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Fracture Behavior of Polymeric Foams Under Mixed-Mode Loading

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ABSTRACT

The present work deals with the crack growth behavior in polymeric foams under mixed-mode loading conditions. Polymeric foams are anisotropic materials and cracks generally propagate under mixed-mode conditions. Due to the anisotropy of the material crack kinking occurs even though the applied load is perpendicular to the crack plane. The strain energy density criterion is used for the determination of the critical load of crack initiation and crack growth path under mixed-mode loading. A stress analysis of the plate is performed by a commercial finite element computer program. Results are obtained for the fracture trajectories for various polymeric foams. The study takes place within the frame of linear elastic fracture mechanics of anisotropic media.

Introduction

Cellular materials have extensively been used in sandwich construction due to their excellent properties, such as high specific modulus and strength, low weight, good thermal insulation and low cost. The mechanical behavior of cellular materials has been studied in [1-4]. It was found that the compressive stress-strain behavior of PVC cellular foams consists of an initial relatively small and stiff elastic regime, an extended stress plateau regime and a final regime in which densification of the material takes place. In the stress plateau regime the cells of the foam collapse, while the average stress remains almost constant during the instability spread through the material. Axial compression produces little lateral spreading resulting to almost zero Poisson's ratio. When all of the cells collapse the material is densified and its stiffness increases. As a consequence of such behavior foams change their volume during plastic compression. This is contrary to dense solids which are incompressible during plastic deformation. On the contrary, the uniaxial stress-strain behavior in tension is nonlinear elastic without any identifiable yield region.
The objective of this work is to study the mixed-mode crack growth behavior in a cross-liked polymeric foam under the commercial name Divinycell H250 with a density of 250 Kg/m^3. The case of a plate with a crack perpendicular to the applied uniaxial stress is analyzed by finite elements. The results of stress analysis are coupled with the strain energy density theory to obtain crack growth trajectories for various values of the angle of orientation of the axes of anisotropy of the material with respect to the loading direction.

Mechanical Characterization of Foam Materials

The study will include many types of fully cross-linked PVC closed-cell foams under the commercial name Divinycell H80, H100, H160, H250 with densities of 80, 100, 160 and 250 kg/m^3, respectively, and balsa wood. Figure 1 shows the stress-strain curves of Divinycell H250 in tension and compression. Note that the uniaxial stress-strain behavior in tension is nonlinear elastic without any identifiable yield region. In uniaxial compression the material is nearly elastic-perfectly plastic in the initial stage of yielding. Mechanical properties of materials studied are shown in Table 1. All Divinycell H80, H100, H160 and H250 materials exhibit axisymmetric anisotropy with much higher stiffness and strength in the cell (3-direction) than the in-plane direction. The ratio of the stiffness in the cell (e-

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direction) to the in-plane direction is of the order of 1.5. The anisotropy of balsa wood is more pronounced with the above ratio equal to 42. All materials display different behavior in tension and compression with tensile strengths much higher than corresponding compressive strengths.

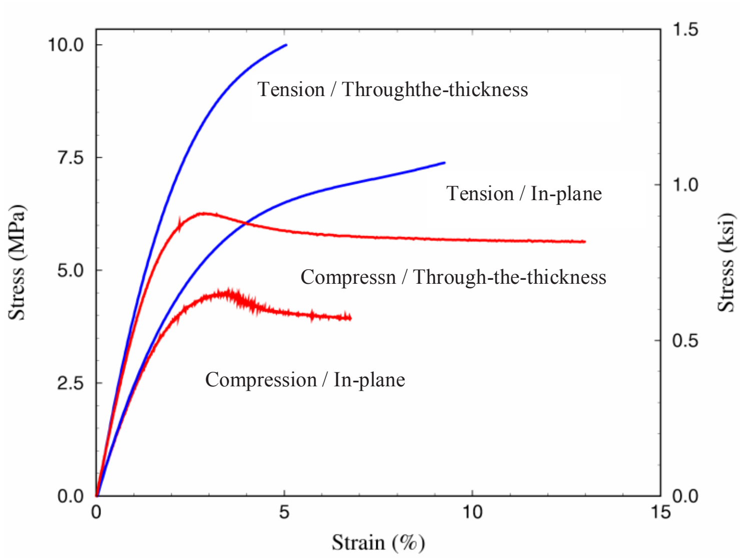

Fig. 1: Stress-strain curves of Divinycell H250 PVC foam, in tension and compression.

