POD based computation of Joint Interface Modes

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Nomenclature

n	number of DOF of FE model
$\mathbf{x}$	nodel DOF vector of FE model
$\mathbf{f}^{B}$	external forces acting on FE model
$\mathbf{f}^{IJ}$	contact forces inside the joint
$\tilde{\mathbf{M}}$	mass matrix of FE model
$\tilde{\mathbf{K}}$	stiffness matrix of FE model
n_B	number of boundary DOF
$\mathbf{x}_B$	vector of boundary DOF
n_{IJ}	number of joint DOF
$\mathbf{x}_{IJ}$	vector of joint DOF
r	number of Ritz vectors for static reduction
$\tilde{\mathbf{X}}$	reduction matrix
$\tilde{\mathbf{K}}^{red}$	reduced stiffness matrix
$\tilde{\mathbf{M}}^{red}$	reduced mass matrix
$\tilde{\mathbf{\Phi}}^{*,red}$	full matrix of eigenvectors
k	considered numbers of eigenvectors
$\tilde{\mathbf{\Phi}}^{red}$	matrix of considered eigenvectors
$\tilde{\mathbf{\Phi}}^{JIM}$	matrix of joint interface modes
m	vector dimension
$\mathbf{y}$	arbitrary vector
$\tilde{\mathbf{Y}}$	matrix with column vectors $\mathbf{y}$
m	number of POM
$\mathbf{u}$	proper orthogonal mode (POM)
i, j	index variables
J	cost function
$\mathbf{v}$	eigenvector
w	ratio of considered energy
$\tilde{\mathbf{A}}$	matrix
$\tilde{\mathbf{\Phi}}^{POD,JIM}$	matrix of POD based JIM

1. Abstract

Recently proposed joint interface modes (JIM), which have been presented at the IMAC 25th, do consider Newton's 3rd law across a joint already at the stage of mode generation which leads to significant improvements in the subsequent mode based computation where nonlinear contact forces are applied.
In the latter publication the computation of the JIM is based on a general eigenvalue problem of a statically reduced mass and stiffness matrix. This approach has certain drawbacks in terms of interpretability and in terms of an 'a priory' - estimation of the required number of JIM.
In this contribution a prober orthogonal decomposition (POD) based method for the computation of the JIM is introduced. In this context the JIM can be interpreted as kind of 'energy modes' so that this procedure holds a meaningful physical interpretation as well as an 'a priory' – estimation of the required number of JIM.

2. Introduction and Motivation

It is a well known fact that the global stiffness and damping properties of a metallic structure, which consists of jointed substructures, are strongly influenced by the local and nonlinear characteristics of the involved joints such as bolted joints or spot welded seams, see exemplarily [1] and [2].
In the industrial practice this kind of problems are typically investigated either by the direct Finite Element Method (FEM) or a Ritz vector based (commonly called 'modal') approach.

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- The strength of the FEM is the high resolution of the domain of interest which leads in general to a satisfying accuracy. The drawback of this method is the huge number of degrees of freedom (DOF) which makes it unfeasible for time integration.
- The strength of a Ritz vector (commonly called 'mode') based approach is the dynamics (time integration) of linear structures. Various Ritz vector based reduction methods have been presented during the last decades. Reviews have been done, among others, by Craig [3], Noor [4] and Zu [5]. Typically, the nonlinearity due to the contact inside a joint has been either neglected or somehow linearized. While this approach preserves the computational efficiency, it may lead to remarkable errors, see exemplarily [6] and [7].

Another approach was introduced by the author and Prof. Irschik at the IMAC 25th [8], 26th [9] and 27th [10]. An extended version of [8] and [9] can be found in [11]. The main idea of the latter approach is the enrichment of existing, well proven mode bases (e.g. Craig – Bampton [12]), by certain problem oriented 'contact modes', which we called joint interface modes (JIM). The JIM represent a generalization of the nodal FE DOF of the involved joint surfaces. It has been demonstrated in [8] and [13] that the convergence is superior to the one of the so called interface modes, where the involved contact surfaces are not related to each other at the time of mode generation via Newton's 3rd law. However, this approach has several drawbacks, namely:

- The JIM are obtained by a generalized eigenvalue problem of statically reduced mass and stiffness matrices. So, in principle, the JIM are computed like vibration modes, which is difficult to interpret in a physical sense.
- Consequently, the eigenvalues of the mentioned eigenvalue problem are more a mathematical quantity than a meaningful physical frequency for the estimation of the required number of JIM. So to say, the method does not provide an 'a – priory' estimation of the required number of JIM.
- For the latter mentioned static reduction Guyans method was suggested in [8]. As an improvement in terms of computational efficiency it has been suggested in [10] to discretize the joint area in a certain number of subareas. An open question up to now was, if the chosen discretization is fine enough to represent the joints mechanical characteristics.

In this paper a modified computation of JIM is suggested. Instead of a generalized eigenvalue problem of the statically reduced mass and stiffness matrix a orthogonal decomposition (POD) of the statically stiffness matrix is suggested. In a first section the former introduced JIM computation approach is briefly reviewed. Then a rough review on POD is given followed by it's application for the computation of JIM. Finally two numerical examples are discussed. It will be demonstrated that the JIM gained by a POD based approach converge as quick as the former method and the Hankel singular values can be used for an a priory estimation of the number of required JIM as well as for the evaluation of the joint area discretization.

