at a graduate level), reading a graduate-level book about analysis on manifolds (by a mathematician) is very often frustrating and unhelpful. This shouldn't be taken to reflect poorly on our friends in the math department or on their pedagogical abilities. It is simply indicative of the wide gulf between two different disciplines.

Nevertheless, despite being different disciplines, the language of physics is mathematics. If you want to understand the inner workings of nature, you have to understand analysis on manifolds, as well as countless other topics, at a graduate level at least. But because physicists are not used to thinking in the way mathematicians

think,¹ they will make much more progress when things are explained in a way that is “friendly to a physicist”, at least at first. For example, if you show a physicist the formal definition of an ideal or of cohomology (when they’ve never encountered those ideas before) they will usually find it very difficult to intuit what they actually are. However, if you say something like “an ideal is essentially all the multiples of something, like all the multiples of 7 on the real line” or “cohomology is essentially a way of measuring how many holes are in something”, and slowly build up the formal definition from there, progress will be much faster. The downside to this approach is that any mathematicians standing nearby will likely get very annoyed because “friendly to a physicist” usually translates to “lacking rigor”.

While mathematicians are correct in pointing out that we often (usually) lack rigor in how we think about their ideas, it is still good for us to do it. For example, after understanding that cohomology is essentially a way of measuring how many holes there are in something, the physicist will have a much easier time parsing through the formal definition. If there are a few non-trivial (but still simple) examples scattered along the way, there is a good chance that the physicist will develop a very good understanding of the “real” mathematical details.

However, with only a few exceptions, there are math books and there are physics books. When physicists write physics books they generally try to go as far as they can with as little advanced math as they can, or they assume that the reader is already familiar with the underlying math. And when mathematicians write math books, they either don’t care about the physical applications or they mention them only briefly and maintain abstract mathematical formalism with some physics vocabulary. While neither of these situations is the fault of the authors, it can often be to the detriment of helping eager physics students get any real intuition for what lines drawn around a hole have to do with magnetic fields.

So that is the context of this series of books. We’re trying to teach math in a way that is ~~lacking rigor~~ friendly to physicists. The goal is that after reading this, one of the many excellent introductory texts on relativistic quantum mechanics, quantum field theory, or particle physics will be much more accessible.

Outline of This Series

This is the first in a series of books intended to teach math to physicists. The current plans are for at least four volumes. Each of the first four volumes will discuss a variety of mathematical topics, but each will have a particular emphasis. Furthermore, a substantial portion of each will discuss, in detail, how specific mathematical ideas are used in particle physics.

The first volume will emphasize algebra, primarily group theory. In the first part we will discuss at length the nature of group theory and the major related

¹The converse is typically very true as well.

ideas, with a special emphasis on Lie groups. The second part will then use these ideas to build a modern formulation of quantum field theory and the tools that are used in particle physics. In keeping with the theme, the formulations and tools will be approached from a heavily algebraic perspective. Finally, the first volume will discuss the structure of the standard model (again, focusing on the algebraic structure) and the attempts to extend and generalize it. As a comment, this does not mean that this volume is *solely* about algebra. We will talk about and use a variety of mathematics (i.e. we'll use analysis, geometry, statistics, etc.) – we'll just be primarily using algebra.

The second volume will emphasize geometry and topology in a fairly classical way. The first part will discuss differential geometry and algebraic topology, and the second part will combine these ideas to discuss more fundamental formulations of classical field theory and electrodynamics, and gravitation.

The third volume will once again emphasize geometry and topology, but in a more modern context – namely through fibre bundles. The major mathematical goal will be to build up a primer on global analysis (mathematical relationships between locally defined geometric data and globally defined topological data). The physical application in the second part of this volume will be a fairly comprehensive and robust overview of gauge theory, and a reformulation of the standard model in modern terms.

Finally, the fourth volume will emphasize real and complex analysis. The physical application will then be the study of particle interactions (a topic that is glaringly, but deliberately, absent from the first volume). This will include detailed discussions of renormalization, scattering amplitudes, decay rates, and all of the other topics that generally make up the major bulk of introductory quantum field theory and particle physics courses.

