

Preface

This book analyzes almost periodic stochastic processes and their applications to various stochastic differential equations, partial differential equations, and difference equations. It is in part a sequel the of authors' recent work [20, 21, 22, 23, 24, 55] on almost periodic stochastic difference and differential equations and has the particularity to be among the few books that are entirely devoted to almost periodic stochastic processes and their applications. The topics treated in it range from existence, uniqueness, boundedness, and stability of solutions to stochastic difference and differential equations.

Periodicity often appears in implicit ways in various natural phenomena. For instance, this is the case when one studies the effects of fluctuating environments on population dynamics. Though one can deliberately periodically fluctuate environmental parameters in controlled laboratory experiments, fluctuations in nature are hardly periodic. Almost periodicity is more likely to accurately describe natural fluctuations [63]. Motivated by this observation, we decided to write this book that is devoted to the study of almost periodic (mild) solutions to stochastic difference and differential equations. Since the beginning of the century, the theory of almost periodicity has been developed in connection with problems related to differential equations, dynamical systems, and other areas of mathematics. The classical books of Bohr [32], Corduneanu [42], Fink [73], and Pankov [151] for instance gave a nice presentation of the concept of almost periodic functions in the deterministic setting as well as pertinent results in the area. Recently, there has been an increasing interest in extending certain classical results to stochastic differential equations in separable Hilbert spaces. This is due to the fact that almost all problems in a real life situation to which mathematical models are applicable are basically stochastic rather than deterministic. Nevertheless, the majority of mathematical methods are based on deterministic models. For instance, the theory of analysis frequently used in deterministic models can often be utilized as a tool to obtain the solutions to stochastic differential equations.

The concept of almost periodicity for stochastic processes was first introduced in the literature by Slutsky [166] at the end of 1930s, who then obtained some reasonable sufficient conditions for sample paths of a stationary process to be almost

periodic in the sense of Besicovitch, that is, B^2 -almost periodic. A few decades later, two other investigations on the almost periodicity of sample paths followed the pioneer work of Slutsky. Indeed, Udagawa [173] investigated sufficient conditions for sample paths to be almost periodic in the sense of Stepanov, and Kawata [109] studied the uniform almost periodicity of sample paths. Next, Swift [167] extended Kawata's results within the framework of harmonizable stochastic processes. Namely, Swift made extensive use of the concept of uniform almost periodicity similar to the one studied by Kawata to obtain some sufficient conditions for harmonizable stochastic processes to be almost periodic.

This book is divided into eight main chapters. It also offers at the end of each chapter some useful bibliographical notes.

Chapter 1 provides the reader with a detailed and somewhat concise account of basic concepts such as Banach and Hilbert spaces as well as some illustrative examples.

Chapter 2 is devoted to the foundations on operator theory, spectral theory, intermediate spaces, semigroups of operators, and evolution families. Some suitable examples are also discussed. The proofs for several of the classical results are given. The technical Lemma 2.2 (Diagana et al. [52]) and Lemma 2.4 (Diagana [62]) will play a key role throughout the book. Detailed proofs of these technical lemmas are discussed at the very end of Chapter 2.

Chapter 3 develops probabilistic tools needed for the analysis of stochastic problems in the book. It begins with a review of the fundamentals of probability including the notion of conditional expectation, which is very useful in the sequel. This chapter also offers an introduction to the mathematical theory of stochastic processes, including the notion of continuity, measurability, stopping times, martingales, Wiener processes, and Gaussian processes. These concepts enable us to define the so-called Itô integral, the Itô formula, and diffusion processes. An extension of Itô integrals to Hilbert spaces and stochastic convolution integrals are also discussed. An investigation of stochastic differential equations driven by Wiener processes is given at the end of the chapter. Special emphasis will be on the boundedness and stability of solutions.

Chapter 4 introduces and develops the concept of p -th mean almost periodicity. In particular, it will be shown that each p -th mean almost periodic process defined on a probability space $(\Omega, \mathcal{F}, \mathbf{P})$ is uniformly continuous and stochastically bounded [132]. Furthermore, the collection of all p -th mean almost periodic processes is a Banach space when it is equipped with its natural norm. Moreover, two composition results for p -th mean almost periodic processes (Theorems 4.4 and 4.5) are established. These two theorems play a crucial role in the study of the existence (and uniqueness) of p -th mean almost periodic solutions to various stochastic differential equations on $L^p(\Omega, \mathcal{H})$ where $\mathcal{H}$ is a real separable Hilbert space.