3. Short review on the former introduced approach for the computation of JIM

A jointed flexible body modeled by the FEM can be represented by the equation of motion in the form of

$$\tilde{\mathbf{M}}\ddot{\mathbf{x}} + \tilde{\mathbf{K}}\mathbf{x} = \mathbf{f}^{B} + \mathbf{f}^{IJ}, \qquad (1)$$

where $\mathbf{x}$ is the (n x 1) vector of nodal DOF and the (n x n) time invariant matrices $\tilde{\mathbf{M}}$ and $\tilde{\mathbf{K}}$ denote the bodies mass and stiffness. The load is a combination of the (n x 1) vectors of external loads $\mathbf{f}^{B}$ and contact forces $\mathbf{f}^{IJ}$.

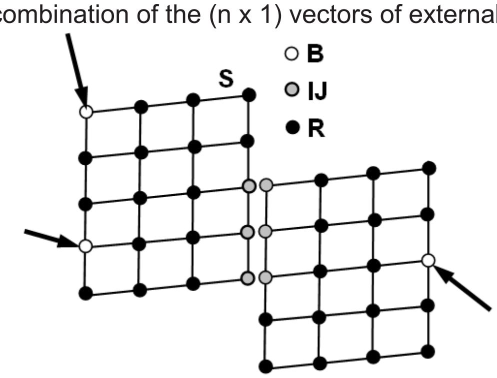

Figure 1: Arbitrary Finite Element structure with a joint

The considered FE structure with n nodal DOF consists of two jointed substructures as outlined in figure 1. Note that for the reasons depicted in [11] the jointed structure is considered as a single structure with 6 rigid body DOF. The external forces $\mathbf{f}^{B}$ are acting on the structure exclusively via the interface B and the according DOF are collected in the (n_B x 1) vector $\mathbf{x}_B$. The nodal DOF of $\mathbf{x}_B$ are outlined in figure 1 by white dots. The non-linear contact forces, which are represented by $\mathbf{f}^{IJ}$, act on the nodal DOF of the joint which are collected in the (n_{IJ} x 1) vector $\mathbf{x}_{IJ}$ and outlined in figure 1 by grey dots. The remaining (n - n_B - n_{IJ}) DOF are denoted as $\mathbf{x}_R$. According to that scheme, the vector of DOF can be written as

$$\mathbf{x} = \left[\mathbf{x}_B^{\,T} \quad \mathbf{x}_{IJ}^{\,T} \quad \mathbf{x}_R^{\,T} \right]^T . \tag{2}$$

In a first step, the number of DOF n is reduced via a linear superposition of Ritz vectors in the form of

$$\mathbf{x} = \tilde{\mathbf{X}}\mathbf{q} , \tag{3}$$

where the (n x r) matrix $\tilde{\mathbf{X}}$ contains r Ritz vectors in its columns. Please refer to [10] for a more detailed description of the reduction procedure. As mentioned, the reduction is based on a discretization of the entire joint area in r subareas.

Based on these reduction the reduced (r x r) mass and stiffness matrixes can be given as

$$\tilde{\mathbf{M}}^{red} = \tilde{\mathbf{X}}^T \mathbf{M} \tilde{\mathbf{X}} \text{ and} \tag{4}$$

$$\tilde{\mathbf{K}}^{red} = \tilde{\mathbf{X}}^T \mathbf{K} \tilde{\mathbf{X}} . \tag{5}$$

The computation of the JIM in the space of $\tilde{\mathbf{X}}$ is based on the eigenvalue problem

$$\left[\tilde{\mathbf{K}}^{red} - \left[\tilde{\mathbf{\Omega}}^{*,red} \right]^2 \tilde{\mathbf{M}}^{red} \right] \tilde{\mathbf{\Phi}}^{*,red} = \tilde{\mathbf{0}} \tag{6}$$

where r eigenvalues are stored in the (r x r) diagonal matrix $\tilde{\mathbf{\Omega}}^{*,red}$ and r eigenvectors in the (r x r) matrix $\tilde{\mathbf{\Phi}}^{*,red}$. The significant reduction of DOF is obtained by considering just the first k eigenvectors for the further considerations where k << r. The considered eigenvectors a collected in the (r x k) matrix $\tilde{\mathbf{\Phi}}^{red}$. The JIM for the entire structure can be computed by applying the reduction rule (3) in the form of

$$\tilde{\mathbf{\Phi}}^{JIM} = \tilde{\mathbf{X}} \tilde{\mathbf{\Phi}}^{red} , \tag{7}$$

where the (n x k) matrix $\tilde{\mathbf{\Phi}}^{JIM}$ contains the JIM for the joint contact.