Strain Energy Density Criterion

The basic quantity in the strain energy density theory for crack problems is the strain energy density function *dW/dV*, which can be put in the form [5-7]:

$$\frac{dW}{dV} = \frac{S}{r} \tag{1}$$

where S is the strain energy density factor and r is the distance measured from the crack tip.

For plane elastic problems under conditions of plane stress the strain energy density function is given by

$$\frac{dW}{dV} = \frac{1}{2}\left(\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \tau_{xy}\gamma_{xy}\right) \tag{2}$$

where σ_x, σ_y, τ_{xy} are the stress and ε_x, ε_y, γ_{xy} are the strain components.

The strain energy density factor S is given by [5-7]:

$$S = A_{11}k_1^2 + 2A_{12}k_1k_2 + A_{22}k_2^2 \tag{3}$$

where

$$A_{11}=\frac{1}{4}\left[\alpha_{11}^{'}A^2+\alpha_{22}^{'}C^2+\alpha_{66}^{'}E^2+2\alpha_{12}^{'}AC+2\alpha_{16}^{'}AE+2\alpha_{26}^{'}CE\right] \tag{4a}$$

$$A_{12}=\frac{1}{4}\left[\alpha_{11}^{'}AB+\alpha_{22}^{'}CD+\alpha_{66}^{'}EF+\alpha_{12}^{'}(AD+BC)+\alpha_{16}^{'}(AE+BE)+2\alpha_{26}^{'}(CE+DE)\right] \tag{4b}$$

$$A_{22}=\frac{1}{4}\left[\alpha_{11}^{'}B^2+\alpha_{22}^{'}D^2+\alpha_{66}^{'}F^2+2\alpha_{12}^{'}BD+2\alpha_{16}^{'}BF+2\alpha_{26}^{'}DF\right] \tag{4c}$$

with

$$A=\mathrm{Re}\left[\frac{s_1 s_2}{s_1-s_2}\left(\frac{s_2}{z_2}-\frac{s_1}{z_1}\right)\right],\quad B=\mathrm{Re}\left[\frac{1}{s_1-s_2}\left(\frac{s_2^2}{z_2}-\frac{s_1^2}{z_1}\right)\right] \tag{5a}$$

$$C=\mathrm{Re}\left[\frac{1}{s_1-s_2}\left(\frac{s_1}{z_2}-\frac{s_2}{z_1}\right)\right],\quad D=\mathrm{Re}\left[\frac{1}{s_1-s_2}\left(\frac{1}{z_2}-\frac{1}{z_1}\right)\right] \tag{5b}$$

$$E=\mathrm{Re}\left[\frac{s_1 s_2}{s_1-s_2}\left(\frac{1}{z_1}-\frac{1}{z_2}\right)\right],\quad F=\mathrm{Re}\left[\frac{1}{s_1-s_2}\left(\frac{s_1}{z_2}-\frac{s_2}{z_2}\right)\right] \tag{5c}$$

and

$$k_1=\frac{K_1}{\sqrt{\pi}}\quad k_2=\frac{K_2}{\sqrt{\pi}}. \tag{6}$$

In the above equations α_{ij} are the compliance coefficients of the anisotropic material relating stress and strain, K_1 and K_2 are the stress intensity factors which dictate the stress field in the neighborhood of the crack tip, $z_1 = x_1 + iy_1$, $z_2 = x_1 - iy_1$ are complex numbers, and the other coefficients are related to the anisotropic behavior of the material [4-6].