So, over the first four volumes, we will cover algebra, geometry, topology, and real and complex analysis – four of the major areas in mathematics. Furthermore, we will have discussed how all of this math ties in to classical field theory, quantum field theory, general relativity, gauge theory, non-perturbative quantum field theory, particle interactions, and renormalization. In other words, the first four volumes are intended to be a fairly comprehensive introduction to modern physics.

However, we do wish to reiterate that while we are hoping to be as comprehensive as possible, we only mean in scope, not in depth. Once again, our goal is *not* to replace any of the standard physics or math texts currently available. Rather, our goal is that these volumes will act as either a primer for those texts (so that after reading these books, you will find those to be highly approachable), or as supplemental references (to assist you when an idea is not clear).

As a final comment before moving on, we do have long term plans for more volumes. We would likely break the theme of having one mathematical emphasis and simply build on the math from the first four volumes as necessary. The tentative plan for the topics in the later volumes is:

- Volume V – Supersymmetry and Supergravity
- Volume VI – Conformal Field Theory and Introductory String Theory

- Volume VII – D-Branes and M-Theory
- Volume VIII – Algebraic Geometry and Advanced Topics in String Theory
- Volume IX – Cosmology and Astrophysics
- Volume X – String Cosmology

As a warning, this series is a “spare-time project” for all of us and the timeline for these volumes will, unfortunately, be quite drawn out.

Outline of This Volume

As we said, the emphasis of this book is algebra, and the physical application is the (algebraic) structure of the standard model. However, because this book is the first in the series, there is quite a bit of additional material that is not entirely vital to the logical flow of this volume or the series as a whole.

Chapter 1 (which is vital to the overall flow) is a primer in the classical prerequisites. In short, it is undergraduate physics using graduate notation. Consequently it is much more cursory than the subsequent chapters. The major idea is to review:

- The variational calculus formulation of classical mechanics
- Special relativity
- Classical field theory (primarily electromagnetism)

Chapter 2 is meant to serve as a reminder that despite all of the math, we are still trying to describe something physical at the end of the day. This chapter is therefore an overview of how experimentalists think about particle physics. We talk about the history of particle physics and how particles and interactions are organized. The content of this chapter is not vital to the overall flow of the book, but is vital to any self-respecting theorist and should be read carefully.

Chapter 3 (which is vital) is meant to serve as an introduction to group theory. There are three major sections:

- Basic group theory. This section focuses on finite, discrete groups because many group theoretic ideas are easier to intuit in this setting. We won’t use finite discrete groups much later in the book (or series), but the ideas illustrated by them will be constantly used.
- Basic Lie group theory.
- A specific Lie group: the Lorentz group of special relativity.

Chapter 4 (which is vital) then begins with the real physical content of this book. Using the algebraic machinery developed previously, we discuss the three major types of fields: spin-0 fields (also called scalar fields or Klein-Gordon fields), spin-1/2 fields (also called spinor fields or Dirac fields), and spin-1 fields (also called vector fields). Then we discuss gauge theory, the algebraic framework that seems to describe all of particle physics. Next is quantization, then symmetry breaking and non-Abelian gauge theory, and finally we look at the standard model itself.

Finally, Chap. 5 (which is not vital, but is highly recommended) is a survey of several of the extensions and generalizations of the standard model, including $SU(5)$ and $SO(10)$, supersymmetry, and approaches to quantum gravity. As a warning, this chapter is meant to be a vast, mountaintop overview of several ideas. It is not meant to be a thorough introduction to anything, and you will likely find that a lot of it leaves you wanting more. We will be coming back to almost every topic in Chap. 5 in later volumes in much greater detail. We encourage you to view it as a way of getting familiar with the generic ideas and vocabulary, not as a way of gaining deep understanding.



<http://www.springer.com/978-1-4419-8266-7>

Symmetry and the Standard Model

Mathematics and Particle Physics

Robinson, M.

2011, XIX, 327 p., Hardcover

ISBN: 978-1-4419-8266-7