In the final Ritz vector based computation the matrix of JIM is suggested as enrichment of established mode bases like the one of Craig-Bampton [12]. An exemplarily final transformation rule can be given as

$$\mathbf{x} = \left[\tilde{\mathbf{\Phi}}^{\text{Craig-Bampton}} \quad \tilde{\mathbf{\Phi}}^{JIM} \right] \mathbf{q} . \tag{8}$$

As already mentioned in the introduction three drawbacks of this method can be given, namely:

- In equation (6) the JIM are obtained by a generalized eigenvalue problem. In principle the JIM are the vibration modes of an artificial structure which is difficult to interpret.
- Consequently, the eigenvalues of (6) are more a mathematical quantity than a physical meaningful frequency. These eigenvalues can not be used as an 'a – priory' estimation of the required number of JIM.
- The reduction (3) determines the quality of the latter JIM. With an increasing number of subareas k the quality of the JIM as well as the computational effort is increasing. The question, whether a particular selection of k is high enough is an open issue.

4. Short Review: POD

In this section, a short introduction in the proper orthogonal decomposition is given based on [14].

Let us assume p independent (m x 1) vectors $\mathbf{y}_1$ to $\mathbf{y}_p$ which are collected in the (m x p) matrix $\tilde{\mathbf{Y}}$. Proper orthogonal decomposition (POD) of rank g delivers g orthonormal (m x 1) vectors $\mathbf{u}_1$ to $\mathbf{u}_g$ which approximate the space spanned by $\tilde{\mathbf{Y}}$ optimal in a Euclidean sense. Note, that the vectors $\mathbf{u}_1$ to $\mathbf{u}_g$ are named proper orthogonal modes (POM).

Proper orthogonal modes of rank g minimize a cost function J in the form of

$$J(\mathbf{u}_1,\cdots,\mathbf{u}_g)=\sum_{j=1}^{m}\left\|\mathbf{y}_j-\sum_{i=1}^{g}\left(\mathbf{y}_j{}^T\mathbf{u}_i\right)\mathbf{u}_i\right\|^2 \longrightarrow \min, \tag{9}$$

so that

$$\mathbf{u}_i{}^T\mathbf{u}_j=\begin{cases}1 & i=j\\ 0 & i\neq j\end{cases}, \tag{10}$$

where the Euclidian norm is defined as

$$\|\mathbf{y}\|=\sqrt{\mathbf{y}^T\mathbf{y}}\ . \tag{11}$$

The POM $\mathbf{u}_1$ to $\mathbf{u}_g$ can be found by the solution of an eigenvalue problem in the form of

$$\tilde{\mathbf{Y}}^T\tilde{\mathbf{Y}}\mathbf{v}_i=\lambda_i\mathbf{v}_i \qquad \text{for i = 1, … ,g} \tag{12}$$

and a subsequent transformation

$$\mathbf{u}_i=\frac{1}{\sqrt{\lambda_i}}\tilde{\mathbf{Y}}\mathbf{v}_i \qquad \text{for i = 1, … ,g.} \tag{13}$$

The maximum error in a Euclidean sense can be given as error of the function J in the form of

$$J(\mathbf{u}_1,\cdots,\mathbf{u}_g)=\sum_{j=1}^{m}\left\|\mathbf{y}_j-\sum_{i=1}^{g}\left(\mathbf{y}_j{}^T\mathbf{u}_i\right)\mathbf{u}_i\right\|^2=\sum_{i=g+1}^{m}\lambda_i\ . \tag{14}$$

The slightly different POD method with a weighted inner product minimizes a cost function in the form of

$$J(\mathbf{u}_1,\cdots,\mathbf{u}_g)=\sum_{j=1}^{m}\alpha_j\left\|\mathbf{y}_j-\sum_{i=1}^{g}\left(\mathbf{y}_j{}^T\tilde{\mathbf{A}}\mathbf{u}_i\right)\mathbf{u}_i\right\|_A^2 \longrightarrow \min, \tag{15}$$

where the matrix $\tilde{\mathbf{A}}$ hast to be symmetric, of the dimension (m x m) and positive semi definite, α_j are nonnegative weights and the induced norm is of the form

$$\|\mathbf{y}\|_W=\sqrt{\mathbf{y}^T W\mathbf{y}}\ . \tag{16}$$

Now the eigenvalue problem takes on the form

$$\tilde{\mathbf{Y}}^T\tilde{\mathbf{A}}\tilde{\mathbf{Y}}\mathbf{v}_i=\lambda_i\mathbf{v}_i \qquad \text{for i = 1, … ,g} \tag{17}$$

and the final POM can be computed along equation (13).

From a mechanical point of view, the characteristics of the matrix $\tilde{\mathbf{A}}$ are the same as the ones of the stiffness matrix of a linear FE model. In that context, the POM can be interpreted as kind of ‘energy’ modes. For more information on the physical interpretation of POD refer to [15].

The singular values λ_i can be used as an estimation of how many number of POM are required because they hold a ratio of the modeled to the total energy contained in the system $\tilde{\mathbf{Y}}$, which is expressed by

$$w(g)=\frac{\sum_{i=1}^{p}\lambda_i}{\sum_{i=1}^{g}\lambda_i} \tag{18}$$

5. POD based Joint Interface Modes

The application of the POD method to the problem under consideration is quite straight forward. The space spanned by $\tilde{\mathbf{Y}}$ is replaced by $\tilde{\mathbf{X}}$ from equation (3) and, in case of POD weighted inner product, the matrix $\tilde{\mathbf{A}}$ of equation (17) is replaced by the stiffness matrix $\tilde{\mathbf{K}}$ from the equation of motion (1).