Consider a plate with a through-the-thickness crack of length 2a that is subjected to a uniaxial stress σ perpendicular to the crack plane. The axıs x´ of orthotropy of the material makes an angle φ with the crack axis, x (Fig. 1). The compliance coefficients $\alpha_{ij}{}'$ referred to the system x'y' (Fig. 1) are related to the coefficients a_{ij} refereed to the system xy by the following equations [5-7]

$$\alpha_{11}^{'}=\alpha_{11}\cos^4\varphi+(2\alpha_{12}+\alpha_{66})\sin^2\varphi\cos^2\varphi+\alpha_{22}\sin^4\varphi+(\alpha_{16}\cos^2\varphi+\alpha_{26}\sin^2\varphi)\sin 2\varphi,$$
$$\alpha_{22}^{'}=\alpha_{11}\sin^4\varphi+(2\alpha_{12}+\alpha_{66})\sin^2\varphi\cos^2\varphi+\alpha_{22}\cos^4\varphi-(\alpha_{16}\cos^2\varphi+\alpha_{26}\cos^2\varphi)\sin 2\varphi,$$
$$\alpha_{12}^{'}=\alpha_{12}+(\alpha_{11}+\alpha_{22}-2\alpha_{12}-\alpha_{66})\sin^2\varphi\cos^2\varphi+\frac{1}{2}(\alpha_{26}-\alpha_{16})\sin 2\varphi\cos 2\varphi,$$
$$\alpha_{66}^{'}=\alpha_{66}+4(\alpha_{11}+\alpha_{22}-2\alpha_{12}-\alpha_{66})\sin^2\varphi\cos^2\varphi+2(\alpha_{26}-\alpha_{16})\sin 2\varphi\cos 2\varphi,$$

$$\alpha_{16}' = [\alpha_{22}\sin^2\varphi - \alpha_{11}\cos^2\varphi + \frac{1}{2}\left(2\alpha_{12} + \alpha_{66}\right)\cos 2\varphi]\sin 2\varphi + \alpha_{16}\cos^2\varphi\left(\cos^2\varphi - 3\sin^2\varphi\right)$$
$$+\alpha_{26}\sin^2\varphi\left(3\cos^2\varphi - \sin^2\varphi\right),$$
$$\alpha_{26}' = [\alpha_{22}\sin^2\varphi - \alpha_{11}\sin^2\varphi - \frac{1}{2}\left(2\alpha_{12} + \alpha_{66}\right)\cos 2\varphi]\sin 2\varphi + \alpha_{16}\sin^2\varphi\left(3\cos^2\varphi - \sin^2\varphi\right)$$
$$+\alpha_{26}\cos^2\varphi\left(\cos^2\varphi - 3\sin^2\varphi\right),$$

(7)

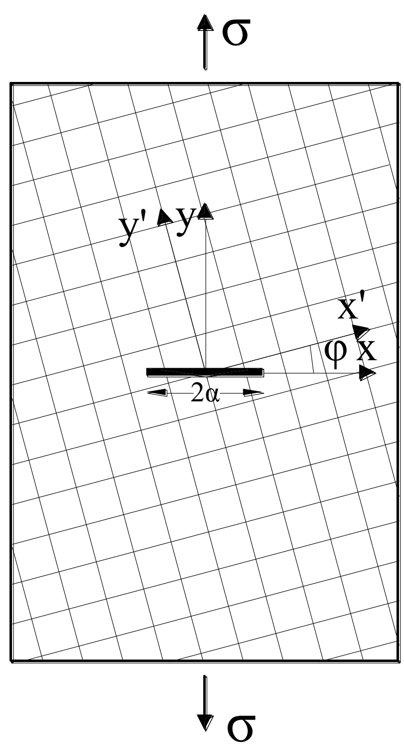

Fig.1 A cracked plate with a crack perpendicular to the applied load at an angle with the direction of the axis of material orthotropy of the material

According to the strain energy density theory unstable crack growth takes place in the radial direction along which S becomes minimum. This condition is mathematically put in the form:

$$\frac{\partial S}{\partial \theta} = 0, \quad \frac{\partial^2 S}{\partial \theta^2} > 0. \qquad (8)$$

This equation is used for the determination of the critical angle θ_c of initial crack growth.

Unstable crack growth occurs when $S_{min}(\theta_c)$ takes its critical value S_c which is an intrinsic material parameter, that is,

$$S_{min}\left(\theta_c\right) = S_c. \qquad (9)$$

Equations (8) and (9) will be used for the determination of the critical quantities at crack instability for the case of Fig. 1.