In Da Prato and Tudor [46], the existence of almost periodic solutions to Eq. (5.3) in the case when the linear operators $A(t)$ are periodic, that is, $A(t + \tau) = A(t)$ for each $t \in \mathbb{R}$ for some $\tau > 0$, was established. In Chapter 5, it goes back to studying the existence of p -th mean almost periodic solutions to the class of nonautonomous stochastic differential equations

$$dX(t) = A(t)X(t) dt + F(t, X(t)) dt + G(t, X(t)) d\mathbb{W}(t), \quad t \in \mathbb{R}, \quad (0.1)$$

where $(A(t))_{t \in \mathbb{R}}$ is a family of densely defined closed linear operators satisfying the well-known Acquistapace–Terreni conditions, $F : \mathbb{R} \times L^p(\Omega, \mathbb{H}) \rightarrow L^p(\Omega, \mathbb{H})$ and $G : \mathbb{R} \times L^p(\Omega, \mathbb{H}) \rightarrow L^p(\Omega, \mathbb{L}_2^0)$ are jointly continuous satisfying some additional conditions, and $\mathbb{W}(t)$ is a Q -Wiener process with values in $\mathbb{K}$. Application to some N -dimensional parabolic stochastic partial differential equations is also discussed. Moreover, Chapter 5 offers some sufficient conditions for the existence of p -th mean almost periodic solutions to the autonomous counterpart of Eq. (0.1).

Chapter 6 offers sufficient conditions for the existence of p -th mean almost periodic mild solutions for the following classes of stochastic evolution equations with infinite delay

$$\begin{aligned} d[X(\omega, t) + f_1(t, X_t(\omega))] &= [\mathcal{A}X(\omega, t) + f_2(t, X_t(\omega))] dt \\ &+ f_3(t, X_t(\omega)) d\mathbb{W}(\omega, t), \quad t \in \mathbb{R}, \quad \omega \in \Omega, \end{aligned} \quad (0.2)$$

where $\mathcal{A} : \mathcal{D} = D(\mathcal{A}) \subset \mathbb{H} \rightarrow \mathbb{H}$ is a sectorial linear operator whose corresponding analytic semigroup is hyperbolic, that is, $\sigma(\mathcal{A}) \cap i\mathbb{R}$ is empty, and $f_1 : \mathbb{R} \times \mathbb{H} \rightarrow \mathbb{H}_\beta$ ($0 < \alpha < \frac{1}{p} < \beta < 1$), $f_2 : \mathbb{R} \times \mathbb{H} \rightarrow \mathbb{H}$, and $f_3 : \mathbb{R} \times \mathbb{H} \rightarrow \mathbb{L}_2^0$ are jointly continuous functions. Chapter 6 also presents some recent results on the existence of p -th mean almost periodic and S^p -almost periodic (mild) solutions to various nonautonomous differential equations using the well-known Schauder fixed point theorem. A few examples are also discussed.

Chapter 7 makes extensive use of abstract results of Chapter 6 to study the existence of square-mean almost periodic solutions to some (non)autonomous second-order stochastic differential equations.

Chapter 8 deals with discrete-time stochastic processes known as random sequences. There, we are particularly interested in the study of almost periodicity of those random sequences and their applications to stochastic difference equations including the so-called Beverton–Holt model.

Almost Periodic Stochastic Processes is aimed at expert readers, young researchers, beginning graduate and advanced undergraduate students, who are interested in the concept of almost periodicity and its applications to stochastic difference and differential equations. The basic background for the understanding of the material presented is timely provided throughout the text.

Last but certainly not least, we are grateful to our families for their continuous support, their encouragement, and especially for their putting up with us during all those long hours we spent away from them, while writing this book.

Paul H. Bezandry and Toka Diagana
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Bezandry, P.H.; Diagana, T.

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