Four different sets of JIM will be numerical investigated, namely:

- POD based on linear discretization of the joint area
- POD with weighted inner product based on linear discretization of the joint area
- POD based on nonlinear discretization of the joint
- POD with weighted inner product based on nonlinear discretization of the joint

5.1. POD based on linear discretization of the joint area

As explained in detail in [10] the joint area is subdivided into r subareas. Each subarea is loaded with a unit pressure distribution on both sides of the joint. This leads to r linear static response computations and the deformations due to this loads are collected in the (n x r) matrix $\tilde{\mathbf{X}}^{\mathrm{Lin}}$.
The POM are computed via the eigenvalue problem

$$\left[\tilde{\mathbf{X}}^{\mathrm{Lin}}\right]^{T}\tilde{\mathbf{X}}^{\mathrm{Lin}}\mathbf{v}_i^{\mathrm{Lin}}=\lambda_i^{\mathrm{Lin}}\mathbf{v}_i^{\mathrm{Lin}} \qquad \text{for i = 1, … , r.} \tag{19}$$

Using $\tilde{\mathbf{V}}^{\mathrm{Lin}}$ as the (r x k) matrix of considered POM, the according JIM can be given as the (n x k) matrix

$$\tilde{\mathbf{\Phi}}^{\mathrm{Lin}}=\tilde{\mathbf{X}}^{\mathrm{Lin}}\tilde{\mathbf{V}}^{\mathrm{Lin}}. \tag{20}$$

5.2. POD with weighted inner product based on linear discretization of the joint area

In difference to the latter approach the stiffness matrix $\tilde{\mathbf{K}}$ is considered for the POM computation in the form of

$$\left[\tilde{\mathbf{X}}^{\mathrm{Lin}}\right]^{T}\tilde{\mathbf{K}}\tilde{\mathbf{X}}^{\mathrm{Lin}}\mathbf{v}_i^{\mathrm{Lin,K}}=\lambda_i^{\mathrm{Lin,K}}\mathbf{v}_i^{\mathrm{Lin,K}} \qquad \text{for i = 1, … , r.} \tag{21}$$

Using $\tilde{\mathbf{V}}^{\mathrm{Lin,K}}$ as the (r x k) matrix of considered POM, the according JIM can be given as the (n x k) matrix

$$\tilde{\mathbf{\Phi}}^{\mathrm{Lin,K}}=\tilde{\mathbf{X}}^{\mathrm{Lin}}\tilde{\mathbf{V}}^{\mathrm{Lin,K}}. \tag{22}$$

5.3. POD based on nonlinear discretization of the joint area

In contrast to the approaches before, the nonlinear contact is considered during the r static response computations and the resulting deformations are collected in the (n x r) matrix $\tilde{\mathbf{X}}^{\mathrm{NoLin}}$.
The POM are computed via the eigenvalue problem

$$\left[\tilde{\mathbf{X}}^{\mathrm{NoLin}}\right]^{T}\tilde{\mathbf{X}}^{\mathrm{NoLin}}\mathbf{v}_i^{\mathrm{NoLin}}=\lambda_i^{\mathrm{NoLin}}\mathbf{v}_i^{\mathrm{NoLin}} \qquad \text{for i = 1, … , r.} \tag{23}$$

Using $\tilde{\mathbf{V}}^{\mathrm{NoLin}}$ as the (r x k) matrix of considered POM, the according JIM can be given as the (n x k) matrix

$$\tilde{\mathbf{\Phi}}^{\mathrm{NoLin}}=\tilde{\mathbf{X}}^{\mathrm{NoLin}}\tilde{\mathbf{V}}^{\mathrm{NoLin}}. \tag{24}$$

5.4. POD with weighted inner product based on linear discretization of the joint area

The POM are computed along

$$\left[\tilde{\mathbf{X}}^{\mathrm{NoLin}}\right]^{T}\tilde{\mathbf{K}}\tilde{\mathbf{X}}^{\mathrm{NoLin}}\mathbf{v}_i^{\mathrm{NoLin,K}}=\lambda_i^{\mathrm{NoLin,K}}\mathbf{v}_i^{\mathrm{NoLin,K}} \qquad \text{for i = 1, … , r.} \tag{25}$$

Using $\tilde{\mathbf{V}}^{\mathrm{NoLin,K}}$ as the (r x k) matrix of considered POM, the according JIM can be given as the (n x k) matrix

$$\tilde{\mathbf{\Phi}}^{\mathrm{NoLin,K}}=\tilde{\mathbf{X}}^{\mathrm{NoLin}}\tilde{\mathbf{V}}^{\mathrm{NoLin,K}}. \tag{26}$$

In the following, these four methods are applied at different structures and compared with respect to each other and with the former approach which based on a generalized eigenvalue problem.

6. Examples

6.1. Generic beam example

6.1.1. FE model

A generic cantilever beam has been modelled according to Figure 2. The entire structure consists of two beam-like substructures, which are denoted with orange and blue colour in Figure 2. The two sub-structures are connected by two beams (Ø 8mm) which could represent two rivets or weld spots. Around the latter connectors where the two substructures do overlap the lap joint is located. Each sub-beam has a dimension of 100mm x 20 mm x 1mm and the overlapping area is of the dimension 40mm x 20 mm. The used material is iron and linear four node shell elements with a dimension of 2mm x 2mm have been used.