Results

Results were obtained for an orthogonal plate with a crack perpendicular to the applied uniaxial stress. The axis of material symmetry made an angle φ with respect to the crack axis (Fig. 1). The stress analysis of the plate was performed by the ABACUS computer program. Figs 2 and 3 present the finite element idealization of the specimen in the vicinity of the crack tip. Fig. 4 presents the contours of strain energy density function near the crack tip. Note that due to material orthotropy the contours are not symmetrical, but inclined with respect to the crack axis. Fig. 5 presents the variation of strain energy density function dW/dV along the circumference of a circle centered at the crack tip for $\varphi = 0$, 30^0 and 60^0. The values of θ at which dW/dV presents local minima are the critical values of the angle of initial crack growth. Fig. 6 presents the variation of θ_c versus the angle φ of the orientation of the axis of material orthotropic symmetry with respect to the crack axis. Note that the critical angle θ_c increases versus φ reaching a maximum value after which it decreases and becomes zero when the crack is along the axis of material symmetry.

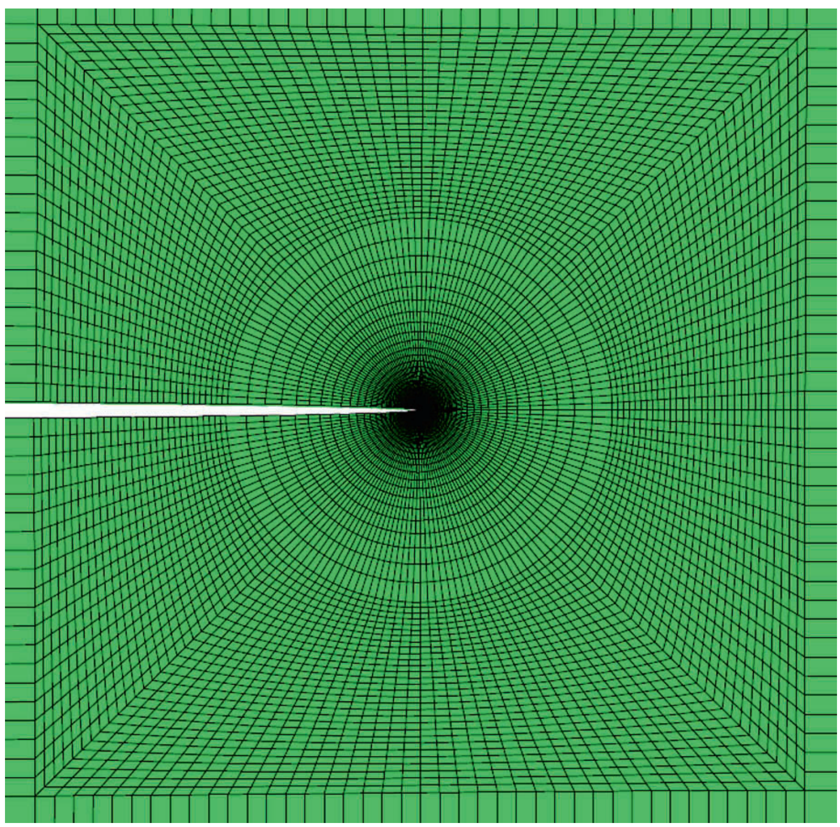

Fig. 2 Finite element mesh near the crack tip

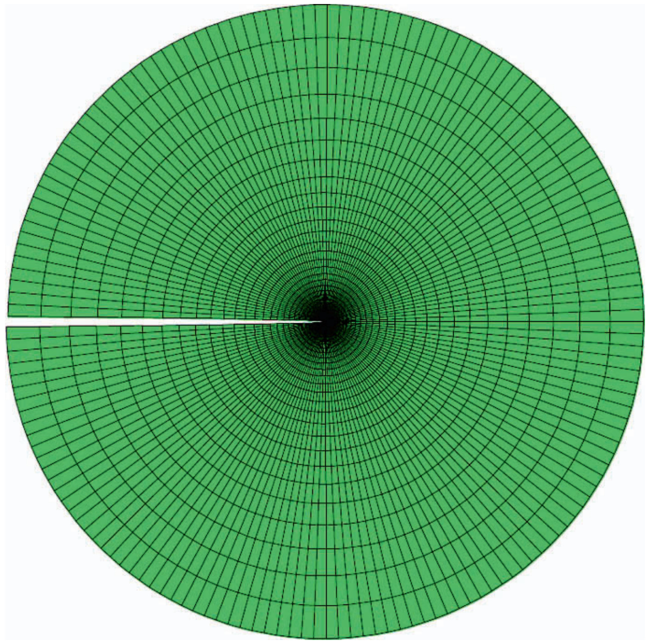

Fig. 3 Detailed mesh

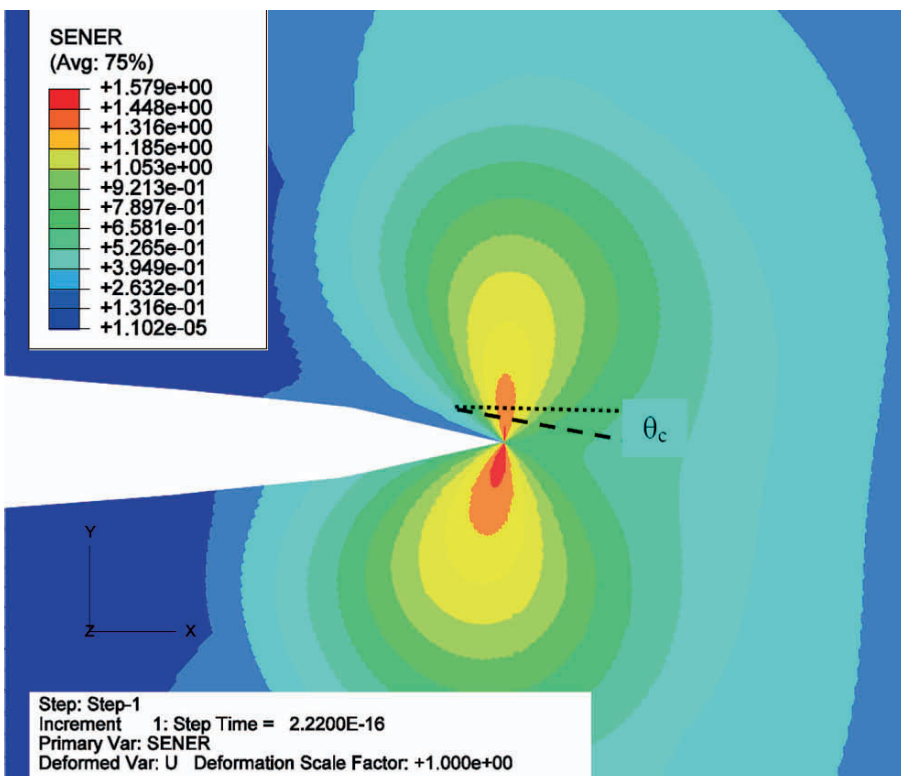

Fig. 4 Contours of strain energy density function near the crack tip for $\varphi = 45^0$