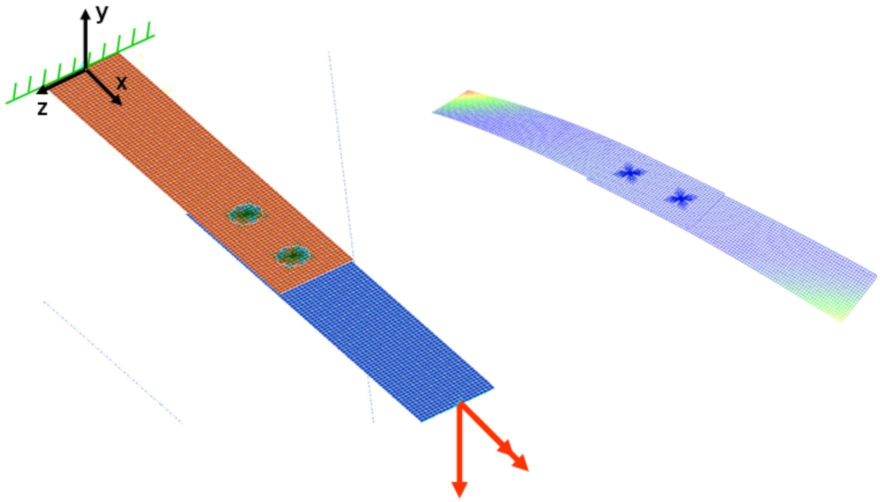

Figure 2: FE model of generic beam example and deformation shape of static force response computation

In a first step five different joint interface mode bases have been computed. One according to the general eigenvalue problem based approach outlined in section 3 and four more according to the POD based approaches of section 5.1 to 5.4 .

Note that the joint interface modes are computed for normal contact (y direction) only. Consequently, no tangential or friction forces are considered in the subsequent mode based analyses.

$\tilde{\Phi}^{JIM}$	$\tilde{\Phi}^{Lin,K}$	$\tilde{\Phi}^{NoLin,K}$	$\tilde{\Phi}^{Lin}$	$\tilde{\Phi}^{NoLin}$

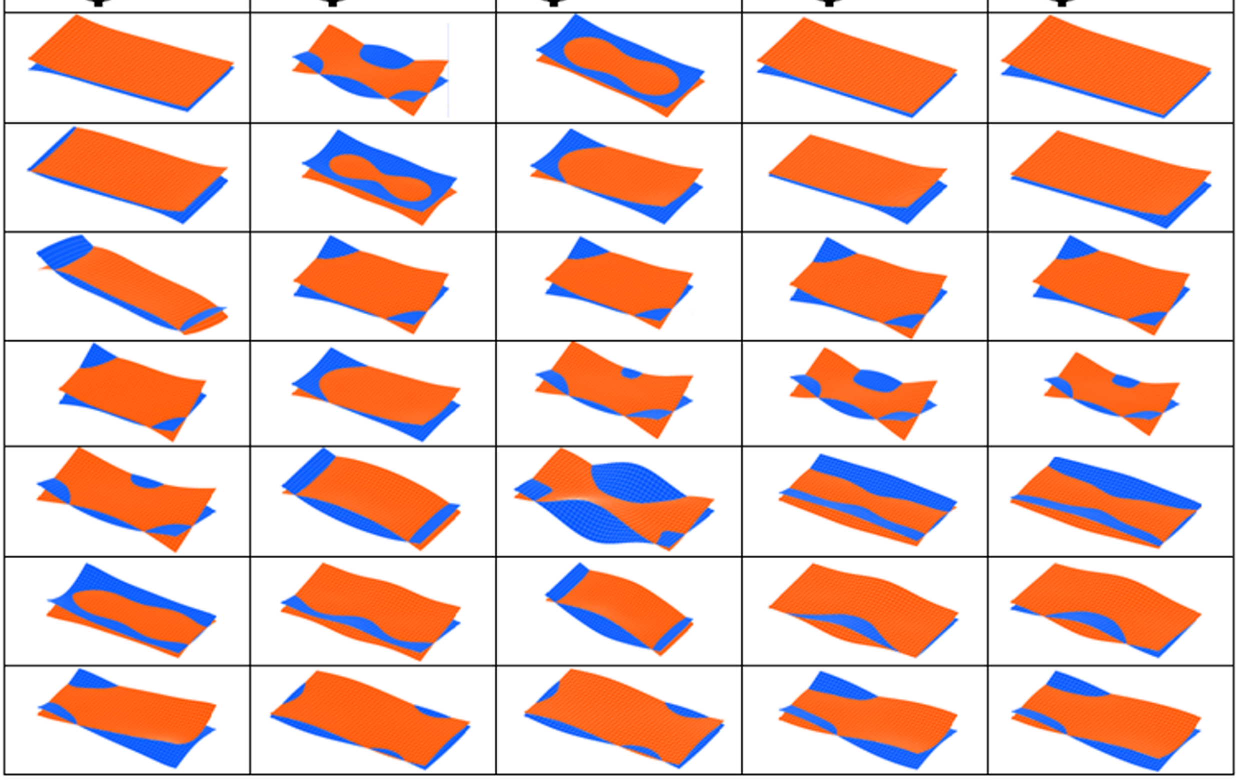

Table 1: First seven JIM according to different approaches

For a convergence analysis of the Euclidean norm of the gap between the contact partners with respect to the considered JIM a series of nonlinear analysis has been performed. The structure is mounted at the location of the reference frame, which is outlined green in Figure 2. The orange arrows in Figure 2 denote a force acting along the y axis and a torque acting around the x axis. The resulting deformation is outlined on the right hand side of Figure 2. Note, that a nonlinear penalty contact model has been implemented for the contact pressure.

6.1.2. Results

Table 1 contains a visualization of the first seven JIM due to the different approaches. Note, that the JIM of the generalized eigenvalue bases approach (first column of Table 1) is sorted by increasing eigenvalues while the others (column 2 to 5) are sorted by decreasing eigenvalues (or Hankel singular values).

Figure 3 contains on the right hand side a convergence analysis of the modelled energy with respect to the number of considered JIM according to equation (18). Using all available JIMs leads to a modelled energy of 100%. It can be seen, that already much less than the available 400 JIM covers almost all energy which can be modelled. On the left hand side the convergence of the Euclidian norm of the gap is depicted.

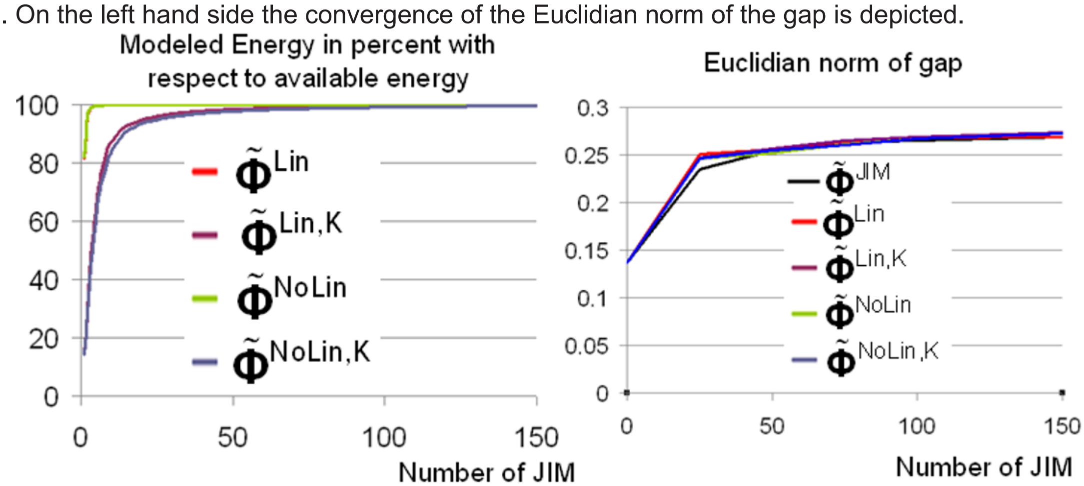

Figure 3: Convergence of modeled energy and of Euclidian gap in the contact with respect to the number of JIM

The latter 400 available JIM indicates, that the joint area is subdivided into 400 subareas in order to determine the mechanical joint characteristics, refer to [11]. In a final computation the JIM according to the POD based method of section 5.2 has been computed with 800, 400, 200, 100, 50, 25 and 13 subareas and the energy distribution has been evaluated in Figure 4. The according convergence analysis of the Euclidian norm of the gap and the joint pressure can be seen in Figure 5.

6.1.3. Conclusions

The conclusions drawn from the Table 1, Figure 3, Figure 4 and Figure 5 are:

- A visual evaluation of the different JIM in Table 1 reveals no significant qualitative difference. This 'intuitive' impression is underlined by the convergence analysis of the gap due to a certain static load. The five approaches lead to almost the same convergence rate.
- Figure 3 shows, that in case of a POD based approach which uses the stiffness matrix $\tilde{\mathbf{K}}$ according to sections 5.2 and 5.4, the convergence of the energy is somehow similar to the convergence of the Euclidian norm of the gap. Consequently, the Hankel singular values can be used as kind of 'a – priory' estimation of the required number of JIM. Note that the POD based approaches without using the stiffness matrix $\tilde{\mathbf{K}}$ (sections 5.1 and 5.3) yields Hankel singular values which do not represent the gap convergence. This is because just the deformations itself are approximated, while in the other case the strain energy is approximated.
- Figure 3 reveals that a nonlinear discretization of the joint area according to [11] is not necessary. There is no gain of accuracy while the nonlinear computation of $\tilde{\mathbf{X}}$ will take much more computational effort as the linear does.
- The question whether a joint is discretized fine enough can be answered by the results depicted in Figure 4 and Figure 5. A necessary criterion for a satisfying discretization seems to be a distinct convergence to the 100% limit. For this particular example, 50 to 100 subareas lead to an energy convergence with distinct convergence characteristics to the 100% limit.