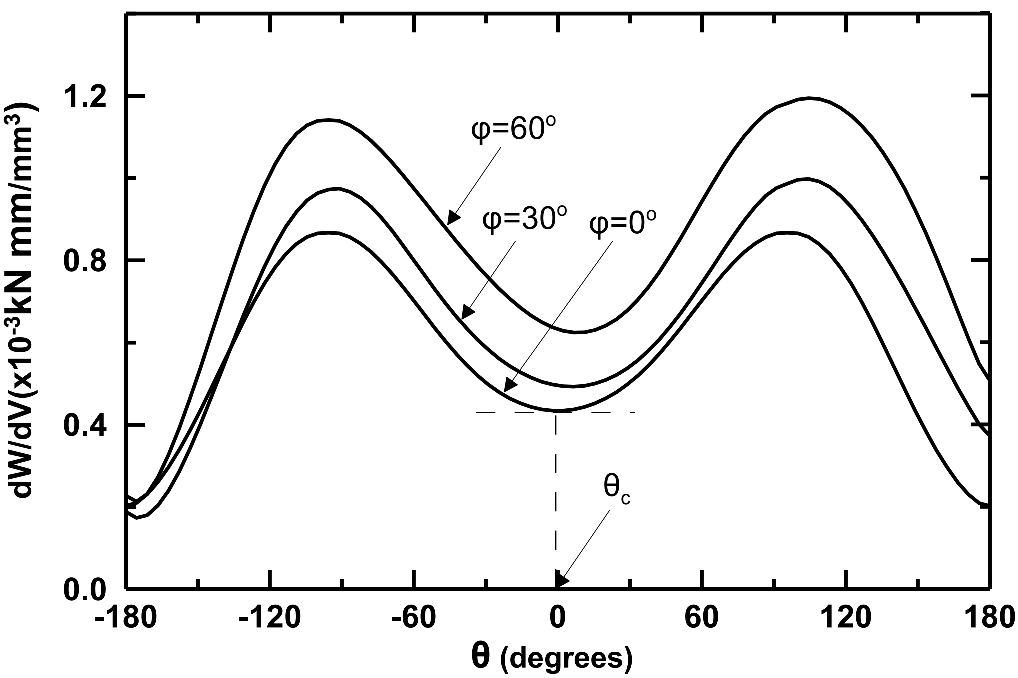

Fig. 5 Variation of strain energy density function dW/dV versus polar angle θ around the circumference of a circle surrounding the crack tip for $\varphi = 0$, 30^0 and 60^0. Crack grows in the direction of local minimum of strain energy density function

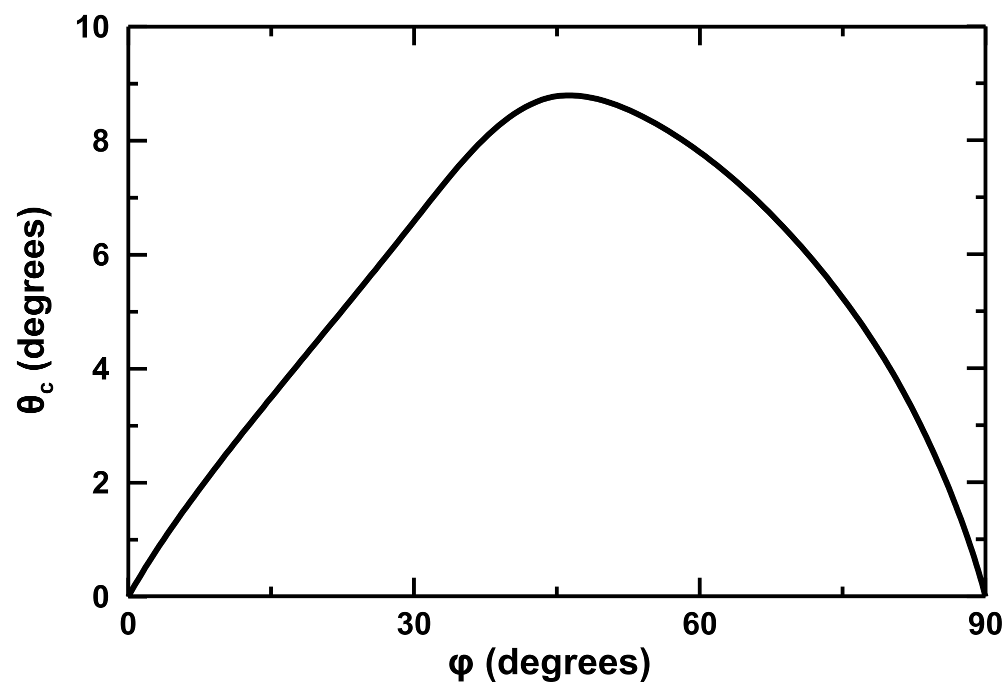

Fig. 6 Critical angle of crack growth θ_c versus angle φ of orientation of axes of material symmetry with respect to the crack axis

Conclusions

The crack growth in polymeric foams which present mechanical anisotropic behavior was studied. The case of a cracked plate subjected to a uniaxial stress perpendicular to the crack plane with the axes of material anisotropy at an angle with respect to the crack plane is analyzed. From the results of stress analysis in conjunction with the strain energy density theory the mixed-mode crack growth behavior of the plate was obtained. Results for the angle of initial crack growth for various orientations of the axes of anisotropy of the material with respect to the loading direction were reported.

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