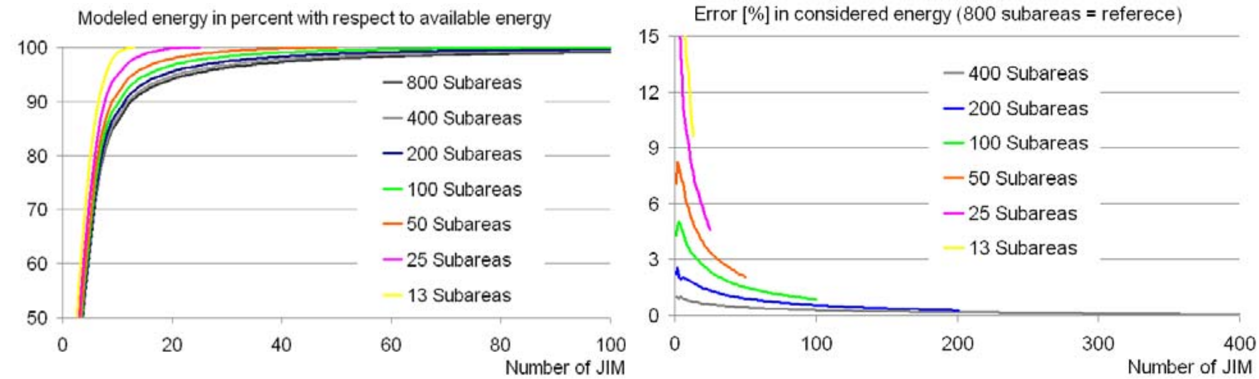

Figure 4: Convergence of modeled energy with respect to the number of JIM and to the number of subareas

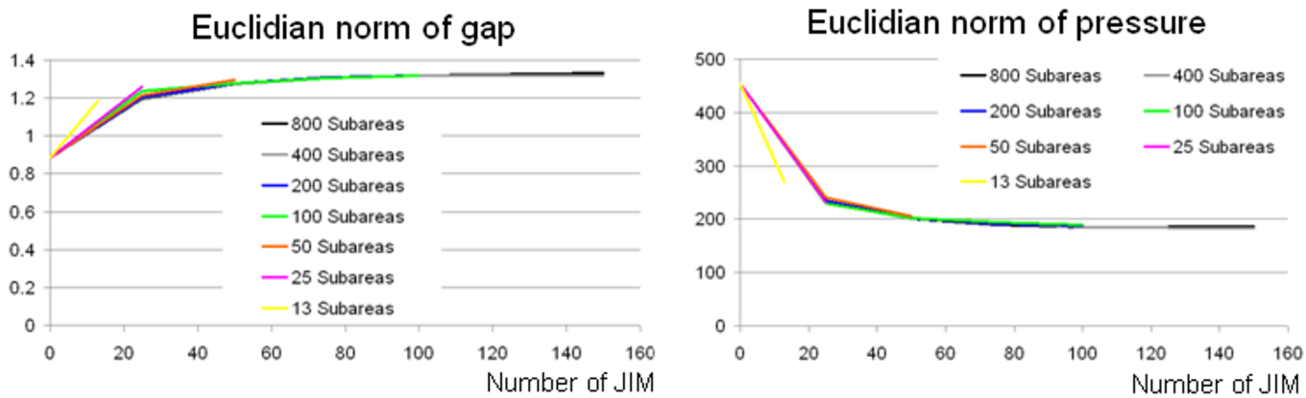

Figure 5: Convergence analysis of Euclidian norm of the gap and the joint pressure with respect to the number of JIM and to the number of subareas

6.2. <u>Hertzian stress example</u>

A Hertz type contact problem, as outlined in Figure 6, has been investigated. The plane strain problem consists of a plane part and a curved part which are moved towards each other so that contact occurs. The joint area for which joint interface modes are computed is outlined with red dots in Figure 6. The JIM have been computed along the approach outlined in section 4 and 5.2. A reference computation has be done using the commercial software ABAQUS using all nodal degrees of freedom.

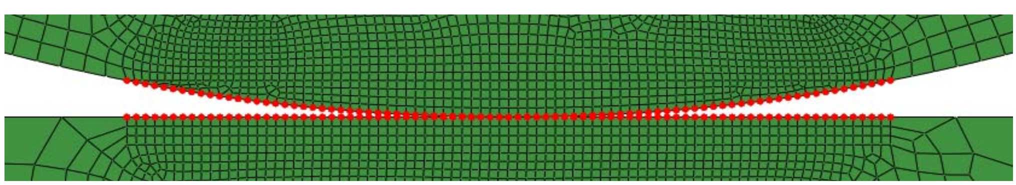
Figure 6: Hertzian contact problem

See Figure 7 for a qualitative convergence study of the von Mises stress in the contact area. It can be seen, that both approaches leads to a similar convergence characteristics and the use of 30 JIM delivers very good results.

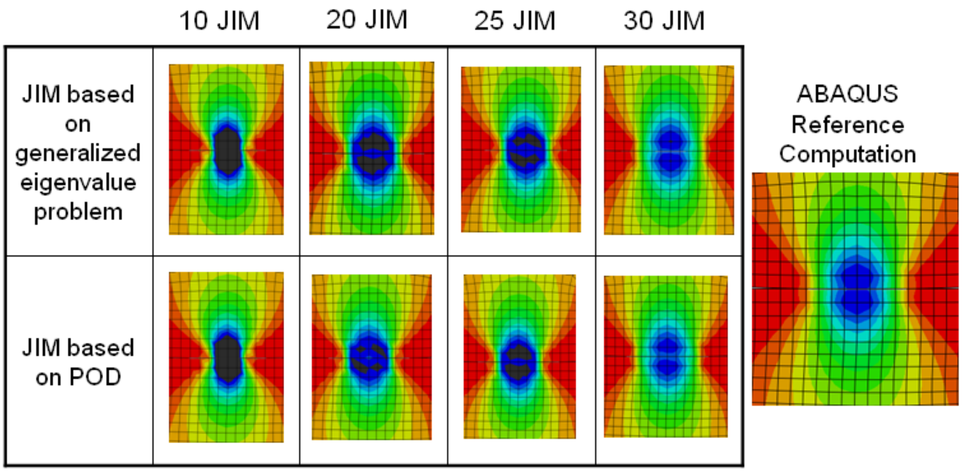

Figure 7: Von Mises Stress distribution in the contact area

7. Conclusion

It has been shown, that the POD based approach can be used for the computation of joint interface modes. Even the convergence characteristic is similar to the already existing method the POD based method has four important practical advantages:

- The POD based JIM have a clear physical meaning. POD with weighted inner product approximates a given subspace in terms of energy.
- Using a POD based method instead of an approach which is based on a generalized eigenvalue problem delivers a useful a-priory estimation of the number of necessary JIM.
- The proposed method gives an indication whether the number of subareas for the computation of the mechanical joint characteristics is high enough.
- Finally it is worth to mention that the POD based approach does not need any system matrixes at all. This is a very important fact when it comes to the commercial realisation of the proposed method which has been done by MAGNA Powertrain. The developed software code is called MAMBA [16].

8. Acknowledgement

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9. Bibliography

[1] Gaul L., Nitsche R., *The role of friction in mechanical joints*, Appl. Mech. Rev.; Vol. 54; No. 2; pp. 93 – 105, March 2001

[2] Ungar E.E., *The status of engineering knowledge concerning the damping of built-up structures*, Journal of Sound and Vibration, Vol 26(1), pp 141-154, 1973

[3] Craig R. J., *A Review of Time-Domain and Frequency-Domain Component Mode Synthesis Methods*, Int. J. Anal. and Exp. Modal Analysis, Vol. 2, No. 2, 1987, pp. 59-72

[4] Noor A. K., *Recent advances and applications of reduction methods*, Appl. Mech. Rev., Vol. 47, No. 5, 1994, pp. 125 – 146.

[5] Zu Q. Q., *Model order reduction techniques*, Springer Verlag London, 2004

[6] W. Witteveen, Irschik H., Riener H., Engelbrechtsmüller M., Plank A., *An efficient mode based approach for the dynamic analysis of jointed and local damped structures: Joint Interface Modes*, Proceedings of ISMA 2008, Leuven, Belgium, pp. 1815-1824

[7] Bograd S., Schmidt A., Gaul L., *Joint damping prediction by thin layer elements*, Proceedings of IMAC 26th, Society of Experimental Mechanics Inc, Bethel, Connecticut, US, 2008, Paper Nr. 288

[8] Witteveen W., Irschik H., *Efficient modal formulation for vibration analysis of solid structures with bolted joints*, Proceedings of IMAC 25th, Society of Experimental Mechanics Inc, Bethel, Connecticut, US, 2007, Paper Nr. 385

[9] Witteveen W., Irschik H., *Joint Interface Modes: Numerical 3D-Benchmark Studies*, Proceedings of IMAC 26th, Society of Experimental Mechanics Inc, Bethel, Connecticut, US, 2008, Paper Nr. 318

[10] Witteveen W., Irschik H., *Efficient Computation of Joint Interface Modes*, Proceedings of IMAC 27th, Society of Experimental Mechanics Inc, Bethel, Connecticut, US, 2009, Paper Nr. 16

[11] Witteveen W., Irschik H., *Efficient Mode-Based Computational Approach for Jointed Structures: Joint Interface Modes*, AIAA Journal, Vol. 47, No. 1, pp. 252-263, 2009

[12] Craig R.R., Bampton M.C.C., *Coupling of Substructures for Dynamic Analysis*, AIAA Journal, Vol. 6, No. 7, pp. 1313 – 1319, 1968

[13] Becker J., Gaul L., *CMS Methods for Efficient Damping Prediction for Structures with Friction*, Proceedings of IMAC 26th, Society of Experimental Mechanics Inc, Bethel, Connecticut, US, 2008, Paper Nr. 74

[14] Volkwein S., *Model reduction using proper orthogonal decomposition*, http://www.uni-graz.at/imawww/volkwein/POD.pdf, cited July 2010

[15] Feeny B. F., Kappagantu R., *On the physical interpretation of proper orthogonal modes in vibrations*, Journal of Sound and Vibration, Vol. 211, Nr. 4, pp 607-616, 1998

[16] www.mamba.ecs.steyr.com

http://www.springer.com/978-1-4419-9304-5

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